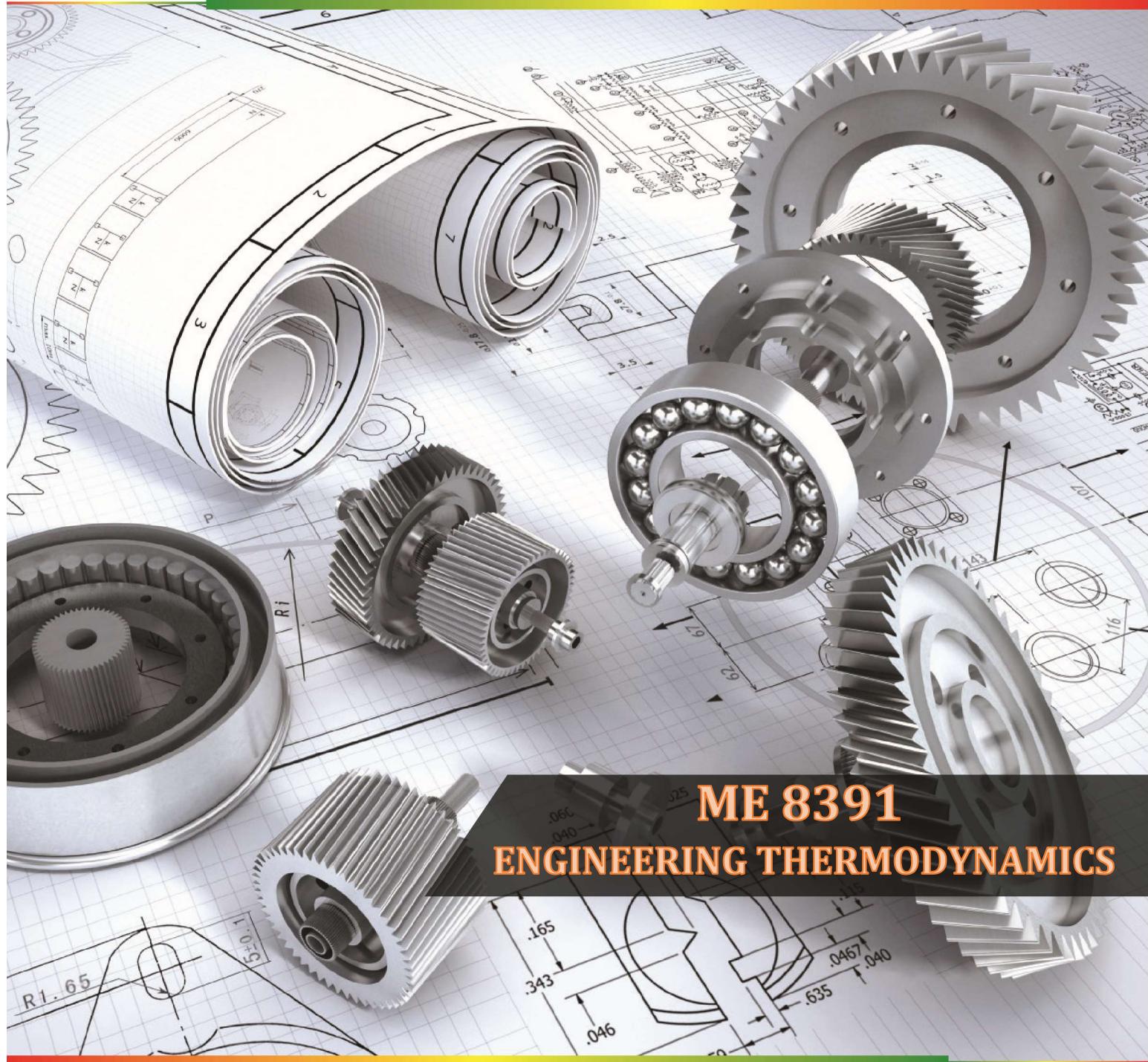




# ROHINI

## COLLEGE OF ENGINEERING & TECHNOLOGY

Approved by AICTE and Affiliated to Anna University, (An ISO Certified Institution)



DEPARTMENT OF  
**MECHANICAL ENGINEERING**

**UNIT-I**  
**Basic Concepts and First Law**

Thermodynamics: is a field of science which deals with energy's possessed by gases and vapours. It also deals with the relationship between heat, work and energy.

Thermodynamic Properties:

① Density ( $\rho$ ): density is defined as the mass which possessed per unit volume of the substance. It is denoted by  $\rho$ , unit is  $\text{kg}/\text{m}^3$ .

$$\rho = \frac{\text{mass}}{\text{Volume}} (\text{kg}/\text{m}^3)$$

2. specific weight ( $w$ ):

specific weight is defined as the weight possessed for unit volume of the substance, unit is  $\text{N}/\text{m}^3$ .

$$\text{Sp. weight } w = \frac{W}{V} (\text{N}/\text{m}^3)$$

3. Sp. Volume ( $V$ )

Sp. Volume is defined as the volume occupied by unit mass of the substances. unit is  $(\text{m}^3/\text{kg})$ .

$$\text{Sp. Vol} = \frac{1}{\rho} = \frac{\text{Volume}}{\text{mass}} (\text{m}^3/\text{kg})$$

4. Sp. Gravity ( $s$ ):

Sp. gravity is defined as the ratio of the density (or) sp. weight of the given substance to the density (or) sp. weight of the standard substance.

$$\text{Sp. gravity } s = \frac{\text{density or sp. wt. of the given substance}}{\text{density or sp. wt. of the std. substance}}$$

std substance of liquid = water

" gas = Air.

Example: For ~~Mercury~~ <sup>Mercury</sup>  $s = \frac{\rho_{\text{Hg}}}{\rho_{\text{water}}} = \frac{13600}{1000} = \underline{13.6}$

S.P. Pressure (P) It is defined as the force exerted per unit area.

$$P = \frac{\text{Force}}{\text{Area}} = \frac{F}{A} \left( \frac{\text{N}}{\text{m}^2} \right).$$

Note :

Various Unit of Pressure:

$$1 \text{ Pascal} = 1 \text{ N/m}^2$$

$$1 \text{ bar} = 10^5 \text{ N/m}^2$$

Atmospheric Pressure: (P<sub>atm</sub>)

It is the pressure exerted by the air on the atmosphere. This value taken at mean sea level.

$$\begin{aligned} \text{Atmospheric pressure} &= 1.01325 \text{ bar} \\ &= 101.325 \text{ KN/m}^2 \\ &= 101325 \text{ N/m}^2 (\text{ex}) \\ &= 760 \text{ mm of Hg.} \end{aligned}$$

Gauge P<sub>r</sub>: P<sub>g</sub>

It is the pressure recorded by the pressure gauge.

Vacuum P<sub>r</sub>: The pressure below the atmospheric pressure is called as vacuum pressure. It is also called as negative P<sub>r</sub>.

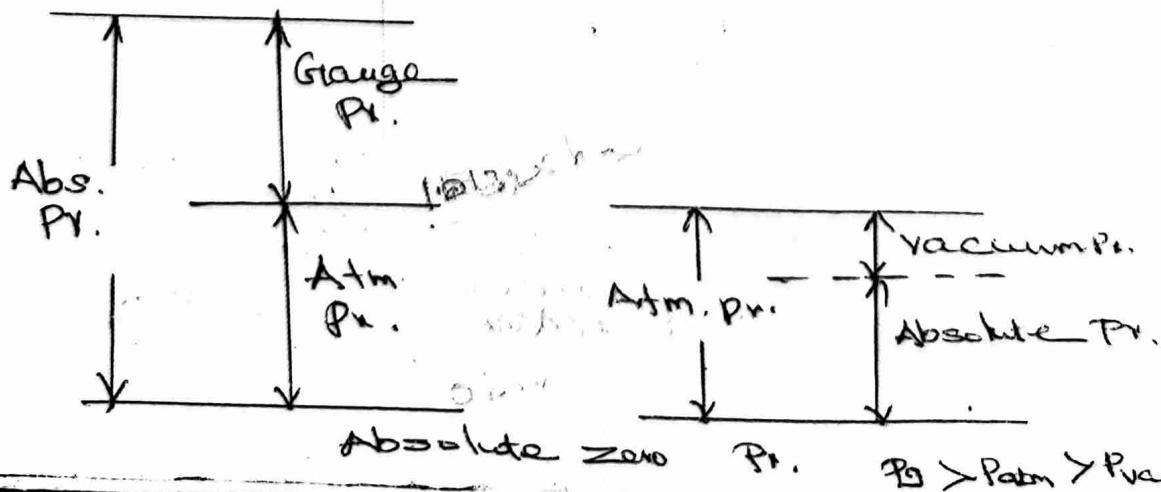
Absolute P<sub>r</sub>: P<sub>abs</sub>

The pressure measured from absolute zero pressure is called absolute pressure.

