4.1. LINEAR WAVESHAPING:

- A linear network is a network made up of linear elements only. A linear network can be described by linear differential equations. The principle of superposition and the principle ofhomogeneity hold good for linear networks. In pulse circuitry, there are a number of waveforms, which appear very frequently. The most important of these are sinusoidal, step, pulse, square wave, ramp, and exponential waveforms. The response of RC, RL, and RLC circuits to these signals is described in this chapter. Out of these signals, the sinusoidal signal has a unique characteristic that it preserves its shape when it is transmitted through a linear network, i.e. under steady state, the output will be a precise reproduction of the input sinusoidal signal. There will only be a change in the amplitude of the signal and there may be a phase shift between the inputand the output waveforms.
- The influence of the circuit on the signal may then be completely specified by the ratio of the output to the input amplitude and by the phase angle between the output and the input. No other periodic waveform preserves its shape precisely when transmitted through a linear network, and in many cases the output signal may bear very little resemblance to the input signal.
- **The process whereby the form of a non-sinusoidal signal is altered by transmission through a linear network is called linear wave shaping.**

4.1.1THE LOW-PASS RC CIRCUIT

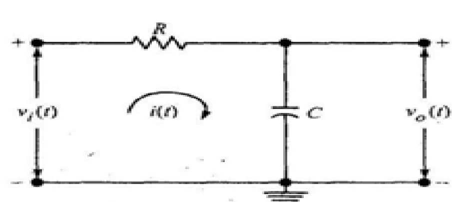
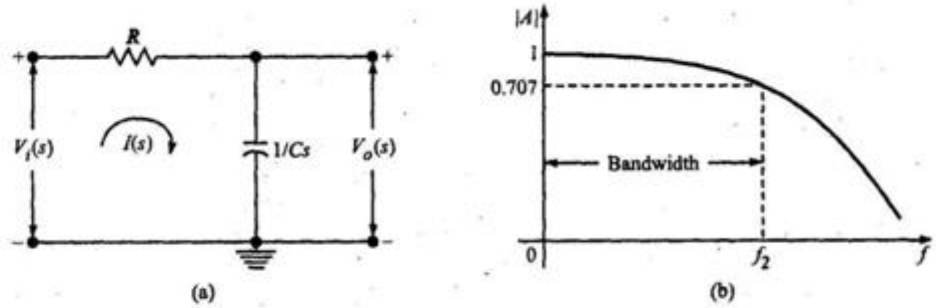


Fig 4.1.1. low-pass RC circuit.

A low-pass circuit is a circuit, which transmits only low-frequency signals and attenuates or stops high-frequency signals.At zero frequency, the reactance of the capacitor is infinity (i.e. the capacitor acts as an open circuit) so the entire input appears at the output, i.e. the input is transmitted to the output with zero attenuation. So the output is the same as the input, i.e. the gain is unity. As the frequency increases the capacitive reactance decreases and so the output decreases. At very high frequencies the capacitor virtually acts as a short-circuit and the output falls to zero.

Sinusoidal Input

- The Laplace transformed low-pass RC circuit is shown in Figure 1.2(a). The gain versus frequency curve of a low-pass circuit excited by a sinusoidal input is shown in Figure 1.2(b). This curve is obtained by keeping the amplitude of the input sinusoidal signal constant and varying its frequency and noting the output at each frequency.
- At low frequencies the output is equal to the input and hence the gain is unity.
- As the frequency increases, the output decreases and hence the gain decreases. The frequency at which the gain is $1/\sqrt{2}$ (= 0.707) of its maximum value is called the cut-off frequency.
- For a low-pass circuit, there is no lower cut-off frequency. It is zero itself. The upper cut-off frequency is the frequency (in the high-frequency range) at which the gain is $1/\sqrt{2}$. i-e- 70.7%, of its maximum value. The bandwidth of the low-pass circuit is equal to the upper cut-off frequency f_2 itself.



(a) Laplace transformed low-pass *RC* circuit and (b) its frequency response.

Fig 4.1.2.Sinusoidal Input

- For the network shown in Figure 1.2(a), the magnitude of the steady-state gain A is given by

$$A = \frac{V_o(s)}{V_i(s)} = \frac{\frac{1}{Cs}}{R + \frac{1}{Cs}} = \frac{1}{1+RCs} = \frac{1}{1+j\omega RC} = \frac{1}{1+j2\pi fRC}$$

$$\therefore \quad |A| = \frac{1}{\sqrt{1+(2\pi fRC)^2}}$$

At the upper cut-off frequency f_2, $|A| = \frac{1}{\sqrt{2}}$

$$\therefore \quad \frac{1}{\sqrt{2}} = \frac{1}{\sqrt{1+(2\pi f_2 RC)^2}}$$

Squaring both sides and equating the denominators,

$$2 = 1 + (2\pi f_2 RC)^2$$

$\therefore$ The upper cut-off frequency, $f_2 = \frac{1}{2\pi RC}$.

So $$A = \frac{1}{1+j\frac{f}{f_2}} \quad \text{and} \quad |A| = \frac{1}{\sqrt{1+\left(\frac{f}{f_2}\right)^2}}$$

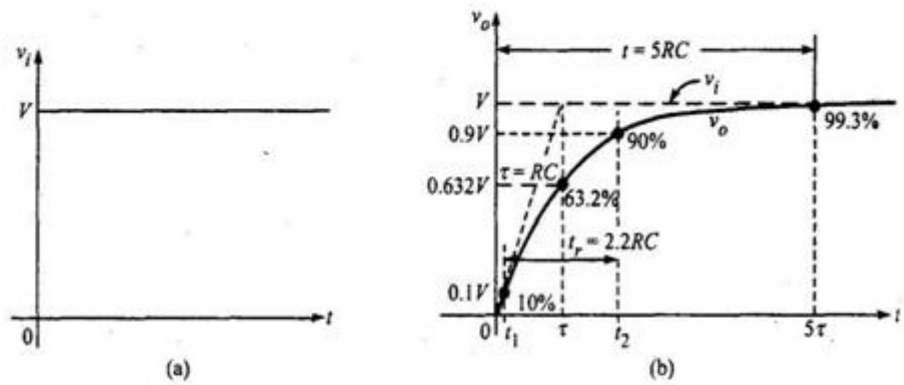
The angle θ by which the output leads the input is given by

$$\theta = \tan^{-1}\frac{f}{f_2}$$

Step-Voltage Input

- A step signal is one which maintains the value zero for all times $t < 0$, and maintains the value V for all times $t > 0$. The transition between the two voltage levels takes place at $t = 0$ and is accomplished in an arbitrarily small time interval.

