Feedback path and optional divider:

Most PLLs also include a divider between the oscillator and the feedback input to the phase detector to produce a frequency synthesizer. A programmable divider is particularly useful in radio transmitter applications, since a large number of transmit frequencies can be produced from a single stable, accurate, but expensive, quartz crystal–controlled reference oscillator.

Some PLLs also include a divider between the reference clock and the reference input to the phase detector. If this divider divides by M, it allows the VCO to multiply the reference frequency by N / M. It might seem simpler to just feed the PLL a lower frequency, but in some cases the reference frequency may be constrained by other issues, and then the reference divider is useful. Frequency multiplication in a sense can also be attained by locking the PLL to the 'N'th harmonic of the signal.

Equations:

The equations governing a phase-locked loop with an analog multiplier as the phase detector may be derived as follows. Let the input to the phase detector be xc(t) and the output of the voltage-controlled oscillator (VCO) is xr(t) with frequency ωr(t), then the output of the phase detector xm(t) is given by

$$x_m(t) = x_c(t) \cdot x_r(t)$$

the VCO frequency may be written as a function of the VCO input y(t) as

$$\omega_r(t) = \omega_f + g_v y(t)$$

where gv is the sensitivity of the VCO and is expressed in Hz / V.

Hence the VCO output takes the form

$$x_r(t) = A_r \cos\left(\int_0^t \omega_r(\tau)\, d\tau\right) = A_r \cos(\omega_f t + \varphi(t))$$

$$\varphi(t) = \int_0^t g_v y(\tau)\, d\tau$$

where

The loop filter receives this signal as input and produces an output

xf(t) = Ffilter(xm(t))

where FFilter is the operator representing the loop filter transformation.

When the loop is closed, the output from the loop filter becomes the input to the VCO thus

y(t) = xf(t) = Ffilter(xm(t))

We can deduce how the PLL reacts to a sinusoidal input signal:

xc(t) = Acsin(ωct).

The output of the phase detector then is:

$$x_m(t) = A_c \sin(\omega_c t) A_r \cos(\omega_f t + \varphi(t)).$$

This can be rewritten into sum and difference components using trigonometric identities:

$$x_m(t) = \frac{A_c A_f}{2} \sin(\omega_c t - \omega_f t - \varphi(t)) + \frac{A_c A_f}{2} \sin(\omega_c t + \omega_f t + \varphi(t))$$

As an approximation to the behaviour of the loop filter we may consider only the difference frequency being passed with no phase change, which enables us to derive a small-signal model of the phase- locked loop. If we can make $\omega_f \approx \omega_c$, then the $\sin(\cdot)$ can be approximated by its argument resulting in: $y(t) = x_f(t) \simeq -A_c A_f \varphi(t)/2$. The phase- locked loop is said to be locked if this is the case.