$$\text{Absolute P}_r = \text{Atm. P}_r + \text{Gauge P}_r.$$

$$\text{Absolute P}_r = \text{Atm. P}_r - \text{Vacuum P}_r.$$

### Pressure Relationships

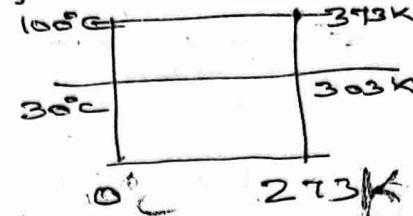


## Temperature:

degree of hotness or coldness.

It is a property which is used to determine the degree of hotness or coldness of the body.

$$\begin{array}{ccc} {}^{\circ}\text{C} & \xrightarrow{+273} & \text{K} \\ \text{K} & \xrightarrow{-273} & {}^{\circ}\text{R} \end{array}$$



## S.T.P. (Standard temp. and P<sub>n</sub>)

standard Temp = 0°C

" P<sub>n</sub> = 1.01325 bar.

## N.T.P.:

## (Normal Temp and P<sub>n</sub>)

Normal Temp = 0°C

" P<sub>n</sub> = 1.01325 bar.

## Sp. heat Capacity (C):

It is defined as the quantity of heat transfer required to raising and lowering the temp. of unit mass of the substance through one degree. unit is (J/kgk).

## Sp. heat Capacity at Constant Vol: (C<sub>v</sub>):

(above same definition) (Same) when the vol. remains constant.

Heat transfer Q = m C<sub>v</sub> (T<sub>2</sub> - T<sub>1</sub>) KJ ~~Heat~~.

## Sp. heat Capacity at constant P<sub>n</sub>: C<sub>p</sub>

(Above same definition with.) when the p<sub>n</sub> remains under

Heat transfer Q = m C<sub>p</sub> (T<sub>2</sub> - T<sub>1</sub>) ~~KJ Heat~~

$$\frac{\text{kg} \times \text{KJ} \times \text{K}}{\text{Kg} \cdot \text{K}} = \text{KJ}$$

- \* Solids and liquids have only one specific heat.  
 \* Gases and Vapours have Two specific heats.  $C_p & C_v$ .  
 $C_p > C_v$

For Example.

For Air

$$C_p = 1.005 \text{ kJ/kgK}$$

$$= 0.287 \text{ kJ/kgK}$$

$$C_v = 0.718 \text{ kJ/kgK}$$

$$\frac{C_p}{C_v} = \gamma \quad (\text{Adiabatic index})$$

$$\gamma = 1.4$$

Types of Thermodynamics: ① Statistical ② Classical.

Microscopic and Macroscopic approaches:

Microscopic approach: Statistical thermodynamics.

In this approach the properties of matter such as Pressure, Velocity, Position and energy of individual molecules are studied or analysed is known as microscopic approach or statistical thermodynamics.

Example: ~~Nuclear power plant~~

~~In Nuclear Power plant we are analysing the atom level.~~

Macroscopic approach: Classical thermodynamics:

In this approach, instead of studying molecular (or) atom level, we are studying and analysing the pressure, velocity, temperature and energy level of total system.

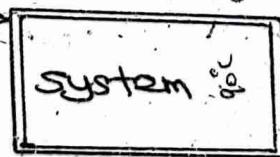
Example: IC engine.

Thermodynamic System:

A quantity of matter or region in space selected for analysis is called System.

Boundary

Ex: Comp, Turb, Boiler, adiab., Ref. Heat exch., mat. exch., etc.



surroundings

Surroundings:

Anything outside the system which affects the behaviour of the system is known as surroundings.

Boundary: It is a imaginary one. The system and surroundings are separated by a boundary.

### Universe:

1. 5kg of steam having specific heat capacity of  $480 \text{ J/kgK}$ . is heated from  $20^\circ\text{C}$  to  $100^\circ\text{C}$ . Estimate the amount of heat required.

Given data:

$$\text{mass } m = 5 \text{ kg}$$

$$T_1 = 20^\circ\text{C} + 273 = 293 \text{ K}$$

$$T_2 = 100^\circ\text{C} + 273 = 373 \text{ K}$$

$$C = 480 \text{ J/kgK} = \frac{480}{1000} = 0.480 \text{ kJ/kgK}$$

To find:

$$\text{Amount of heat } Q = ?$$

Solution:

$$Q = m C (\Delta T)$$

$$Q = m C (T_2 - T_1)$$

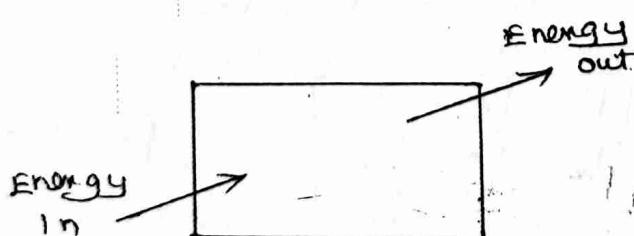
$$Q = 5 \times 0.480 (373 - 293) ; \quad \frac{\text{kg} \times \text{kJ}}{\text{kgK}} \times \text{K}$$

$$Q = 192 \text{ kJ}$$

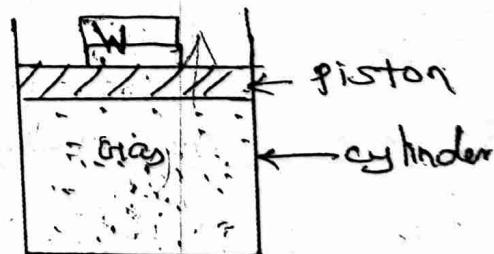
Types of System: The system is mainly classified into three category:

- \* closed system
- \* Open System
- \* Isolated system

### Closed System:

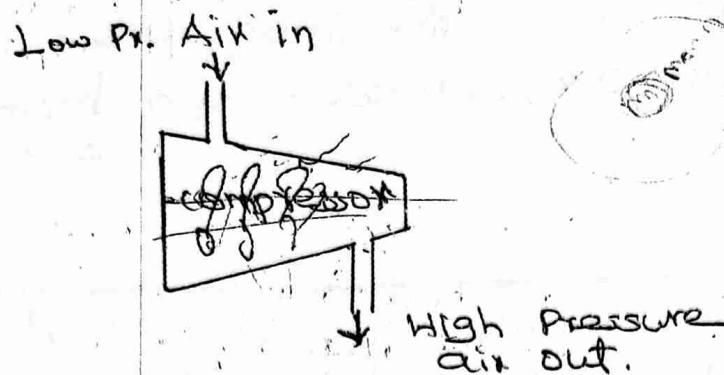


No mass transfer.



It is the system where no mass transfer, but only Energy transfer (heat and work) taking place across the boundary.

Open System: In open system both mass and energy transfer takes place.



Isolated System:

(2) A system which is not affected by surroundings is called isolated system.

(3) There is no mass and energy transfer takes place.

Example: Entire Universe.

State and Properties of a System:

State: The exact condition of the system at ~~at~~ any particular moment is called state. The state is identified by the properties of the system such as pressure, volume and temperature etc.

Properties: are classified into two types;

- \* Intensive properties
- \* Extensive properties.

(1) Intensive Properties:

It is the property which do not dependent on the mass of the system.

Example: Pressure, Temperature, specific volume, density etc.