- Thus, in Figure (a), $v_i = 0$ immediately before $t = 0$ (to be referred to as time $t = 0^-$) and $v_i = V$, immediately after $t = 0$ (to be referred to as time $t = 0^+$). In the low-pass RC circuit shown in Figure 1.1, if the capacitor is initially uncharged, when a step input is applied, since the voltage across the capacitor cannot change instantaneously, the output will be zero at $t = 0$, and then, as the capacitor charges, the output voltage rises exponentially towards the steady-state value V with a time constant RC as shown in Figure 4.1.3(b).



(a) Step input and (b) step response of the low-pass *RC* circuit.

Fig.4.1.3 Step Voltage Input

- Let V' be the initial voltage across the capacitor. Writing KVL around the IOOD in Figure

$$v_i(t) = Ri(t) + \frac{1}{C}\int i(t)\,dt$$

Differentiating this equation,

$$\frac{dv_i(t)}{dt} = R\,\frac{di(t)}{dt} + \frac{1}{C}i(t)$$

Since $$v_i(t) = V, \quad \frac{dv_i(t)}{dt} = 0$$

$$\therefore \quad 0 = R\frac{di(t)}{dt} + \frac{1}{C}i(t)$$

Taking the Laplace transform on both sides,

$$0 = R[sI(s) - I(0^+)] + \frac{1}{C}I(s)$$

$$\therefore \quad I(0^+) = I(s)\left(s + \frac{1}{RC}\right)$$

The initial current $I(0^+)$ is given by

$$I(0^+) = \frac{V - V'}{R}$$

$$\therefore \quad I(s) = \frac{I(0^+)}{s + \frac{1}{RC}} = \frac{V - V'}{R\left(s + \frac{1}{RC}\right)}$$

and $$V_o(s) = V_i(s) - I(s)R = \frac{V}{s} - \frac{(V - V')R}{R\left(s + \frac{1}{RC}\right)} = \frac{V}{s} - \frac{V - V'}{s + \frac{1}{RC}}$$

Taking the inverse Laplace transform on both sides,

$$v_o(t) = V - (V - V')e^{-t/RC}$$

where V' is the initial voltage across the capacitor ($V_{initial}$) and V is the final voltage (V_{final}) to which the capacitor can charge.

So, the expression for the voltage across the capacitor of an RC circuit excited by a step input is given by

$$v_o(t) = V_{final} - (V_{final} - V_{initial})e^{-t/RC}$$

If the capacitor is initially uncharged, then $v_o(t) = V(1 - e^{-t/RC})$

Expression for rise time

- When a step signal is applied, the rise time t_r is defined as the time taken by the output voltage waveform to rise from 10% to 90% of its final value: It gives an indication of how fast the circuit can respond to a discontinuity in voltage. Assuming that the capacitor in Figure 1.1 is initially uncharged, the output voltage shown in Figure 1.3(b) at any instant of time is given by

$$v_o(t) = V(1 - e^{-t/RC})$$

At $t = t_1$, $v_o(t) \neq 10\%$ of $V = 0.1\,\text{V}$

$$\therefore \quad 0.1V = V(1 - e^{-t_1/RC})$$

$$\therefore \quad e^{-t_1/RC} = 0.9 \quad \text{or} \quad e^{t_1/RC} = \frac{1}{0.9} = 1.11$$

$$\therefore \quad t_1 = RC \ln (1.11) = 0.1RC$$

At $t = t_2$, $v_o(t) = 90\%$ of $V = 0.9\,\text{V}$

$$\therefore \quad 0.9V = V(1 - e^{-t_2/RC})$$

$$\therefore \quad e^{-t_2/RC} = 0.1 \quad \text{or} \quad e^{t_2/RC} = \frac{1}{0.1} = 10$$

$$\therefore \quad t_2 = RC \ln 10 = 2.3RC$$

$$\therefore \quad \text{Rise time, } t_r = t_2 - t_1 = 2.2RC$$

- This indicates that the rise time t_r is proportional to the time constant RC of the circuit. The larger the time constant, the slower the capacitor charges, and the smaller the time constant, the faster the capacitor charges.

Relation between rise time and upper 3-dB frequency

- We know that the upper 3-dB frequency (same as bandwidth) of a low-pass circuit is

$$f_2 = \frac{1}{2\pi RC} \quad \text{or} \quad RC = \frac{1}{2\pi f_2}$$

$$\therefore \quad \text{Rise time, } t_r = 2.2RC = \frac{2.2}{2\pi f_2} = \frac{0.35}{f_2} = \frac{0.35}{\text{BW}}$$

- Thus, the rise time is inversely proportional to the upper 3-dB frequency.
- The time constant (T= RC) of a circuit is defined as the time taken by the output to rise to 63.2% of the amplitude of the input step. It is same as the time taken by the output to rise to 100% of the amplitude of the input step, if the initial slope of rise is maintained. See Figure 1.3(b).
- The Greek letter T is also employed as the symbol for the time constant.

Pulse Input

- The pulse shown in Figure 1.4(a) is equivalent to a positive step followed by a delayed negative step as shown in Figure 1 .4(b). So, the response of the low-pass RC circuit to a pulse for times less than the pulse width t_p is the same as that for a step input and is given by $v_0(t) = V(1 - e^{-t/RC})$. The responses of the low-pass RC circuit for time constant RC » t_p, RC smaller than t_p and RC very small compared to t_p are shown in Figures 1.5(a), 1.5(b), and1.5(c) respectively.
- If the time constant RC of the circuit is very large, at the end of the pulse, the output voltage will be

$$V_p(t) = V(1 - e^{-t_p/RC}),$$

and the output will decrease to zero from this value with a time constant RC as shown in Figure 1.5(a).

