

UNIT–II FOURIER SERIES

Harmonic Analysis

The process of finding the Fourier series for a function given by numerical values is known as harmonic analysis.

$$f(x) = \frac{a_0}{2} + \sum_{n=1}^{\infty} (a_n \cos nx + b_n \sin nx), \text{ where}$$

$$\text{ie, } f(x) = (a_0/2) + (a_1 \cos x + b_1 \sin x) + (a_2 \cos 2x + b_2 \sin 2x) + (a_3 \cos 3x + b_3 \sin 3x) + \dots \text{---(1)}$$

$$\text{Here } a_0 = 2 \left[\text{mean values of } f(x) \right] = \frac{2 \sum f(x)}{n}$$

$$= 2 \left[\text{mean values of } f(x) \cos nx \right] = \frac{2 \sum f(x) \cos nx}{n} \quad a_n$$

$$b_n = 2 \left[\text{mean values of } f(x) \sin nx \right] = \frac{2 \sum f(x) \sin nx}{n}$$

In (1), the term $(a_1 \cos x + b_1 \sin x)$ is called the **fundamental or first harmonic**, the term $(a_2 \cos 2x + b_2 \sin 2x)$ is called the **second harmonic** and so on.

Example 16

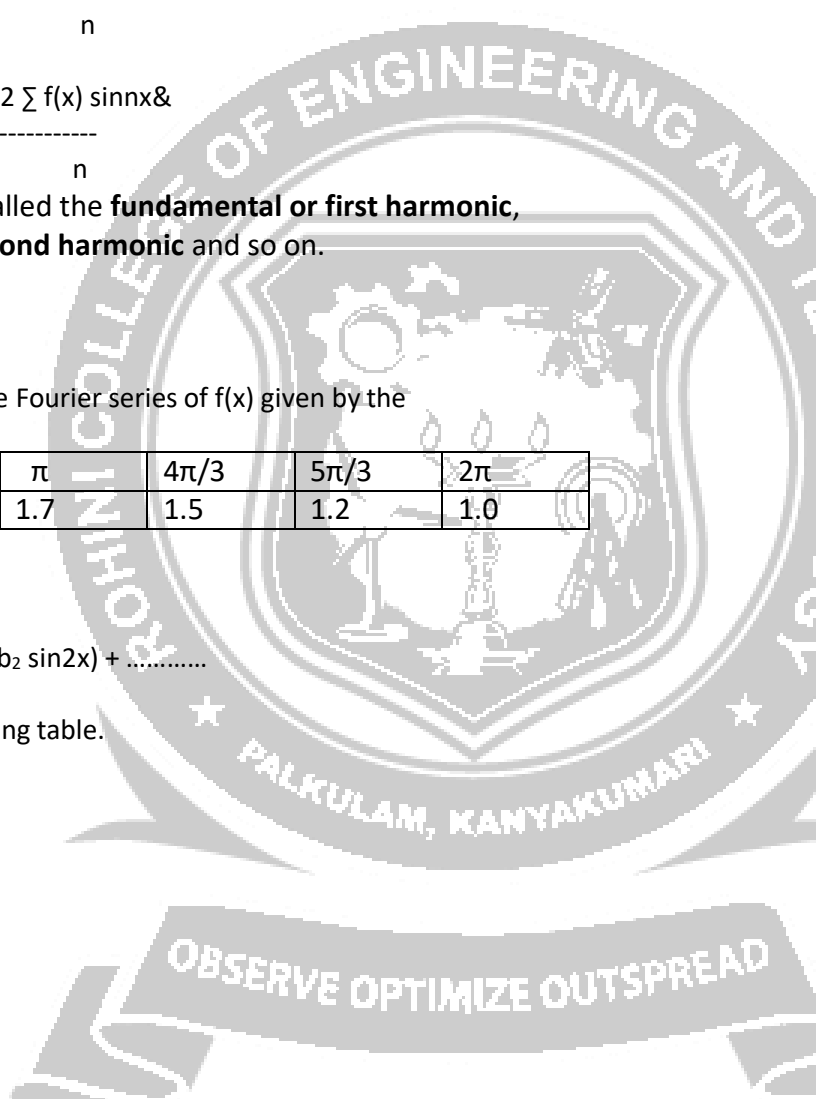
Compute the first three harmonics of the Fourier series of $f(x)$ given by the following table.