Note: These properties of the part of the system is equal to the these properties of the whole of the system.

$$P = P_A = P_B$$

$$\frac{P_1}{P_2} = \frac{P_A}{P_B}$$

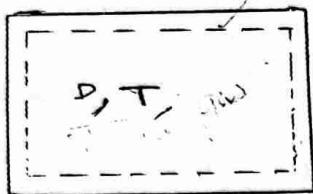
Extensive properties:

$$T = T_A = T_B$$

It is the property which dependent on the mass of the system.

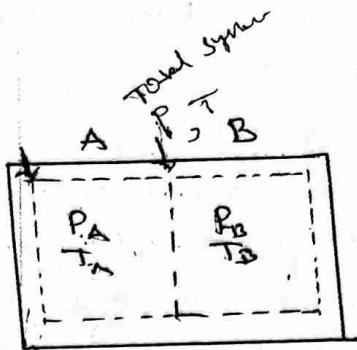
Example: mass, volume, weight, total energy etc.

$$Y = Y_A + Y_B$$



$$P_A = P$$

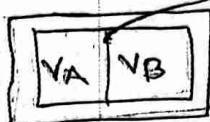
$$T_A = T$$



$P_A = P_B = P$  consider  
we have  
segment mass  
mass goes  
and so  
on

These properties of the whole of the system is equal to the sum of the properties of the parts of the system.

$$V = V_A + V_B$$

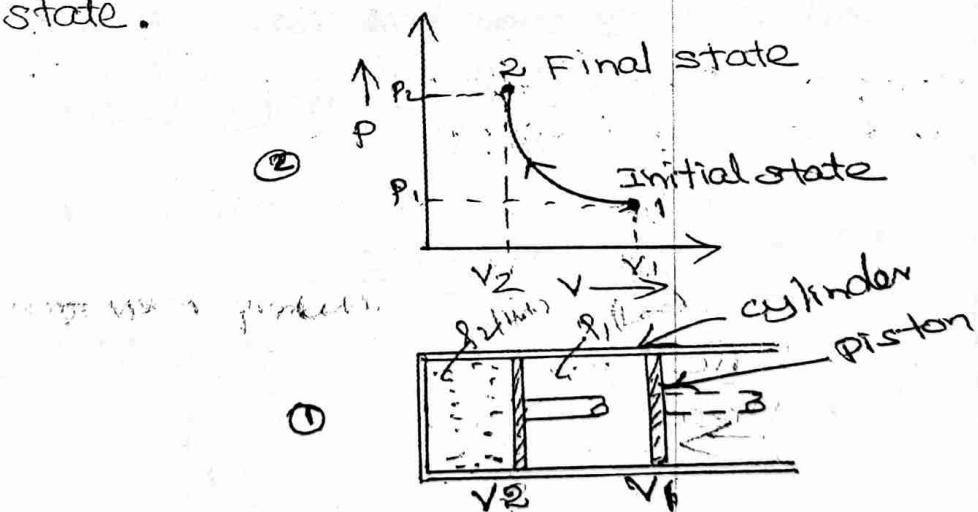


## The Law of Thermodynamics:

- \* Zeroth Law of Thermodynamics
- \* First Law of Thermodynamics
- \* Second Law of Thermodynamics.

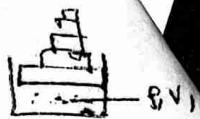
## Thermodynamic process:

Process is the change of state undergone by a system from one equilibrium state to another equilibrium state.



- \* Quasi-static process.
- \* Reversible and irreversible process.
- \* Flow and Non flow process.

## Quasi-Static Process:



(A System Passes through an infinite number of continuous equilibrium states and attains the original state when the process is reversed.

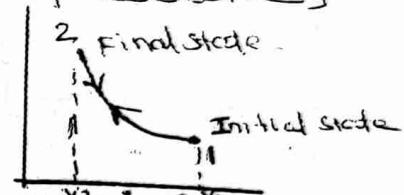
This process is called quasi-static process.

It is reversible process and very slow process.

## Reversible and Irreversible Process:

A system passes through an infinite number of continuous equilibrium states, and it traces the same path when this process is reversed. This process is known as reversible process.

Example: Constant volume, constant pressure, isothermal, ~~isentropic~~ process etc.



## Irreversible process:

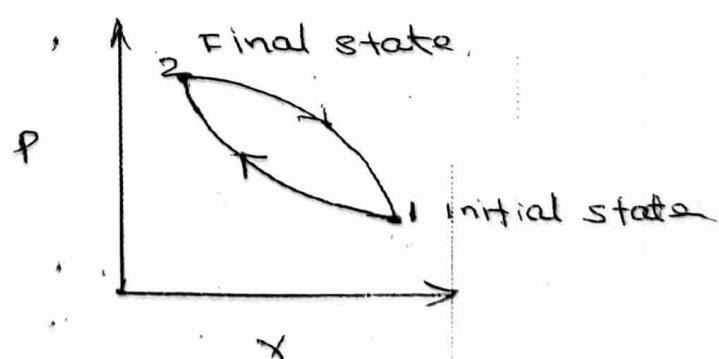
A system passes through an infinite number of continuous non equilibrium states, and it does not trace the same path when the process is reversed. This process is known as irreversible process.

Example: Free expansion process, throttling process, and steady flow process.

Process There are many factors that make a irreversible.

The most common causes of irreversibility are \* Friction loss

\* Heat loss.



## Flow and Non flow Process:

Flow Process: In a flow process the working fluid enters the system and leaves it to atmosphere after doing the work. In this system both energy and mass transfer takes place.

Example: Boiler, Turbine, Compressor etc.

Non flow process: In a non flow process, the same working fluid is recirculated again and again. In this system only energy crosses the boundary in the form of heat and work.

Example: Constant volume process, constant pressure process, isothermal process and isentropic process etc.



## Thermodynamic equilibrium:

A System is said to be in equilibrium, if it does not tend to undergo any change of state.

### a) Mechanical equilibrium:

A System is said to be in mechanical equilibrium, when there is (no unbalanced forces acting on it).

### b) Thermal equilibrium:

A System is said to be in thermal equilibrium, when there is no temperature difference throughout the system.

### c) Chemical equilibrium:

A System is said to be in chemical equilibrium, when there is (no chemical reaction throughout the system).



## Thermodynamic Cycle:

A series of state changes such that the final state is identical with the initial state is known as Cycle.

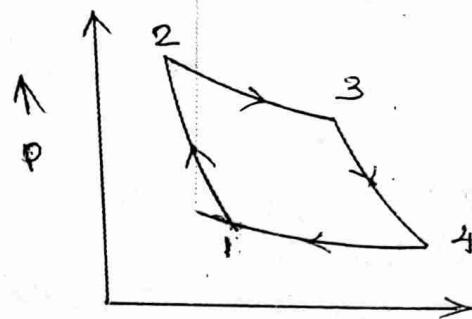
There are two types of cycles.

(a) Closed cycle.

(b) Open Cycle.

### Closed cycle:

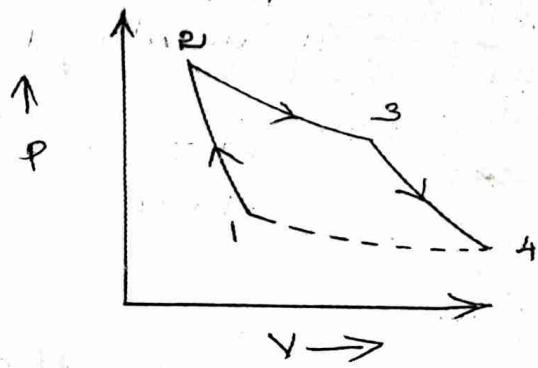
In a closed cycle, the working substance is recirculated again and again within the system itself, without taking any mass transfer.



P-V diagram of a closed cycle.

### Open Cycle:

In an open cycle, the working substance is exhausted to atmosphere after completing the process. So here both mass and energy transfer takes place.



### Point function:

Some of the properties such as pressure, volume and temperature are not dependent upon the path followed by the system. These properties are called as point function (or) state function.

Example: Pressure, Volume, Temperature etc.

at any pt. prop are

The prop. P & V are not dependent upon the path.

## Path function:

Some properties such as work transfer and heat transfer, etc are dependent upon the path followed by the system. These properties are called as path function.