- Observe that the pulse waveform is distorted when it is passed through a linear network. The output will always extend beyond the pulse width t_p, because whatever charge has accumulated across the capacitor C during the pulse cannot leak off instantaneously.

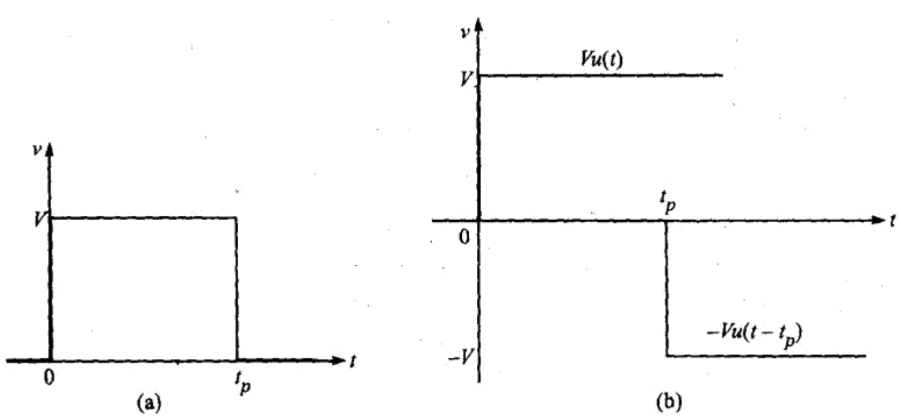


Figure 1.4 (a) A pulse and (b) a pulse in terms of steps.

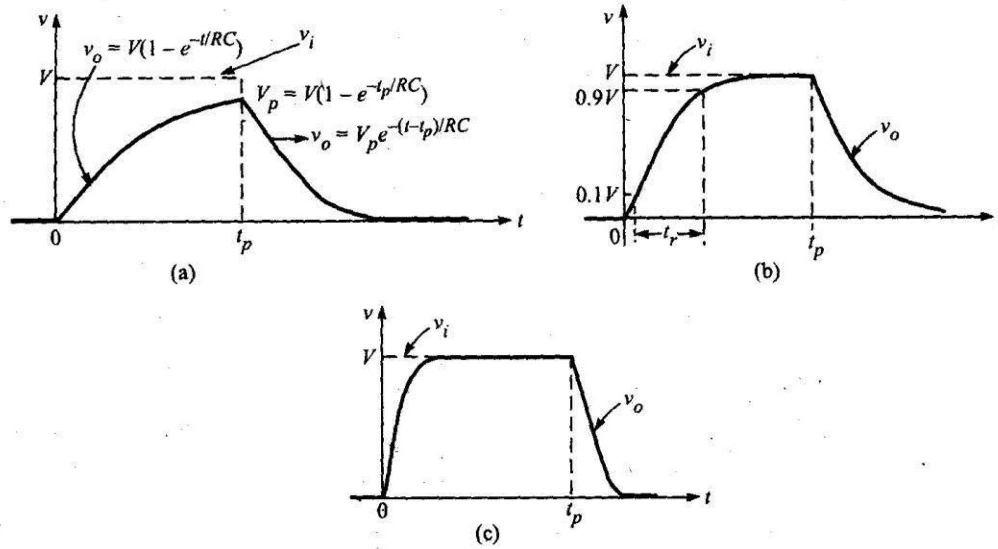


Figure 1.5 Pulse response for (a) $RC >> t_p$, (b) $RC < t_p$, and (c) $RC << t_p$.

- If the time constant RC of the circuit is very small, the capacitor charges and discharges very quickly and the rise time t_r will be small and so the distortion in the wave shape is small.
- For minimum distortion (i.e. for preservation of wave shape), the rise time must be small compared to the pulse width t_p. If the upper 3-dB frequency /2 is chosen equal to the reciprocal of the pulse width t_p, i.e. if $f_2 = 1/t_p$ then $t_r = 0.35t_p$ and the output is as shown in Figure 1.5(b), which for many applications is a reasonable reproduction of the input. As a rule of thumb, we can say:
- **A pulse shape will be preserved if the 3-dB frequency is approximately equal to the reciprocal of the pulse width.**
- Thus to pass a 0.25 μ.s pulse reasonably well requires a circuit with an upper cut-off frequency of the order of 4 MHz.

Square-Wave Input

- A square wave is a periodic waveform which maintains itself at one constant level V' with respect to ground for a time T_1 and then changes abruptly to another level V'', and remains constant at that level for a time T_2, and repeats itself at regular intervals of $T = T_1 + T_2$.

- A square wave may be treated as a series of positive and negative steps.
- The shape of the output waveform for a square wave input depends on the time constant of the circuit. If the time constant is very small, the rise time will also be small and a reasonable reproduction of the input may be obtained.
- For the square wave shown in Figure 1.6(a), the output waveform will be as shown in Figure 1.6(b) if the time constant RC of the circuit is small compared to the period of the input waveform. In this case, the wave shape is preserved. If the time constant is comparable with the period of the input square wave, the output will be as shown id Figure 1.6(c). The output rises and falls exponentially.
- If the time constant is very large compared to the period of the input waveform, the output consists of exponential sections, which are essentially linear as indicated inFigure 1.6(d). Since the average voltage across R is zero, the dc voltage at the output is the same as that of the input. This average value is indicated as V&. in all the waveforms

Taking the inverse Laplace transform on both sides,

$$v_o(t) = -\alpha RC + \alpha t + \alpha RCe^{-t/RC}$$
$$= \alpha(t - RC) + \alpha RCe^{-t/RC}$$

If the time constant RC is very small, $e^{-t/RC} \approx 0$

$$\therefore \quad v_o(t) = \alpha(t - RC)$$

$$= \frac{V}{2}\, \frac{1 - e^{-T/2RC} - e^{-T/2RC} + e^{-T/RC}}{1 - e^{-T/RC}}$$

$$= \frac{V}{2}\, \frac{(1 - e^{-T/2RC})^2}{(1 + e^{-T/2RC})(1 - e^{-T/2RC})}$$

$$= \frac{V}{2} \left(\frac{1 - e^{-T/2RC}}{1 + e^{-T/2RC}} \right)$$

$$= \frac{V}{2} \left(\frac{e^{T/2RC} - 1}{e^{T/2RC} + 1} \right)$$

$$= \frac{V}{2} \left(\frac{e^{2x} - 1}{e^{2x} + 1} \right) = \frac{V}{2} \tanh x$$

where $x = \frac{T}{4RC}$ and T is the period of the square wave.

