TIME VARYING FIELDS AND MAXWELL'S EQUATIONS

Introduction:

In our study of static fields so far, we have observed that static electric fields are produced by electric charges, static magnetic fields are produced by charges in motion or by steady current. Further, static electric field is a conservative field and has no curl, the static magnetic field is continuous and its divergence is zero. The fundamental relationships for static electric fields among the field quantities can be summarized as:

$$\nabla \times \vec{E} = 0 \qquad (4.1a)$$

$$\nabla \cdot \vec{D} = \rho_v \qquad (4.1b)$$

For a linear and isotropic medium,

$$\vec{D} = \varepsilon \vec{E} \qquad (4.1c)$$

Similarly for the magnetostatic case

$$\nabla . \vec{B} = 0 \qquad (4.2a)$$

$$\nabla \times \vec{H} = \vec{J} \qquad (4.2b)$$

$$\vec{B} = \mu \vec{H} \qquad (4.2c)$$

It can be seen that for static case, electric field $\vec{E}$ and $\vec{D}$ magnetic field vectors $\vec{B}$ and $\vec{H}$ form separate pairs.

In this chapter we will consider the time varying scenario. In the time varying case we will observe that a changing magnetic field will produce a changing electric field and vice versa.

We begin our discussion with Faraday's Law of electromagnetic induction and then present the Maxwell's equations which form the foundation for the electromagnetic theory.

Faraday's Law of electromagnetic Induction

Michael Faraday, in 1831 discovered experimentally that a current was induced in a conducting loop when the magnetic flux linking the loop changed. In terms of fields, we can say that a time varying magnetic field produces an electromotive force (emf) which causes a current in a closed circuit. The quantitative relation between the induced emf (the voltage that arises from conductors moving in a magnetic field or from changing magnetic fields) and the rate of change of flux linkage developed based on experimental observation is known as Faraday's law. Mathematically, the induced emf can be written as

$$\text{Emf} = -\frac{d\phi}{dt} \quad \text{Volts} \qquad (4.3)$$

where ϕ is the flux linkage over the closed path.

A non zero $\frac{d\phi}{dt}$ may result due to any of the following: (a) time changing flux linkage astationary closed path.

(a) relative motion between a steady flux a closed path.

(b) a combination of the above two cases.

The negative sign in equation (4.3) was introduced by Lenz in order to comply with thepolarity of the induced emf. The negative sign implies that the induced emf will cause a current flow in the closed loop in such a direction so as to oppose the change in the linking magnetic flux which produces it. (It may be noted that as far as the induced emf is concerned, the closed path forming a loop does not necessarily have to be conductive).

If the closed path is in the form of N tightly wound turns of a coil, the change in the magnetic flux linking the coil induces an emf in each turn of the coil and total emf is the sum of the induced emfs of the individual turns, i.e.,

$$Emf = -N\frac{d\phi}{dt} \quad Volts \qquad (4.4)$$

By defining the total flux linkage as

$$\lambda = N\phi \qquad (4.4)$$

The emf can be written as

$$Emf = -\frac{d\lambda}{dt} \qquad (4.6)$$

Continuing with equation (4.3), over a closed contour '*C*' we canwrite

$$Emf = \oint_C \vec{E}.\vec{dl} \qquad (4.7)$$

where $\vec{E}$ is the induced electric field on the conductor to sustain the current. Further, total flux enclosed by the contour '*C* ' is given by

$$\phi = \int_S \vec{B}.\vec{ds} \qquad (4.8)$$

Where *S* is the surface for which '*C*' is the contour. From (4.7) and using (4.8) in (4.3) we can write

$$\oint_C \vec{E}.\vec{dl} = -\frac{\partial}{\partial t}\oint_S \vec{B}.\vec{ds} \qquad (4.9)$$

By applying stokes theorem

$$\int_s \nabla \times \vec{E}.d\vec{s} = -\int_s \frac{\partial \vec{B}}{\partial t}.d\vec{s}$$

Therefore, we can write (4.10)

$$\nabla \times \vec{E} = -\frac{\partial \vec{B}}{\partial t} \quad (4.11)$$

which is the Faraday's law in the point form

$$\frac{d\phi}{dt}$$

We have said that non zero can be produced in a several ways. One particular case is when a time varying flux linking a stationary closed path induces an emf. The emf induced in a stationary closed path by a time varying magnetic field is called atransformer emf **.**

Ideal transformers

As shown in figure 2.1, a transformer consists of two or more numbers of coils coupled magnetically through a common core. Let us consider an ideal transformer whose winding has zero resistance, the core having infinite permittivity and magnetic losses are zero.