## Difference between path and point functions:

Path function	Point function.
1. Properties are dependent upon the path followed by the system	1. Properties are dependent on state only.
2. Example: Work and heat.	2. All properties are point function excluding work and heat. Example: pressure, volume, Temperature
3. The total amount of both work and heat is the sum of cyclic integral of the path.	3. The total amount of any point function property is the algebraic sum of the individual property.
4. It considers the direction of processes	4. It does not consider the direction of process.
5. The Cyclic integral of path function is non-zero,	5. The cyclic integral of point function is zero.

## Heat transfer: ( $Q$ )

Heat is defined as the energy crossing the boundary of the system due to the temperature difference between system and surroundings.

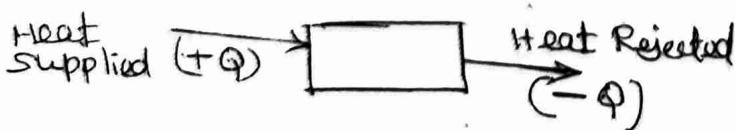
$$Q = mC(T_2 - T_1) = mC \Delta T, \text{ J or kJ}$$

$C$  - sp. heat.

### Sign convention:

If  $Q$  is positive, the heat is supplied or transferred to a system.

If  $Q$  is negative, the heat is rejected or transferred from a system.



① Work Transfer: Work is an energy interaction between a system and its surroundings, usually the energy can cross the boundary of any system.

$$\text{WORK} = \text{Force} \times \text{distance}$$

$$W = F \times x \quad \text{Nm (or) Joule.}$$

$$(or) W = P(V_2 - V_1)$$

Power: is defined as the workdone per unit time.  
unit of power  $J/s$  = watts.

### Sign convention:

- (i) If work is done by the system,  $w$  is positive.
- (ii) If work is done on the system,  $w$  is negative.



Example: Compressor, turbine, fan, heat pump, boiler, engine etc.

### Modes of work Transfer:

(i) Mechanical work.

(ii) Non mechanical work.

#### Mechanical forms of Work:

1. Shaft Work — propeller shaft.

2. Paddle Wheel Work.

3. Spring Work.

4. Elastic Solid bar Work.

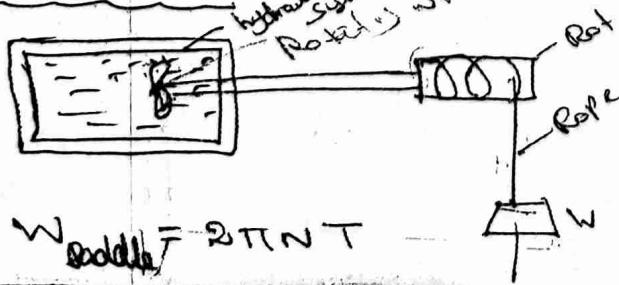
5. Gravitational Work.

#### Non mechanical work:

1. Electrical Work

2. Magnetic Work.

#### Paddle wheel work:



$$T = \frac{F}{r} \cdot x \quad r = \text{radial distance}$$

problem: Pg. 1.48  
Ex. 1.

- 1.48. A paddle wheel fixed to the shaft of a engine revolves in a closed hollow vessel containing water. Engine rpm = 650, Load applied = 60 kg at a leverage of 1.2 m. Find the paddle wheel work.

$$\text{G.D} \quad N = 650 \text{ rpm.}$$

$$W = 60 \text{ kg} = 60 \times 10 = 600 \text{ N}$$

$$\text{length } L = 1.2 \text{ m.}$$

To find: the paddle wheel work

$$W_{\text{paddle}} = ?$$

Sol:

$$W_{\text{paddle}} = 2\pi NT$$

$$T = W \times l = 600 \times 1.2 \\ = 720 \text{ N-m.}$$

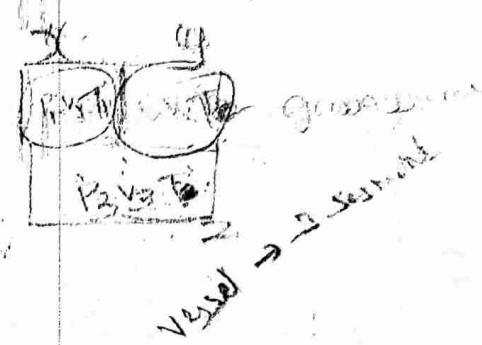
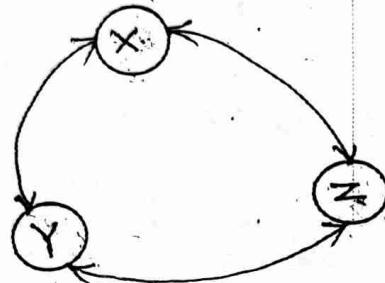
$$W_{\text{paddle}} = 2\pi \times 650 \times 720 \\ = 2940530 \text{ J.}$$

$$W_{\text{paddle}} = 2940.53 \text{ KJ}$$

## ② The law of Thermodynamics:

- \* Zeroth law of Thermodynamics
- \* First law of Thermodynamics
- \* Second law of Thermodynamics.

Zeroth law of Thermodynamics: States that "When two bodies are in thermal equilibrium with a third body separately, they are also in thermal equilibrium with each other."



First LAW OF THERMODYNAMICS: States that "when a system undergoes a cyclic process then the net heat transfer is equal to the net work transfer."

$$\oint dQ = \oint dW \quad \Rightarrow Q = \sum W$$

It may be stated "Heat and Work are mutually convertible."  $Q = W$  For cyclic process.

According to the LAW OF CONSERVATION OF ENERGY:

"Energy may be neither created nor destroyed but it can be transferred from one form to another form."

For any thermodynamic System

The First law of thermodynamics can be written in the form,

Heat transfer = Work done + Change in Internal energy

$$Q = W + \Delta U$$

Limitations of First law of thermodynamics:

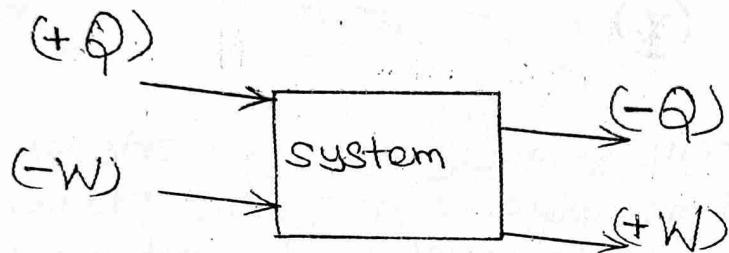
1. First law of thermodynamics does not specify the direction of flow of heat and work, i.e. whether the heat flows from hot body to a cold body or from a cold body to hot body.

2. The heat and work are mutually convertible. The work can be converted fully into heat energy but the heat energy cannot be converted fully into mechanical work.

### PMM-1 (Perpetual Motion Machine):

A machine which violates the first law of thermodynamics is known as PMM-1. This kind of machine is impossible.

### Sign Convention:



### Characteristic gas equation:

The general gas equation for Ideal gas

$$\frac{PV}{T} = \text{Constant}$$

$$\frac{PV}{T} = R$$

$$PV = RT$$

P - be the pressure

V - be the volume

T - be the Temperature in K

R - be the Gas constant.

If we consider mass "m"       $R = 0.287 \text{ kJ/kg K}$ .  
then the equation becomes.

$$PV = mRT$$

### Ideal gas eqn.

$$\left[ \frac{PV}{T} = C ; \frac{P_1 V_1}{T_1} = \frac{P_2 V_2}{T_2} \right] = \frac{P_2 V_2}{T_2} - - - - -$$

1. A closed system receives an input heat of 450 kJ and increases the internal energy of the system for 325 kJ. Determine work done by the system.

Given data:

$$Q = 450 \text{ kJ}$$

$$\Delta U = 325 \text{ kJ}$$

To find:

$$W = ?$$

Solution:

$$Q = W + \Delta U$$

$$W = Q - \Delta U = 450 - 325$$

$$W = 125 \text{ kJ}$$

2. During the compression stroke of reciprocating compressor, the work done to the air in the cylinder is 95 kJ/kg and 43 kJ/kg of heat is rejected to the surroundings. Determine the change in internal energy.