Now, $$V_2 = -V_1 = -\frac{V}{2}\left(\frac{1 - e^{-T/2RC}}{1 + e^{-T/2RC}} \right) = \frac{V}{2} \left(\frac{1 - e^{T/2RC}}{1 + e^{T/2RC}} \right)$$

➢

1.1.5 Ramp Input

When a low-pass RC circuit shown in Figure 1.1 is excited by a ramp input, i.e.

$v_i(t) = \alpha t$, where α is the slope of the ramp

we have, $$V_i(s) = \frac{\alpha}{s^2}$$

From the frequency domain circuit of Figure 1.2(a), the output is given by

$$V_o(s) = V_i(s)\, \frac{\frac{1}{Cs}}{R + \frac{1}{Cs}} = \frac{\alpha}{s^2} \cdot \frac{1}{1 + RCs} = \frac{\alpha}{RC}\, \frac{1}{s^2\left(s + \frac{1}{RC}\right)}$$

$$= \frac{\alpha}{RC} \left[\frac{-(RC)^2}{s} + \frac{RC}{s^2} + \frac{(RC)^2}{s + \frac{1}{RC}} \right]$$

i.e. $$V_o(s) = \frac{-\alpha RC}{s} + \frac{\alpha}{s^2} + \frac{\alpha RC}{s + \frac{1}{RC}}$$

When the tme constant is very small relative to the total ramp time T, the ramp will be transmitted with minimum distortion. The output follows the input but is delayed by one time constant RC from the input (except near the origin where there is distortion) as shown in Figure 1.7(a). If the time constant is large compared with the sweep duration, i.e. if RCIT » 1, the output will be highly distorted as shown in Figure 1.7(b).

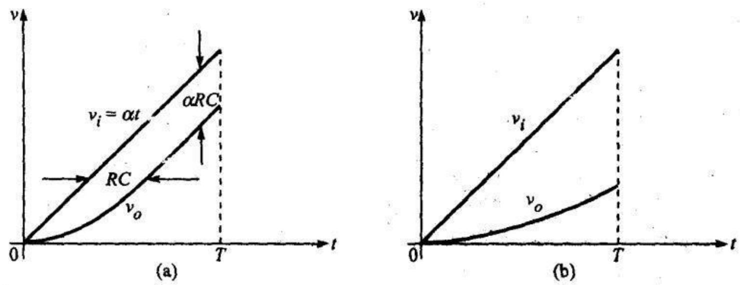


Figure 1.7 Response of a low-pass *RC* circuit for a ramp input for (a) *RC/T* << 1 and (b) *RC/T* >> 1.

Expanding $e^{-t/RC}$ in to an infinite series in t/RC in the above equation for $v_o(t)$,

$$v_o(t) = \alpha(t - RC) + \alpha RC\left(1 - \frac{t}{RC} + \left(\frac{t}{RC}\right)^2 \frac{1}{2!} - \left(\frac{t}{RC}\right)^3 \frac{1}{3!} + \dots\right)$$

$$= \alpha t - \alpha RC + \alpha RC - \alpha t + \frac{\alpha t^2}{2RC} - \dots$$

$$\approx \frac{\alpha t^2}{2RC} \approx \frac{\alpha}{RC}\left(\frac{t^2}{2}\right)$$

- This shows that a quadratic response is obtained for a linear input and hence the circuit acts as anintegrator for RC/T » 1.
- The transmission error e_t for a ramp input is defined as the difference between theinput and the output divided by the input at the end of the ramp, i.e. at t = T.

For RC/T « 1,

$$e_t = \left.\frac{\alpha t - (\alpha t - \alpha RC)}{\alpha t}\right|_{t=T}$$

$$= \frac{\alpha RC}{\alpha T} = \frac{RC}{T} = \frac{1}{2\pi f_2 T}$$

- where f_2 is the upper 3-dB frequency. For example, if we desire to pass a 2 ms pulse with less than 0.1% error, the above equation yields f_2 > 80 kHz and RC < 2 μ.s.

4.1.2.THE LOW-PASS RC CIRCUIT AS AN INTEGRATOR

- If the time constant of an RC low-pass circuit is very large, the capacitor charges very slowly and so almost all the input voltage appears across the resistor for small values of time.
- Then, the current in the circuit is vffylR and the output signal across C is

$$v_o(t) = \frac{1}{C}\int i(t)\,dt = \frac{1}{C}\int \frac{v_i(t)}{R}\,dt = \frac{1}{RC}\int v_i(t)\,dt$$

Hence the output is the integral of the input, i.e. if $v_i(t) = \alpha t$, then

$$v_o(t) = \frac{\alpha t^2}{2RC}$$

- As time increases, the voltage drop across C does not remain negligible compared with that across R and the output will not remain the integral of the input. The output will change from a quadratic to a linear function of time.
- **If the time constant of an RC low-pass circuit is very large in comparison with the. time required for the input signal to make an appreciable change, the circuit acts as an integrator.**
- A criterion for good integration in terms of steady-state analysis is as follows: The low-pass circuit acts as an integrator provided the time constant of the circuit RC > 15T, where T is the period of the input sine wave.
- When RC > 15T, the input sinusoid will be shifted at least by 89.4° (instead of the ideal 90° shift required for integration) when it is transmitted through the network.
- An RC integrator converts a square wave into a triangular wave. Integrators are almost invariably preferred over differentiators in analog computer applications for the following reasons:

I. It is easier to stabilize an integrator than a differentiator because the gain of an integrator decreases with frequency whereas the gain of a differentiator increases with frequency.

