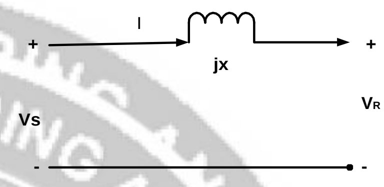
Power Flow under Steady State:-

Consider a short transmission line with negligible resistance.

V_S = per phase sending end voltage
V_R = per phase receiving end voltage
V_s leads V_R by an angle δ
x = reactance of per transmission line



(Fig.3-A short transmission line)

On the per phase basis power on sending end,

$$S_S = P_S + j\,Q_S = V_S I^* \quad \ldots\ldots (29)$$

From Fig.3 I is given as

$$I = \frac{V_S - V_R}{jx}$$

or

$$I^* = \frac{V_S^* - V_R^*}{-jx} \quad \ldots\ldots (30)$$

From equation (29) and (30), we get

$$S_S = \frac{V_S(V_S^* - V_R^*)}{-jx} \quad \ldots\ldots (31)$$

Now $V_R = |V_R| \angle 0^0$ so, $V_R = V_R^* = |V_R|$

$V_S = |V_S| \angle \delta = |V_S| e^{j\delta}$

Equation (31) becomes

$$S_S = P_S + jQ_S = \frac{|V_S||V_R|}{x} \sin\delta + j\frac{1}{x}\left(|V_S|^2 - |V_S||V_R| \cos\delta\right)$$

So $$P_S = \frac{|V_S||V_R|}{x} \sin\delta \quad \ldots\ldots (32)$$

and $$Q_s = \frac{|V_s|^2 - |V_S||V_R| \cos\delta}{x} \quad \ldots\ldots (33)$$

Similarly, at the receiving end we have

$$S_R = P_R + j\,Q_R = V_R I^* \quad \ldots\ldots (34)$$

Proceeding as above we finally obtain

$$Q_R = \frac{|V_S||V_R|\cos\delta - |V_R|^2}{x} \quad \text{......} (35)$$

$$P_R = \frac{|V_S||V_R|}{x}\sin\delta \quad \text{......} (36)$$

Therefore for lossless transmission line,

$$P_S = P_R = \frac{|V_S||V_R|}{x}\sin\delta \quad \text{......} (37)$$

In a similar manner, the equation for steady-state power delivered by a lossless synchronous machine is given by

$$P_e = P_d = \frac{|E_g||V_t|}{x_d} \sin \delta$$

$$= P_{max} \sin \delta \quad \ldots(38)$$

Where $|E_g|$ is the rms internal voltage, $|V_t|$ is the rms terminal voltage, x_d is the direct axis reactance (or the synchronous reactance in a round rotor machine) and δ is the electrical power angle.

Steady-state Stability:-

The steady state stability limit of a particular circuit of a power system defined as the maximum power that can be transmitted to the receiving end without loss of synchronism.

Now consider equation (18),

$$M_{(pu)} . \frac{d^2\delta}{dt^2} = (P_i - P_e) \quad \ldots(39)$$

Where $M_{(pu)} = \frac{H}{\pi f}$

And $P_e = \frac{|E_g||V_t|}{x_d} \sin \delta = P_{max} \sin \delta \quad \ldots (40)$

Let the system be operating with steady power transfer of $P_{e0} = P_i$ with torque angleδ_0. Assume a small increment ΔP in the electric power with the input from the prime mover remaining fixed at P_i causing the torque angle to change to $(\delta_0 + \Delta\delta)$. Linearizing the operating point (P_{e0}, δ_0) we can write

$$\Delta P_e = \left(\frac{6P_e}{6ð}\right)_0 \Delta\delta \quad \ldots (41)$$

The excursions of $\Delta\delta$ are then described by

$$M \frac{d^2\Delta\delta}{dt^2} = P_i - (P_{e0} + \Delta P_e) = -\Delta P_e \quad \ldots (42)$$

or $$M \frac{d^2\Delta\delta}{dt^2} + \left[\frac{6P_e}{6ð}\right]_0 \Delta\delta = 0 \quad \ldots (43)$$

or $$\left[Mp^2 + \left(\frac{6P_e}{6ð}\right)_0\right]\Delta\delta = 0 \quad \ldots (44)$$

Where $p = \frac{d}{dt}$

The system stability to small changes is determined from the characteristic equation

$$Mp^2 + \left(\frac{\partial P_e}{\partial \delta}\right)_0 = 0 \quad \text{...(45)}$$

Where two roots are

$$p = \pm\left[\frac{-\left(\partial P_e/\partial\delta\right)_0}{M}\right]^{\frac{1}{2}} \quad \text{................................. (46)}$$

As long as $\left(\partial P_e/\partial \delta\right)_0$ is positive, the roots are purely imaginary and conjugate and system behavior is oscillatory about δ_0. Line resistance and damper windings of machine cause the system oscillations to decay. The system is therefore stable for a small increment in power so long as $\left(\partial P_e/\partial \delta\right)_0 > 0$.