Given data:

$$W = -95 \text{ kJ/kg}$$

$$Q = -43 \text{ kJ/kg}$$

To find:

$$\text{Change in internal energy } \Delta U = ?$$

Solution:

$$Q = W + \Delta U$$

$$\Delta U = Q - W = (-43) - (-95) = -43 + 95$$

$$\boxed{\Delta U = 52 \text{ kJ/kg}}$$

3. A gas whose original pressure, volume and Temperature were 140 kN/m<sup>2</sup>, 0.1 m<sup>3</sup> and 25°C respectively. It is compressed such that new pressure is 700 kN/m<sup>2</sup> and its new temperature is 60°C. Determine the new volume of the gas.

Given Data:

$$P_1 = 140 \text{ kN/m}^2$$

$$V_1 = 0.1 \text{ m}^3$$

$$T_1 = 25 + 273 \text{ K} = 298 \text{ K}$$

$$P_2 = 700 \text{ kN/m}^2$$

$$T_2 = 60^\circ\text{C} + 273 = 333 \text{ K}$$

To find:

$$V_2 = ?$$

Solution:

From ideal gas equation

$$\frac{PV}{T} = C$$

$$\frac{P_1 V_1}{T_1} = \frac{P_2 V_2}{T_2}$$

$$\frac{140 \times 0.1}{298} = \frac{700 \times V_2}{333}$$

$$V_2 = 2.234 \times 10^{-3} \text{ m}^3$$

4. A piston and cylinder machine contains a fluid system which powers through a complete cycle of four processes. During the cycle, the sum of all heat transfers is  $-170 \text{ kJ}$ . The system completes 100 cycles per minute. Complete the following table showing the method of each item, and compute the net rate of work output in KW.

process	$Q(\text{kJ}/\text{min})$	$W(\text{kJ}/\text{min})$	$\Delta E(\text{kJ}/\text{min})$
a-b	0	-2,170	-
b-c	21,000	0	-
c-d	-2,100	-	-36,600
d-a	-	-	-

Given data:

$$\Sigma Q = -170 \text{ kJ} \text{ (For one cycle)}$$

$$\text{number of cycles/min} = 100$$

Solution: Total heat transfer for 100 cycle =  $-170 \times 100$

$$\Sigma Q = -17000 \text{ kJ/min.}$$

Solution:

Process a-b:

From First law of thermodynamics  $Q = W + \Delta E$

$$Q = W + \Delta E$$

$$0 = 2170 + \Delta E$$

$$\Delta E = -2170 \text{ kJ/min}$$

Process b-c:

$$Q = W + \Delta E$$

$$21,000 = 0 + \Delta E$$

$$\Delta E = 21,000 \text{ kJ/min}$$

Process C-d:  $Q = \dot{W} + \Delta E$

$$-2100 = W + (-36600)$$

$$-2100 = W - 36600$$

$$W = -2100 + 36600$$

$$W = 34500 \text{ kJ/min}$$

process d-a:

$$\Sigma Q = Q_{a-b} + Q_{b-c} + Q_{c-d} + Q_{d-a}$$

$$-17000 = 0 + 21000 - 2100 + Q_{d-a}$$

$$Q_{d-a} = -35900 \text{ kJ/min}$$

We know that in Cyclic process

$$\Sigma Q = \Sigma W = -17000$$

$$-17000 = W_{a-b} + W_{b-c} + W_{c-d} + W_{d-a}$$

$$-17000 = 2170 + 0 + 34500 + W_{d-a}$$

$$W_{d-a} = -53670 \text{ kJ/min}$$

$$Q_{d-a} = W_{d-a} + \Delta E_{d-a}$$

$$-35900 = -53670 + \Delta E$$

$$\Delta E = 17770 \text{ kJ/min}$$

Net rate of work output,  $\Sigma W$  in KW

$$\Sigma W = -17000 \text{ kJ/min} = -\frac{17000}{60} \text{ KJ/sec}$$

$$\Sigma W = -283.3 \text{ KW}$$

KJ/sec

1 J =  $10^3$  KJ

Result:

process	Q (kJ/min)	W (kJ/min)	$\Delta E$ (kJ/min)
a-b	0	2170	-2170 ✓
b-c	21000	0	21000
c-d	-2100	34500	+36600
d-a	-35900	-53670	17770



KW

for

25 People attended a farewell party in a small Room of size 10x8m and has a 5m ceiling. Each person gives up about 350KJ of heat per hour. Assuming that the Room is completely sealed off and insulated. Calculate the air temperature rise occurring within 10 minutes. Assume  $C_v$  of air = 0.718 KJ/kg K and  $R = 0.287 \text{ KJ/kgK}$  and each person occupies a volume of  $0.05 \text{ m}^3$ .  
G.I.D (Assume Air  $P = 1.01325 \text{ bar}$  and  $T = 20^\circ\text{C}$ )

No of Person	= 25
Room size	= $10 \times 8 \text{ m}$
Ceiling height	= 5 m
Heat / Person	= 350 KJ/hr
Time $t$	= 10 min.
$C_v$	= 0.718 KJ/kg K
$R$	= 0.287 KJ/kgK

$$1 \text{ bar} = 10^5 \text{ N/m}^2$$

$$\text{Each person Volume } V_p = 0.05 \text{ m}^3$$

$$\text{To find: } P = 1.013 \text{ bar} = 101.3 \text{ KN/m}^2$$

$$T = 20^\circ\text{C} + 273 = 293 \text{ K}$$

$$\text{Air Temperature rise, } (\Delta T)_{10 \text{ min.}}$$

Solution:

$$\text{Heat gained by air } Q = m C_v \Delta T$$

$$\checkmark m, \checkmark Q$$

$m$  - mass of air

$$PV = mRT$$

$$m = \frac{PV_a}{RT}$$

$V_a$  - Volume of air.

nk

/m<sup>2</sup>)

area

$$\text{Volume of air} = V_r - V_p$$

$$\text{Volume of room} = V_r = 10 \times 8 \times 5 = 400 \text{ m}^3$$

$$\text{Volume of person } V_p = 25 \times 0.05 = 1.25 \text{ m}^3$$

system  
(g)  
at  
s/kg K)

$$\text{Volume of air} V_a = V_r - V_p = 400 - 1.25 = 398.75 \text{ m}^3$$

$$\text{mass of air } m = \frac{P V_a}{R T} = \frac{101.3 \times 398.75}{0.287 \times 293} = \frac{40393.375}{84.091} = 480.35 \text{ kg}$$

Q, Heat gained by air = ?  
From the First law of thermodynamics.

$$Q = W + \Delta U$$

$W = 0$  (constant volume process)

$$Q = \Delta U$$

Internal Energy

$$W = P(V_2 - V_1) = 0$$

$Q = \text{Internal Energy}$

$$Q = 950 \times 2.5 = 8750 \text{ kJ/hr.}$$

$$= \frac{8750}{60} \text{ kJ/min} = 145.83 \text{ kJ/min}$$

Heat gained in 10 min ( $Q = 145.83 \times 10 = 1458.3 \text{ kJ}$ )

$$Q = m C_v (\Delta T)_{\text{min}}$$

$$1458.3 = 480.3 \times 0.718 \times (\Delta T)$$

$$1458.3 = 344.855 \Delta T$$

$$\boxed{\Delta T = 4.22^\circ\text{C}}$$

## First law of Thermodynamics - Application to closed system (Non flow process)

### Non flow process:

In a non flow process, the same working fluid is recirculated again and again. It does not leave the system after doing the work.

The non flow processes are

- ① Constant Volume (or) Iso-Choric process.
- ② Constant pressure (or) Iso-baric process.
- ③ Constant temperature (or) Isothermal process.
- ④ Hyperbolic process.
- ⑤ Reversible Adiabatic (or) Isentropic process
- ⑥ Polytropic process
- ⑦ Free Expansion process
- ⑧ Throttling process.



### Important Terms:

#### ① Work (W):

For any non-flow process, the mechanical work is mathematically given by

$$W = \int p \cdot dv$$

$p$  - pressure ( $N/m^2$ )  
 $v$  - volume ( $m^3$ ).