II. An integrator is less sensitive to noise voltages than a differentiator because of its limited bandwidth.

III. The amplifier of a differentiator may overload if the input waveform changes very rapidly.

IV. It is more convenient to introduce initial conditions in an integrator.

4.1.3.THE HIGH-PASS RC CIRCUIT

- Figure shows a high-pass RC circuit. At zero frequency the reactance of the capacitor is infinity and so it blocks the input and hence the output is zero. Hence, this capacitor is called the blocking capacitor and this circuit, also called the capacitive coupling circuit, is used to provide dc isolation between the input and the output.
- As the frequency increases, the reactance of the capacitor decreases and hence the output and gain increase.
- At very high frequencies, the capacitive reactance is very small so a very small voltage appears, across C and, so the output is almost equal to the input and the gain is equal to 1.
- Since this circuit attenuates low-frequency signals and allows transmission of high-frequency signals with little or no attenuation, it is called a high-pass circuit.

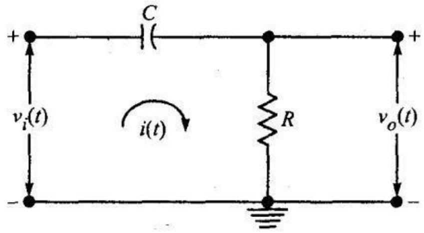


Figure 1.30 The high-pass *RC* circuit.

Sinusoidal Input

- Figure shows the Laplace transformed high-pass RC circuit. The gain versus frequency curve of a high-pass circuit excited by a sinusoidal input is shown in Figure .
- For a sinusoidal input v,, t;he output signal v_0 increases in amplitude with increasing frequency. The frequency at which the gain is 1/V2 of its maximum value is called the lower cut-off or lower 3-dB frequency. For a high-pass circuit, there is no upper cut-off frequency because all

  high frequency signals are transmitted with zero attenuation. Therefore, $f_2 - f_1$.

  Hence bandwidth B.W= f_2 - fi =∞

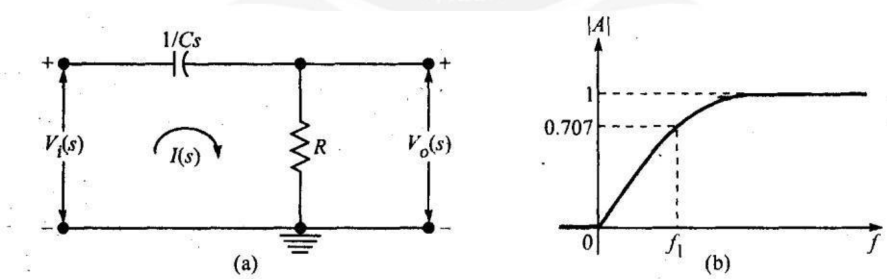


Figure 1.31 (a) Laplace transformed high-pass circuit and (b) gain versus frequency plot.

Expression for the lower cut-off frequency

- For the high-pass RC circuit shown in Figure 1.31 (a), the magnitude of the steady-state

  gain A, and the angle θ by which the output leads the input are given by

$$A = \frac{V_o(s)}{V_i(s)} = \frac{R}{R + \frac{1}{Cs}} = \frac{1}{1 + \frac{1}{RCs}}$$

Putting $s = j\omega$; $A = \dfrac{1}{1 - j\dfrac{1}{\omega RC}} = \dfrac{1}{1 - j\dfrac{1}{2\pi fRC}}$

$\therefore$ $|A| = \dfrac{1}{\sqrt{1 + \left(\dfrac{1}{2\pi fRC}\right)^2}}$ and $\theta = -\tan^{-1}\dfrac{1}{2\pi fRC}$

At the lower cut-off frequency f_1, $|A| = 1/\sqrt{2}$

$$\therefore \quad \frac{1}{\sqrt{1 + \left(\frac{1}{2\pi f_1 RC}\right)^2}} = \frac{1}{\sqrt{2}}$$

Squaring and equating the denominators,

$$\frac{1}{2\pi f_1 RC} = 1 \quad \text{i.e.} \quad f_1 = \frac{1}{2\pi RC}$$

This is the expression for the lower cut-off frequency of a high-pass circuit.

Relation between f1 and tilt

- The lower cut-off frequency of a high-pass circuit is /i = \/2nRC. The lower cut-off frequency produces a tilt.For a 10% change in capacitor voltage, the time or pulse width involved is

$$t = 0.1RC = \text{PW}$$

$$\therefore \quad \frac{\text{PW}}{RC} = 0.1 = \text{Fractional tilt}$$

$$\therefore \quad \text{Fractional tilt} = \frac{\text{PW}}{RC} = 2\pi f_1 \cdot \text{PW}$$

- This equation applies only when the tilt is 10% or less. When the tilt exceeds 10%, the voltage should be treated as exponential instead of linear and the equation $V_o = V_f - (V_f - V_i)e^{-t/RC}$ should be applied.

Step Input

- When a step signal of amplitude V volts shown in Figure 1.32(a) is applied to the high- pass RC circuit of Figure 1.30, since the voltage across the capacitor cannot change instantaneously the output will be just equal to the input at t = 0 (for f < 0, v,- = 0 and v_a = 0). Later when the capacitor charges exponentially, the output reduces exponentially with the same time constant RC.

- The expression for the output voltage for t > 0 is given by

$$v_o(t) = V_f - (V_f - V_{in})e^{-t/RC} = 0 - (0 - V)e^{-t/RC} = Ve^{-t/RC}$$

- Figure 1.32(b) shows the response of the circuit for large, small, and very small time constants.For t > 5r, the output will reach more than 99% of its final value.
- Hence although the steady state is approached asymptotically, for most applications we may assume that the final value has been reached after 5f.
- If the initial slope of the exponential is maintained, the output falls to zero in a time t = T.
- The voltage across a capacitor can change instantaneously only when an infinite current passes through it, because for any finite current i(t) through the capacitor, the instantaneous change in voltage across the capacitor is given by $\frac{1}{C}\int_0^0 i(t)\,dt = 0$.

Pulse Input

- A pulse of amplitude V and duration t_p shown in Figure 1.4(a) is nothing but the sum of a positive step of amplitude V starting at t = 0 and a negative step of amplitude V starting at t_p as shown in Figure 1.4(b).