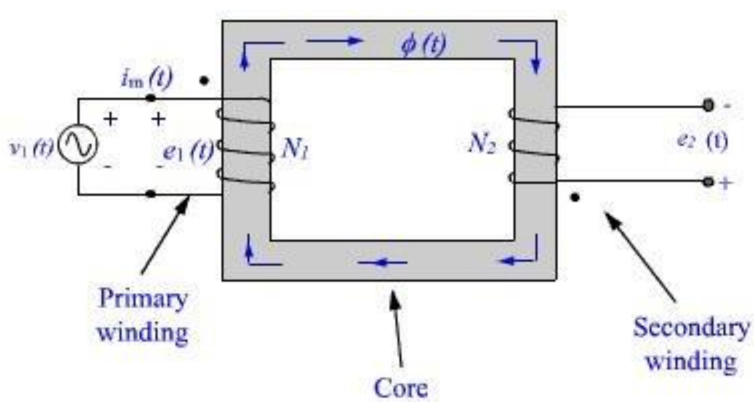

Fig 2.1: Transformer with secondary open
(www.brainkart.com/subject/Electromagnetic-Theory_206/)

These assumptions ensure that the magnetization current under no load condition is vanishingly small and can be ignored. Further, all time varying flux produced by the primary winding will follow the magnetic path inside the core and link to the secondary coil without any leakage. If N_1 and N_2 are the number of turns in the primary and the secondary windings respectively, the induced emfs are

$$e_1 = N_1 \frac{d\phi}{dt} \qquad (4.12a)$$

$$e_2 = N_2 \frac{d\phi}{dt} \qquad (4.12b)$$

(The polarities are marked, hence negative sign is omitted. The induced emf is +ve atthe dotted end of the winding.)

$$\therefore \frac{e_1}{e_2} = \frac{N_1}{N_2} \qquad (4.13)$$

i.e., the ratio of the induced emfs in primary and secondary is equal to the ratio of their turns. Under ideal condition, the induced emf in either winding is equal to their voltage rating.

$$\frac{v_1}{v_2} = \frac{N_1}{N_2} = a \qquad (4.14)$$

where 'a' is the transformation ratio. When the secondary winding is connected to a load, the current flows in the secondary, which produces a flux opposing the original flux. The net flux in the core decreases and induced emf will tend to decrease from the no load value. This causes the primary current to increase to nullify the decrease in the flux and induced emf.

The current continues to increase till the flux in the core and the induced emfs are restoredto the no load values. Thus the source supplies power to the primary winding and the secondary winding delivers the power to the load. Equating the powers

$$i_1 v_1 = i_2 v_2 \quad (4.14)$$

$$\frac{i_2}{i_1} = \frac{v_1}{v_2} = \frac{e_1}{e_2} = \frac{N_1}{N_2} \quad (4.16)$$

Further,

$$i_2 N_2 - i_1 N_1 = 0 \quad (4.17)$$

i.e., the net magnetomotive force (mmf) needed to excite the transformer is zero underideal condition.

Motional EMF:

Let us consider a conductor moving in a steady magnetic field as shown in the fig 2.2.

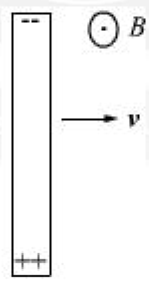

Fig 2.2 Motional EMF:
(www.brainkart.com/subject/Electromagnetic-Theory_206/)

If a charge Q moves in a magnetic field $\vec{B}$, it experiences a force

$$\vec{F} = Q\vec{v} \times \vec{B} \quad (4.18)$$

This force will cause the electrons in the conductor to drift towards one end and leave the other end positively charged, thus creating a field and charge separation continuous until electric and magnetic forces balance and an equilibrium is reached very quickly, the net force on the moving conductor is zero.

$\frac{\vec{F}}{Q} = \vec{v} \times \vec{B}$ can be interpreted as an induced electric field which is called the motional electric field. This emf is called the motional emf.

$$\vec{E}_m = \vec{v} \times \vec{B} \qquad (4.19)$$