#### ② Internal energy: (U)

It is defined as the heat energy stored in the gas at a given temperature.

$$U = mC_v T$$

$$\Delta U = mC_v \Delta T$$

where,  $m$  be the mass of the system  
 $\text{in}(kg)$

$C_v$  - Specific heat Capacity at constant volume ( $J/Kg K$ )

$\Delta T$  - change in Temperature ( $K$ )

$$\Delta U = mC_v (T_2 - T_1)$$

$T_2$  - Final Temp.

$T_1$  - Initial Temp.

## Enthalpy (H):

It is defined as the total heat of the substance which is equal to the sum of internal energy (U) and product of pressure and volume.

$$\text{Enthalpy } H = U + PV$$

For unit mass, it can be written as

$$\text{Specific enthalpy } h = U + PV$$

Enthalpy is also function of temperature

$$\text{Enthalpy } H = mC_p T$$

$$\text{Change in Enthalpy, } \Delta H = mC_p \Delta T \quad (\because \Delta T = T_2 - T_1)$$

## Entropy (S):

It is defined as the quantitative measure of the amount of thermal energy not available to do work.

Loss of available energy is called entropy.

$$ds = \frac{dq}{T}$$

Q - heat transfer

T - Temp. in (K)

Work obtained from the system called available energy.

## Note:

- Area under P-V diagram represents the Workdone. Area under T-S diagram represent the heat transfer during the process.

## Ideal gas / Real gas

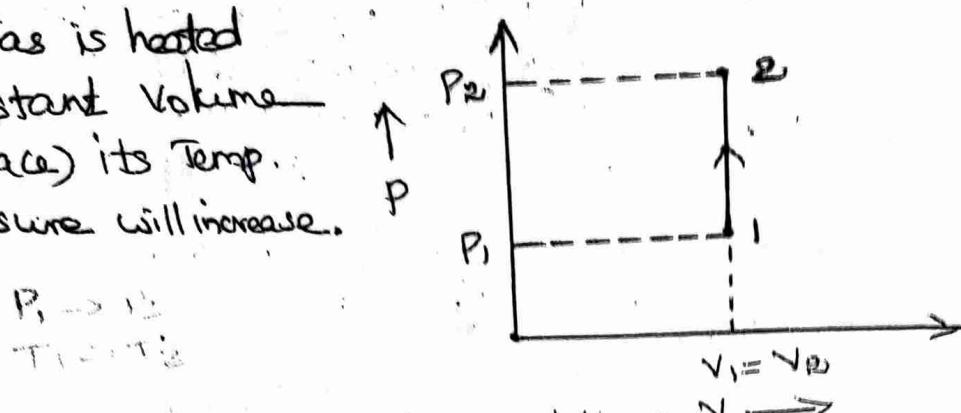
A gas which obeys the gas laws that gas is called ideal gas. It is the imaginary gas.

## Real gas:

A gas which does not obey the gas laws that gas is called a Real gas. All the gases are real gases.

## Constant Volume Process (or) Iso-choric Process:

When a gas is heated at a constant volume (i.e. fixed space) its Temp. and pressure will increase.



(i) Relation between  $P, V$  and  $T$ : Relation

From characteristic gas equation,

$$\frac{PV}{T} = C$$

$$\frac{P_1 V_1}{T_1} = \frac{P_2 V_2}{T_2}$$

In this process  $V_1 = V_2$ , const. volume.

$$\frac{P_1}{T_1} = \frac{P_2}{T_2}$$

(or)

$$\frac{P_1}{P_2} = \frac{T_1}{T_2}$$

(ii) Work done ( $W$ ):

Work done during any process is given by

$$W = \int p \cdot dV = p \int dV = p [V_2 - V_1] = p [0]$$

$$W = 0$$

$$p[V]$$

(iii) Change in internal energy ( $\Delta U$ )

$$\Delta U = mC_V (T_2 - T_1).$$

(iv) Change in Enthalpy ( $\Delta H$ ):

$$\Delta H = mC_p (T_2 - T_1).$$

(v) Heat transfer ( $Q$ )

$$Q = W + \Delta U$$

$$Q = 0 + \Delta U$$

$$Q = mC_V (T_2 - T_1)$$

## ② Constant Pressure (or) Iso-baric process:

When a gas is heated at constant pressure, both Volume and Temp. will increase.

$$V_1 \rightarrow V_2$$

### (i) PVT relation

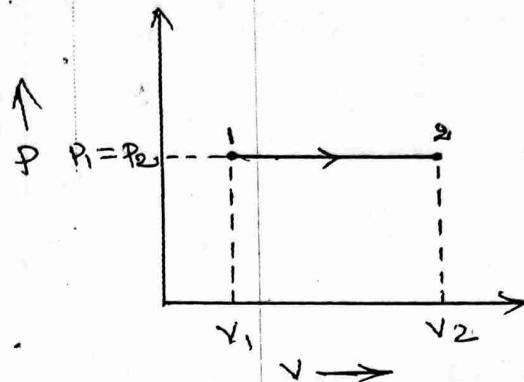
$$\frac{PV}{T} = C$$

$$\frac{P_1 V_1}{T_1} = \frac{P_2 V_2}{T_2}$$

$$P_1 = P_2 \quad (\because \text{pressure} = \text{constant})$$

$$\frac{V_1}{T_1} = \frac{V_2}{T_2}$$

$$(ii) \frac{V_1}{V_2} = \frac{T_1}{T_2}$$



### (ii) Work done (W):

$$W = \int_p dV$$

$$= \int_{V_1}^{V_2} p dV = p [V]_{V_1}^{V_2}$$

$$W = p [V_2 - V_1]$$

$$W = P_2 V_2 - P_1 V_1$$

$$W = mRT_2 - mRT_1$$

$$\therefore PV = mRT$$

$$W = mR(T_2 - T_1)$$

### (iii) Change in Internal energy ( $\Delta U$ ):

$$\Delta U = mC_V(T_2 - T_1)$$

### (iv) Change in Enthalpy ( $\Delta H$ ):

$$\Delta H = mC_P(T_2 - T_1)$$

### (v) Heat Transferred (Q):

$$Q = W + \Delta U$$

$$= mR(T_2 - T_1) + mC_V(T_2 - T_1)$$

$$= m(C_P - C_V)(T_2 - T_1) + mC_V(T_2 - T_1)$$

$$= m[C_P T_2 - C_P T_1 - C_V T_2 + C_V T_1] + m[C_V T_2 - C_V T_1]$$

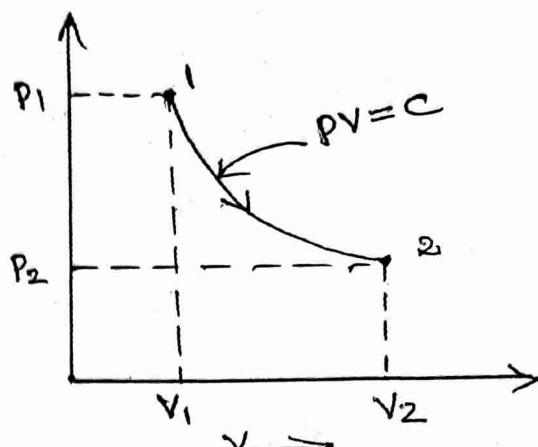
$$= mC_P T_2 - mC_P T_1 - mC_V T_2 + mC_V T_1 + mC_V T_2 - mC_V T_1$$

$$Q = mC_P(T_2 - T_1)$$

### ③ Constant Temperature (or) Isothermal Process:

When A gas is ~~isothermal~~ expanded in a vessel from  $V_1$  to  $V_2$ .

~~T = C~~  
It's Volume is now  
 $N \rightarrow V_2$  &  $P \rightarrow P_2$   
 $PV = C$



In this process, the temperature of the working substance remains constant, the pressure decreases and the volume increases. Hence there is no change in temperature. There is no change in internal energy and enthalpy because both the function of Temperature. And Isothermal process is a very slow process.