- So, the response of the circuit for 0 < t < t,, for the pulse input is the same as

that for a step input and is given by $v_0(t) = Ve^{-t/RC}$.

- At t = t_p, $v_o(t) = V = Ve^{-t/RC}$. At t = t_p,
- since the input falls by V volts suddenly and since the voltage across the capacitor cannot change instantaneously, the output also falls suddenly by V volts to V_p - V. Hence at $t = t^+$,
- $v_a(t) = Ve^{-t_p/RC} - V$. Since V_p< V, V_p- V is negative. So there is an undershoot at

$t = t_p$ and hence

- for $t > t_p$, the output is negative. For $t > t_p$, the output rises exponentially towards zero with a time constant RC according to the expression $(Ve^{-t_p/RC} - V)e^{-(t-t_p)/RC}$-

- The output waveforms for RC » t_p, RC comparable to t_p and RC « t_p are shown in Figures 1.33(a), (b), and (c) respectively. There is distortion in the outputs and the distortion is the least when the time constant is very large.
- Observe that there is positive area and negative area in the output waveforms. The negative area will always be equal to the positive area.
- Hence a high-pass circuit with a very small time constant is called a peaking circuit and this process of converting pulses into pips by means of a circuit of short time constant is called peaking.

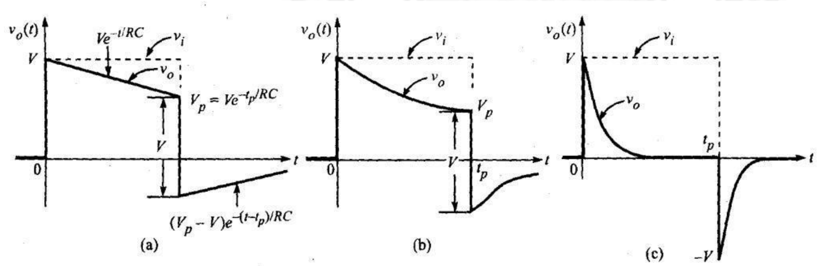


Figure 1.33 Pulse response for (a) $RC >> t_p$, (b) RC comparable to t_p and (c) $RC << t_p$.

Square-Wave Input

A square wave shown in Figure 1.34(a) is a periodic waveform, which maintains itself at one constant level V with respect to ground for a time T{ and then changes abruptly to another level V" and remains constant at that level for a time T_2, and then repeats itself at regular intervals of $T = T1 + T_2$. A square wave may be treated as a series of positive and negative steps. The shape of the output depends on the time constant of the circuit. Figures 1.34(b), 1.34(c), 1.34(d), and 1.34(e) show the output waveforms of the high-pass RC circuit under .steady-state conditions for the cases (a) RC » T, (b) RC > T, (c) RC - T, and (d) RC « T respectively. When the time constant is arbitrarily large (i.e. RC/T1 and RC/T2 are very very large incomparison to unity) the output is same as the input but with zero dc level. When RC > T, the output is in the form of a tilt. When RC is comparable to T, the output rises and falls exponentially. When RC « T (i.e. RCIT\ and RC/T2 are very small in comparison to unity), the output consists of alternate positive and negative spikes. In this case the peak-to-peak amplitude of the output is twice the peak-to-peak value of the input.In fact, for any periodic input waveformunder steady-state conditions, the average level of the output waveform from the high-pass circuit of Figure 1.30 is always zero independently of the dc level of the input. The proof is as follows: Writing KVL around the loop of Figure 1.30,

$$v_i(t) = \frac{1}{C}\int i(t)\,dt + v_o(t)$$

$$= \frac{1}{RC}\int v_o(t)\,dt + v_o(t) \qquad \left(\because\ i(t) = \frac{v_o(t)}{R}\right)$$

Differentiating,

$$\frac{dv_i(t)}{dt} = \frac{v_o(t)}{RC} + \frac{dv_o(t)}{dt}$$

Multiplying by *dt* and integrating this equation over one period *T*,

$$\int_{t=0}^{t=T} dv_i(t) = \int_{t=0}^{t=T} \frac{v_o(t)\,dt}{RC} + \int_{t=0}^{t=T} dv_o(t)$$

i.e. $$v_i(T) - v_i(0) = \frac{1}{RC}\int_0^T v_o(t)\,dt + v_o(T) - v_o(0)$$

Under steady-state conditions, the output waveform (as well as the input signal) is repetitive with a period T so that $v_o(T) = v_o(0)$ and $v_i(T) = v_i(0)$.

- Under steady-state conditions, the output waveform (as well as the input signal) is repetitivewith a period T so that v0(T) = v0(0) and v,(T) = v,(0).

  Hence $$\int_0^T v_o(t)\,dt = 0.$$

- Since this integral represents the area under the output waveform over one cycle, we can say that the average level of the steady-state output signal is always zero.

- This canalso be proved based on frequency domain analysis as follows.The periodic input signal may be resolved into a Fourier series consisting of a constant term and an infinite number of sinusoidal components whose frequencies are multiples of / = 1/T.
- Since the blocking capacitor presents infinite impedance to the dc input voltage, none of these dc components reach the output under steady-state conditions. Hence the output signal is a sum of sinusoids whose frequencies are multiples of This waveform is therefore periodic with a fundamental period T but without a dc component.With respect to the high-pass circuit of Figure 1.30, we can say that:

1. The average level of the output signal is always zero, .independently of the average level of the input. The output must consequently extend in both negative and positive directions with respect to the zero voltage axis and the area of the part of the waveform above the zero axis must equal the area which is below the zero axis.

2. When the input changes abruptly by an amount V, the output also changes abruptly byan equal amount and in the same direction.

3. During any finite time interval when the input maintains a constant level, the outputdecays exponentially towards zero voltage.