x:	0	$\pi/3$	$2\pi/3$	π	$4\pi/3$	$5\pi/3$	2π
f(x):	1.0	1.4	1.9	1.7	1.5	1.2	1.0

We exclude the last point $x = 2\pi$.

$$\text{Let } f(x) = (a_0/2) + (a_1 \cos x + b_1 \sin x) + (a_2 \cos 2x + b_2 \sin 2x) + \dots$$

To evaluate the coefficients, we form the following table.



x	f(x)	cosx	sinx	cos2x	sin2x	cos3x	sin3x
0	1.0	1	0	1	0	1	0
$\pi/3$	1.4	0.5	0.866	-0.5	0.866	-1	0
$2\pi/3$	1.9	-0.5	0.866	-0.5	-0.866	1	0
π	1.7	-1	0	1	0	-1	0
$4\pi/3$	1.5	-0.5	-0.866	-0.5	0.866	1	0
$5\pi/3$	1.2	0.5	-0.866	-0.5	-0.866	-1	0

$$\text{Now, } a_0 = \frac{2 \sum f(x)}{6} = \frac{2 (1.0 + 1.4 + 1.9 + 1.7 + 1.5 + 1.2)}{6} = 2.9$$

$$a_1 = \frac{2 \sum f(x) \cos x}{6} = -0.376$$

$$a_2 = \frac{2 \sum f(x) \cos 2x}{6} = -0.1$$

$$a_3 = \frac{2 \sum f(x) \cos 3x}{6} = 0.033$$

$$b_1 = \frac{2 \sum f(x) \sin x}{6} = 0.17$$

$$b_2 = \frac{2 \sum f(x) \sin 2x}{6} = -0.06$$

$$b_3 = \frac{2 \sum f(x) \sin 3x}{6} = 0$$

$$\therefore f(x) = 1.45 - 0.37 \cos x + 0.17 \sin x - 0.1 \cos 2x - 0.06 \sin 2x + 0.033 \cos 3x + \dots$$

Example 17

Obtain the first three coefficients in the Fourier cosine series for y, where y is given in the following table:

x:	0	1	2	3	4	5
y:	4	8	15	7	6	2

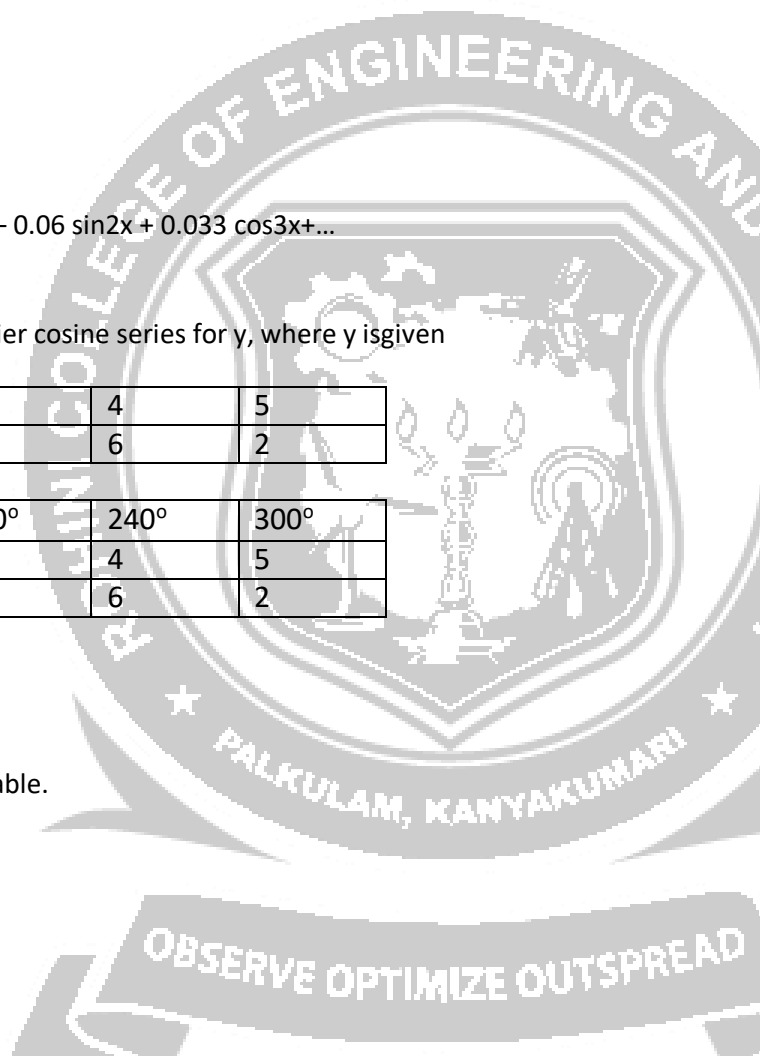
Taking the interval as 60° , we have

θ :	0°	60°	120°	180°	240°	300°
x:	0	1	2	3	4	5
y:	4	8	15	7	6	2

$\therefore$ Fourier cosine series in the interval $(0, 2\pi)$ is

$$y = (a_0/2) + a_1 \cos \theta + a_2 \cos 2\theta + a_3 \cos 3\theta + \dots$$

To evaluate the coefficients, we form the following table.



θ°	$\cos\theta$	$\cos 2\theta$	$\cos 3\theta$	y	$y \cos\theta$	$y \cos 2\theta$	$y \cos 3\theta$
0°	1	1	1	4	4	4	4
60°	0.5	-0.5	-1	8	4	-4	-8
120°	-0.5	-0.5	1	15	-7.5	-7.5	15
180°	-1	1	-1	7	-7	7	-7
240°	-0.5	-0.5	1	6	-3	-3	6
300°	0.5	-0.5	-1	2	1	-1	-2
Total				42	-8.5	-4.5	8

$$\text{Now, } a_0 = 2 (42/6) = 14a_1$$

$$= 2 (-8.5/6) = -2.8$$

$$a_2 = 2 (-4.5/6) = -1.5a_3$$

$$= 2 (8/6) = 2.7$$

$$\therefore y = 7 - 2.8 \cos\theta - 1.5 \cos 2\theta + 2.7 \cos 3\theta + \dots$$

Example 18

The values of x and the corresponding values of $f(x)$ over a period T are given below. Show that $f(x) = 0.75 + 0.37 \cos\theta + 1.004 \sin\theta$, where $\theta = (2\pi x)/T$

$x:$	0	$T/6$	$T/3$	$T/2$	$2T/3$	$5T/6$	T
$y:$	1.98	1.30	1.05	1.30	-0.88	-0.25	1.98

We omit the last value since $f(x)$ at $x = 0$ is known. Here

$$\theta = \frac{2\pi x}{T}$$

T

When x varies from 0 to T , θ varies from 0 to 2π with an increase of $2\pi/6$.

$$\text{Let } f(x) = F(\theta) = (a_0/2) + a_1 \cos\theta + b_1 \sin\theta.$$

To evaluate the coefficients, we form the following table.

θ	y	$\cos\theta$	$\sin\theta$	$y \cos\theta$	$y \sin\theta$
0	1.98	1.0	0	1.98	0
$\pi/3$	1.30	0.5	0.866	0.65	1.1258
$2\pi/3$	1.05	-0.5	0.866	-0.525	0.9093
π	1.30	-1	0	-1.3	0
$4\pi/3$	-0.88	-0.5	-0.866	0.44	0.762
$5\pi/3$	-0.25	0.5	-0.866	-0.125	0.2165
	4.6			1.12	3.013

$$\text{Now, } a_0 = 2 (\sum f(x) / 6) = 1.5$$

$$a_1 = 2 (1.12 / 6) = 0.37$$

$$a_2 = 2 (3.013 / 6) = 1.004$$

$$\text{Therefore, } f(x) = 0.75 + 0.37 \cos\theta + 1.004 \sin\theta$$