#### (i) PVT relations:

$$\frac{PV}{T} = C$$

$$\frac{P_1 V_1}{T_1} = \frac{P_2 V_2}{T_2}$$

$$\boxed{T_1 = T_2}$$

$$P_1 V_1 = P_2 V_2$$

$$\boxed{\frac{P_1}{P_2} = \frac{V_2}{V_1}}$$

#### (ii) Work done (W):

$$W = \int_{V_1}^{V_2} P \cdot dV$$

From Characteristics gas eqn.

$$\frac{PV}{T} = C$$

$$\frac{P_1 V_1}{T_1} = \frac{P_2 V_2}{T_2}; \quad P_1 V_1 = P_2 V_2$$

$$PV = C$$

$$P = \frac{C}{V}$$

$$PV = C.$$

Substituting the value of "P" in eqn ①.

$$W = \int_{V_1}^{V_2} \frac{C}{V} dV = C \int_{V_1}^{V_2} \frac{dV}{V}$$

$$\therefore \int \frac{1}{x} dx = \ln x$$

$$W = C [\ln V_2]^2$$

$$W = C [\ln V_2 - \ln V_1]$$

$$W = C \ln \frac{V_2}{V_1}$$

$$W = PV \ln \frac{V_2}{V_1}$$

$$W = mRT_1 \ln \left( \frac{V_2}{V_1} \right)$$

$$\text{or } P_1 V_1$$

$$P_1 V_1 \ln \left( \frac{V_2}{V_1} \right)$$

$$P_1 V_1 = C$$

$$P_1 V_1 = P_2 V_2$$

(iii) Change in internal energy ( $\Delta U$ ):

$$\Delta U = m C_v (T_2 - T_1)$$

$$\Delta U = 0$$

$$\therefore T_1 = T_2$$

(iv) Change in Enthalpy ( $\Delta H$ ):

$$\Delta H = m C_p (T_2 - T_1)$$

$$\Delta H = 0$$

$$\therefore T_1 = T_2$$

(v) Heat transfer ( $Q$ ):

$$Q = W + \Delta U$$

$$Q = W$$

$$\therefore \Delta U = 0$$

$$Q = P_1 V_1 \ln \left( \frac{V_2}{V_1} \right)$$

$$Q = m R T_1 \ln \left( \frac{V_2}{V_1} \right)$$

$$\log Q = \frac{T}{\Delta U}$$

$$\log Q = \frac{1}{\Delta U}$$

## ② Hyperbolic process

The hyperbolic process follows the law of  $PV = C$ . This process is identical to isothermal process.

Isothermal process  $\rightarrow$  applicable to gases only

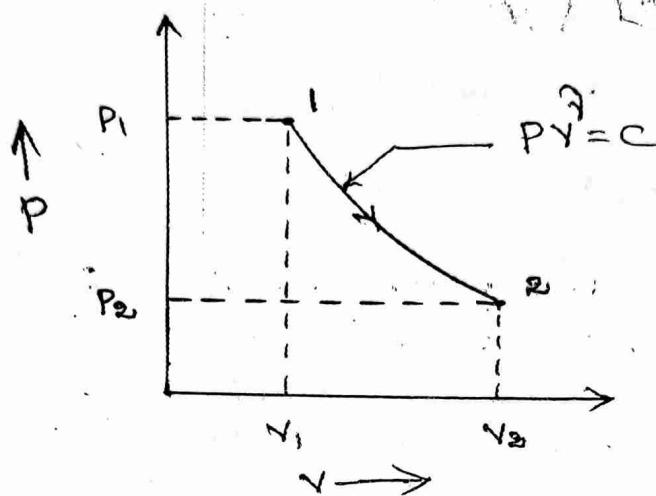
Hyperbolic process  $\rightarrow$  applicable to vapors

⑤ Reversible Adiabatic or Isentropic process  
 This process follows the law  $PV^{\gamma} = \text{constant}$ .

where

$\gamma$  - Adiabatic index

Adiabatic process is the process in which no heat transfer takes place. ( $Q=0$ )



### ① PVT Relation:

The relationship between "P" and "V"

From adiabatic eqn

$$PV^{\gamma} = C$$

$$P_1 V_1^{\gamma} = P_2 V_2^{\gamma}$$

$$\frac{P_2}{P_1} = \left(\frac{V_1}{V_2}\right)^{\gamma}$$

T,V-relation

$$\frac{T_2}{T_1} = \left(\frac{V_1}{V_2}\right)^{\gamma-1}$$

T,P-relation:

$$\frac{T_2}{T_1} = \left(\frac{P_2}{P_1}\right)^{\frac{1}{\gamma}}$$

Work done (W):

$$W = \int_{1}^{2} P \cdot dV$$

$$W = \int_{1}^{2} \frac{C}{V^{\gamma}} \cdot dV$$

$$= C \int_{1}^{2} V^{-\gamma} dV$$

$$= C \left[ \frac{V^{1-\gamma}}{1-\gamma} \right]_1^2$$

$$PV^{\gamma} = C$$

$$P = \frac{C}{V^{\gamma}}$$

$$\int x^{\alpha} dx = \frac{x^{\alpha+1}}{\alpha+1}$$

$$\frac{PV}{T} = C \quad \text{--- ①}$$

$$\frac{PV}{T^{\gamma}} = C \quad \text{--- ②}$$

Divide the eqn ① by ②

$$\frac{PV^{\gamma}}{PV} = C$$

$$\frac{V^{\gamma}}{V} = C$$

$$V^{\gamma-1} = C$$

$$V^{\gamma-1} T = C$$

$$V^{\gamma-1} T_1 = V_2^{\gamma-1} T_2$$

$$\frac{T_2}{T_1} = \left(\frac{V_1}{V_2}\right)^{\gamma-1}$$

$$= \frac{C}{\gamma-1} \left[ V^{\gamma+1} \right]_1^\gamma = \frac{C}{\gamma-1} \left[ V_2^{\gamma+1} - V_1^{\gamma+1} \right]$$

$$= \frac{C V_2^{\gamma+1} - C V_1^{\gamma+1}}{\gamma-1} \quad \therefore P V^\gamma = C$$

$$= \frac{P_2 V_2^{\gamma} V_2^{-\gamma+1} - P_1 V_1^{\gamma} V_1^{-\gamma+1}}{\gamma-1} = \frac{P_2 V_2 - P_1 V_1}{\gamma-1}$$

$$W = \frac{P_1 V_1 - P_2 V_2}{\gamma-1}$$

$$\gamma-\gamma+1 = 1$$

(OK).

$$W = \frac{m R (T_2 - T_1)}{\gamma-1}$$

$$\therefore Q = 0$$

$$Q = W + \Delta U$$

$$Q = W + \Delta U$$

$$\Delta U = -W = - \left[ \frac{P_1 V_1 - P_2 V_2}{\gamma-1} \right]$$

$$\Delta U = \frac{P_2 V_2 - P_1 V_1}{\gamma-1}$$

(OK)

$$\Delta U = m C_v (T_2 - T_1)$$

(iv) Change in Enthalpy  $\Delta H$ :

$$\Delta H = m C_p (T_2 - T_1)$$

(v) Heat transfer  $Q$ :

$$Q = 0$$



=

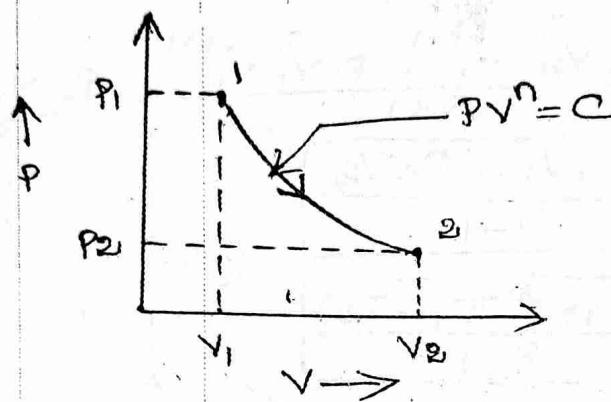
## ⑥ Poly Tropic process

This process follows the law  $PV^n = \text{Constant}$ ,

where

n - polytropic index

n = zero to infinite.