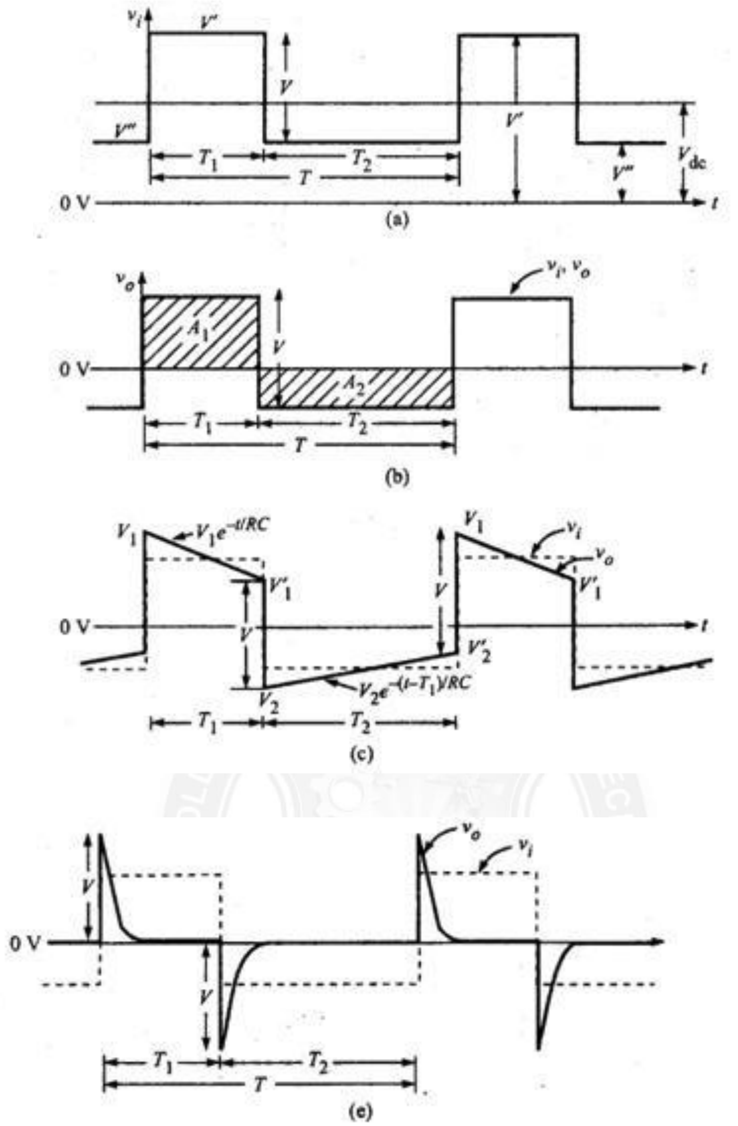


Figure 1.34 (a) A square wave input, (b) output when RC is arbitrarily large, (c) output when $RC > T$, (d) output when RC is comparable to T, and (e) output when $RC << T$.

- Under steady-state conditions, the capacitor charges and discharges to the same voltage levels in each cycle. So the shape of the output waveform is fixed.

For $0 < t < T_1$, the output is given by $v_{o1} = V_1 e^{-t/RC}$

At $t = T_1$, $v_{o1} = V_1' = V_1 e^{-T_1/RC}$

For $T_1 < t < T_1 + T_2$, the output is $v_{o2} = V_2 e^{-(t-T_1)/RC}$

At $t = T_1 + T_2$, $v_{o2} = V_2' = V_2 e^{-T_2/RC}$

Also $V_1' - V_2 = V$ and $V_1 - V_2' = V$

From these relations V_1, V_1', V_2 and V_2' can be computed.

Expression for the percentage tilt

- We will derive an expression for the percentage tilt when the time constant RC of the circuit is very large compared to the period of the input waveform, i.e. RC » T. For a symmetricalsquare wave with zero average value

$$V_1 = -V_2, \text{ i.e. } V_1 = |V_2|, \; V_1' = -V_2', \text{ i.e. } V_1' = |V_2'|, \text{ and } T_1 = T_2 = \frac{T}{2}$$

The output waveform for $RC >> T$ is shown in Figure 1.35. Here,

$$V_1' = V_1 e^{-T/2RC} \quad \text{and} \quad V_2' = V_2 e^{-T/2RC}$$

$$V_1 - V_2' = V$$

i.e.

$$V_1 - V_2 e^{-T/2RC} = V_1 + V_1 e^{-T/2RC} = V$$

$\therefore$

$$V_1 = \frac{V}{1 + e^{-T/2RC}} \quad \text{or} \quad V = V_1(1 + e^{-T/2RC})$$

$$\% \text{ tilt}, P = \frac{V_1 - V_1'}{\frac{V}{2}} \times 100\% = \frac{V_1 - V_1 e^{-T/2RC}}{V_1(1 + e^{-T/2RC})} \times 200\% = \frac{1 - e^{-T/2RC}}{1 + e^{-T/2RC}} \times 200\%$$

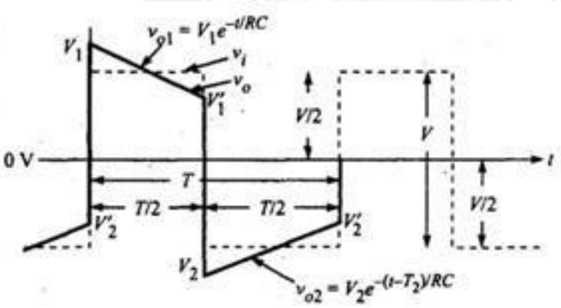


Figure 1.35 Linear tilt of a symmetrical square wave when $RC >> T$.

When the time constant is very large, i.e. $\frac{T}{RC} << 1$

$$P = \frac{1 - \left[1 + \left(\frac{-T}{2RC}\right) + \left(\frac{-T}{2RC}\right)^2 \frac{1}{2!} + \ldots\right]}{1 + 1 + \left(\frac{-T}{2RC}\right) + \left(\frac{-T}{2RC}\right)^2 \frac{1}{2!} + \ldots} \times 200\%$$

$$= \frac{\frac{T}{2RC}}{2} \times 200\%$$

$$= \frac{T}{2RC} \times 100\%$$

$$= \frac{\pi f_1}{f} \times 100\%$$

where $f_1 = \frac{1}{2\pi RC}$ is the lower cut-off frequency of the high-pass circuit.