When $\left(\partial P_e/\partial \delta\right)_0$ is negative, the roots are real, one positive and the other negative but of equal magnitude. The torque angle therefore increases without bound upon occurrence of a small power increment and the synchronism is soon lost. The system is therefore unstable for $\left(\partial P_e/\partial \delta\right)_0 < 0$.

$\left(\partial P_e/\partial \delta\right)_0$ is known as **synchronizing coefficient**. This is also called **stiffness** of synchronous machine. It is denoted as S_p. This coefficient is given by

$$_p = \frac{\partial P_e}{\partial \delta}\Big|_{\delta=\delta_0} = P_{max}\ s\,\delta_0 \quad \text{............................ (47)}$$

If we include damping term in swing equation then equation (43) becomes

$$M\frac{d^2\Delta\delta}{dt^2} + D\frac{d\Delta\delta}{dt} + \left[\frac{\partial P_e}{\partial \delta}\right]_0 \Delta\delta = 0$$

or

$$\frac{d^2\Delta\delta}{dt^2} + \frac{D}{M}\frac{d\Delta\delta}{dt} + \frac{1}{M}\left[\frac{\partial P_e}{\partial \delta}\right]_0 \Delta\delta = 0$$

or

$$\frac{d^2\Delta\delta}{dt^2} + \frac{D\pi f}{H}\frac{d\Delta\delta}{dt} + \frac{S_p\pi f}{H}\Delta\delta = 0$$

or

$$\frac{d^2\Delta\delta}{dt^2} + 2\ _n\frac{d\Delta\delta}{dt} + \ _n^2\Delta\delta = 0 \quad \text{.................................. (48)}$$

Where $m_n = \sqrt{\frac{fS_p}{H}}$ and $r = \frac{D}{2}\sqrt{\frac{f}{HS_p}}$ .... (49)

So damped frequency of oscillation, $m_d = m_n\sqrt{1 - r^2}$ .... (50)

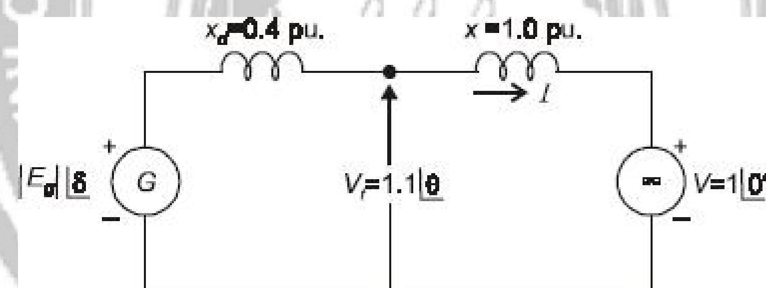
And Time Constant, $T = \frac{1}{c\omega_n} = \frac{2H}{\pi f D}$…………………………….. (51)

Example2:-

Find the maximum steady-state power capability of a system consisting of a generator equivalent reactance of 0.4pu connected to an infinite bus through a series reactance of 1.0 p.u. The terminal voltage of the generator is held at1.10 p.u. and the voltage of the infinite bus is 1.0 p.u.

Solution:-

Equivalent circuit of the system is shown in Fig.4.



(Fig.4 Equivalent circuit of example2)

$$|E_g|L\delta = {}_t + jx_d$$ ....(i)

$$I = \frac{V_t - V}{jx} = \frac{1.1L\theta - 1L0°}{j1}$$ ……………………………….. (ii)

Using equation (i) and (ii)

$$|E_g|L\delta = 1.1L\theta + j0.4(\frac{1.1L\theta - 1L0°}{j1})$$

$$\therefore |E_g|L\delta = 1.1\cos\theta + j1.1\sin\theta + 0.4x1.1L\theta - 0.4$$

$$\therefore |E_g|L\delta = (1.4\cos\theta - 0.4) + j1.4\sin\theta$$ .... (iii)

Maximum steady-state power capability is reached when $\delta = 90^o$, i.e., real part of equation is zero. Thus

$$1.54\cos\theta - 0.4 = 0$$

$$\therefore \theta = 74.9°$$

$$\therefore |E_g| = 1.54\sin 74.9° = 1.486 \text{ pu.}$$

$$\therefore V_t = 1.1L74.9°$$

$$\therefore P_{max} = \frac{|E_g||V|}{(x_d} \quad 1.48x1.0$$