Ramp Input

- A waveform which is zero for t < 0 and which increases linearly with time for / > 0 iscalled a ramp or sweep voltage.
- When the high-pass RC circuit of Figure 1.30 is excited by a ramp input v/(r) = at, where a isthe slope of the ramp, then

For times t which are very small in comparison with RC, we have

$$v_o(t) = \alpha RC\left[1-\left\{1+\left(\frac{-t}{RC}\right)+\left(\frac{-t}{RC}\right)^2\frac{1}{2!}+\left(\frac{-t}{RC}\right)^3\frac{1}{3!}+\dots\right\}\right]$$

$$= \alpha RC\left[\frac{t}{RC}-\frac{t^2}{2(RC)^2}+\dots\right]$$

$$= \alpha t-\frac{\alpha t^2}{2RC}=\alpha t\left(1-\frac{t}{2RC}\right)$$

$$V_i(s) = \frac{\alpha}{s^2}$$

From the Laplace transformed circuit of Figure 1.31(a),

$$V_o(s) = V_i(s)\frac{R}{R+\frac{1}{Cs}}=\frac{\alpha}{s^2}\frac{RCs}{1+RCs}$$

$$= \frac{\alpha}{s\left(s+\frac{1}{RC}\right)}=\alpha RC\left(\frac{1}{s}-\frac{1}{s+\frac{1}{RC}}\right)$$

Taking the inverse Laplace transform on both sides,

$$v_o(t) = \alpha RC(1 - e^{-t/RC})$$

- Figure 1.36 shows the response of the high-pass circuit for a ramp input when (a) RC» T, and (b) RC « T, where T is the duration of the ramp. For small values of T, the output signal falls away slightly from the input as shown in the Figure 1.36(a).

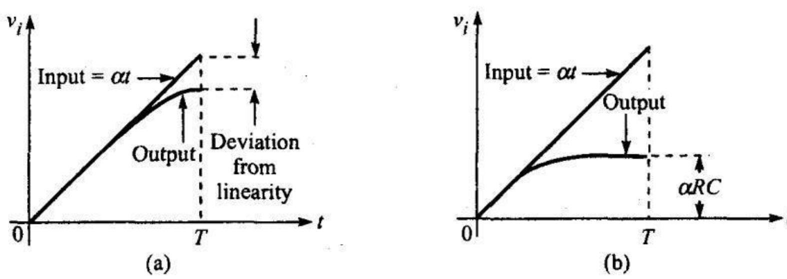


Figure 1.36 Response of the high-pass circuit for a ramp input when (a) $RC >> T$ and (b) $RC << T$.

- When a ramp signal is transmitted through a linear network, the output departs from the input. A measure of the departure from linearity expressed as the transmission error e, is defined as the difference between the input and the output divided by the input. The transmission error at a time

$$e_t = \left.\frac{v_i - v_o}{v_i}\right|_{t=T} \approx \left.\frac{\alpha t - \alpha t\left(1 - \frac{t}{2RC}\right)}{\alpha t}\right|_{t=T} \approx \frac{T}{2RC} = \pi f_1 T$$

where $f_1 = \dfrac{1}{2\pi RC}$ is the lower 3-dB frequency of the high-pass circuit.

- t = T is then For large values of t in comparison with RC, the output approaches the constant value aRC as indicated in Figure 1.36(b).

4.1.4.THE HIGH-PASS RC CIRCUIT AS A DIFFERENTIATOR

- When the time constant of the high-pass RC circuit is very very small, the capacitor charges very quickly; so almost all the input v,(0 appears across the capacitor and the voltage across the resistor will be negligible compared to the voltage across the capacitor. Hence the current is determined entirely by the capacit nce. Then the current

$$i(t) = C\frac{dv_i(t)}{dt}$$

and the output signal across R is

$$v_o(t) = RC\frac{dv_i(t)}{dt}$$

- Thus we see that the output is proportional to the derivative of the input.
- **The high-pass RC circuit acts as a differentiator provided the RC time, constant of the circuit is very small in comparison with the time required for the input signal to make an appreciable change.**
- The derivative of a step signal is an impulse of infinite amplitude at the occurrence of the discontinuity of step. The derivative of an ideal pulse is a positive impulse followed by a delayed negative impulse, each of infinite amplitude and occurring at the points of discontinuity. The derivative of a square wave is a waveform which is uniformly zero except, at the points of

discontinuity. At these points, precise differentiation would yield impulses of infinite amplitude, zero width and alternating polarity.

- For a square wave input, an RC high-pass circuit with very small time constant will produce an output, which is zero except at the points of discontinuity. At these points of discontinuity, there will be peaks of finite amplitude V. This is because the voltage across R is not negligible compared with that across C.
- An RC differentiator converts a triangular wave into a square wave.For the ramp v_i = at, the value of RC(dv/dt) = aRC. This is true except near the origin. The output approaches the proper derivative value only after a lapse of time corresponding to several time constants. The error near θ= 0 is again due to the fact that in this region the voltage across R is not negligible compared with that across C.
- If we assume that the leading edge of a pulse can be approximated by a ramp, then we can measure the rate of rise of the pulse by using a differentiator. The peak output is measured on an oscilloscope, and from the equation = aRC, we see that this voltage divided by the product RC gives the slope a.

$$\tan\theta = \frac{X_C}{R} = \frac{1}{\omega RC}$$

- A criteria for good differentiation in terms of steady-state sinusoidal analysis is, that if a sine wave is applied to the high-pass RC circuit, the output will be a sine wave shifted by a leading angle θ such that: with the output being proportional to sin(a>t + θ). In order to have true differentiation, we must obtain cos ωt. In other words,θ must equal 90°.

- This result can be obtained only if R =,0 or C = 0.
- However, if ωRC =0.01, then 1/ωRC = 100 and θ = 89.4°, which is sufficiently close to 90° for most purposes. If ωRC = 0.1, then 90 - 84.3° and

for some applications this may be close enough to 90°.If the peak value of input is V_m, the output is and if ωRC « 1, then the output is approximately V_mωRC cos (at. This result agrees with the expected value RC(dv_t/dt).

- output amplitude is 0.01 times the input amplitude.

$$v_o = \frac{V_m R}{\sqrt{R^2 + \frac{1}{\omega^2 C^2}}} \sin(\omega t + \theta)$$