### (i) P-V-T relation.

In the same manner as already derived  
in the P-V relation: Adiabatic process.

$$P_1 V_1^n = C$$

$$P_1 V_1^n = P_2 V_2^n$$

P-V relation

$$\frac{P_2}{P_1} = \left[ \frac{V_1}{V_2} \right]^n$$

### T-V-relation

$$\frac{T_2}{T_1} = \left[ \frac{V_1}{V_2} \right]^{n-1}$$

### T-P-relation

$$\frac{P_2}{T_1} = \left[ \frac{P_2}{P_1} \right]^{\frac{n-1}{n}}$$

### This work done: (W)

$$W = \int p dV$$

$$= \int_{V_1}^{V_2} \frac{C}{V^n} dV$$

$$= C \int_{V_1}^{V_2} \frac{1}{V^n} dV = C \int_{V_1}^{V_2} V^{-n} dV$$

$$= C \left[ \frac{V^{-n+1}}{-n+1} \right]_{V_1}^{V_2} = \frac{C}{-n+1} \left[ V_2^{-n+1} - V_1^{-n+1} \right]$$

$$W = \frac{P V^n}{-n+1} \left[ \frac{-n+1}{V_2} - \frac{-n+1}{V_1} \right]$$

$$\therefore PV^n = C$$

$$P = \frac{C}{V^n}$$

$$W = \frac{P_2 V_2^{\gamma} - P_1 V_1^{\gamma}}{\gamma - 1} = \frac{P_2 V_2 - P_1 V_1}{\gamma - 1}$$

$$W = \frac{P_1 V_1 - P_2 V_2}{\gamma - 1}$$

(or)

$$W = \frac{m R (T_2 - T_1)}{\gamma - 1}$$

$$\frac{m R (T_2 - T_1)}{1 - \gamma}$$

(iii) Change in internal energy ( $\Delta U$ ):

$$\Delta U = m C_V (T_2 - T_1) \quad Q \neq 0$$

(iv) Change in Enthalpy:  $\Delta H$

$$\Delta H = m C_P (T_2 - T_1)$$

(v) Heat transfer ( $Q$ ):

$$\begin{aligned} Q &= W + \Delta U \\ &= \frac{P_1 V_1 - P_2 V_2}{\gamma - 1} + m C_V (T_2 - T_1) \\ &= \frac{P_2 V_2 - P_1 V_1}{1 - \gamma} + m C_V (T_2 - T_1) \\ &= \frac{m R T_2 - m R T_1}{1 - \gamma} + m C_V (T_2 - T_1) \\ &= \frac{m R (T_2 - T_1)}{1 - \gamma} + m \frac{R}{\gamma - 1} (T_2 - T_1) \\ &= m R (T_2 - T_1) \left[ \frac{1}{1 - \gamma} + \frac{1}{\gamma - 1} \right] \\ &= m R (T_2 - T_1) \left[ \frac{(\gamma - 1) + (1 - \gamma)}{(1 - \gamma)(\gamma - 1)} \right] \\ &= m R (T_2 - T_1) \left[ \frac{1 - \gamma + 1 - \gamma}{(1 - \gamma)(\gamma - 1)} \right] \\ &= m R (T_2 - T_1) \left[ \frac{2 - 2\gamma}{(1 - \gamma)(\gamma - 1)} \right] \\ &= \frac{m R (T_2 - T_1)}{1 - \gamma} \left[ \frac{2 - 2\gamma}{2 - 1} \right] \end{aligned}$$

$$Q = W \left[ \frac{2 - 2\gamma}{2 - 1} \right]$$

$$\begin{aligned} C_p - C_v &= R \quad \text{--- ①} \\ \text{case } &\frac{C_p}{C_v} = \gamma \quad \text{--- ②} \\ C_p &= \gamma C_v \\ \gamma C_v - C_v &= R \\ C_v (\gamma - 1) &= R \\ C_v &= \frac{R}{\gamma - 1} \end{aligned}$$

### Value of "n"

$$P_1 V_1^n = C$$

$$P_2 V_2^n = C$$

Taking logarithms on both sides,

$$\log P_1 V_1^n = \log P_2 V_2^n$$

$$\log_{10} P_1 + n \log_{10} V_1 = \log_{10} P_2 + n \log_{10} V_2$$

$$n \log_{10} V_1 - n \log_{10} V_2 = \log_{10} P_2 - \log_{10} P_1$$

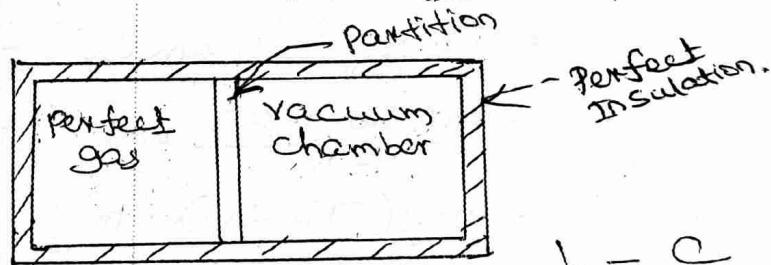
$$n \log_{10} \left( \frac{V_1}{V_2} \right) = \log_{10} \left( \frac{P_2}{P_1} \right)$$

$$n = \frac{\log_{10} \left( \frac{P_2}{P_1} \right)}{\log_{10} \left( \frac{V_1}{V_2} \right)}$$

This same term for Adiabatic index  
 $\gamma = \frac{\log(P_2/V_2)}{\log(V_1/P_1)}$

### Free Expansion process

If a fluid is allowed to expand suddenly into a vacuum chamber through an orifice of large dimension, it is known as free expansion process.



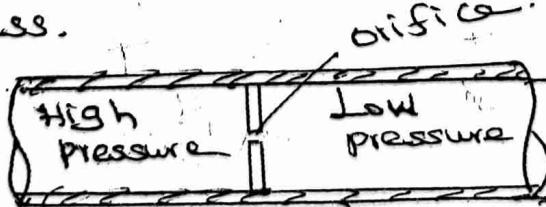
Work done  $W = 0 \checkmark$

Heat transfer  $Q = 0 \checkmark$

Change in  $\Delta U = 0$

$$h = c$$

### Throttling process: When a fluid expands through a minute orifice, the process is called as throttling process.



Work done  $W = 0 \checkmark$

Heat transfer  $Q = 0 \checkmark$

Change in  $\Delta U = 0$

Perfect insulation.

Process	Work done (W) kJ	Change in Internal Energy ( $\Delta U$ ) kJ	Change in Enthalpy (AH) kJ	Heat transfer (Q) kJ	P, V, T Relations
Constant Volume process $V = C$	zero	$mC_V(T_2 - T_1)$	$mC_V(T_2 - T_1)$	$mC_V(T_2 - T_1)$	$\frac{P_1}{T_1} = \frac{P_2}{T_2}$
Constant Pressure process $P = C$	$P_2V_2 - P_1V_1$ (or) $mR(T_2 - T_1)$	$mC_V(T_2 - T_1)$	$mC_p(T_2 - T_1)$	$mC_p(T_2 - T_1)$	$\frac{V_1}{T_1} = \frac{V_2}{T_2}$
Const. Temperature (or) Isothermal process $T = C$	$P_1V_1 \ln\left(\frac{V_2}{V_1}\right)$ (or) $mRT_1 \ln\left(\frac{V_2}{V_1}\right)$	zero	zero	$P_2V_2 \ln\left(\frac{V_2}{V_1}\right)$ (or) $mRT_2 \ln\left(\frac{V_2}{V_1}\right)$	$P_1V_1 = P_2V_2$
Reversible Adiabatic Process Isentropic Process $PV^\gamma = C$	$\frac{P_1V_1 - P_2V_2}{\gamma - 1}$ (or) $\frac{mR(T_1 - T_2)}{\gamma - 1}$	$\frac{P_2V_2 - P_1V_1}{\gamma - 1}$ (or) $\frac{mR(T_2 - T_1)}{\gamma - 1}$	$mC_p(T_2 - T_1)$	zero	$\frac{P_2}{P_1} = \left(\frac{V_1}{V_2}\right)^{\gamma}$ $\frac{T_2}{T_1} = \left(\frac{V_1}{V_2}\right)^{\gamma - 1}$ $\frac{T_2}{T_1} = \left(\frac{P_2}{P_1}\right)^{\frac{1}{\gamma}}$
Polytropic Process $PV^n = C$	$\frac{P_1V_1 - P_2V_2}{n-1}$ (or) $\frac{mR(T_1 - T_2)}{n-1}$	$mC_V(T_2 - T_1)$	$mC_p(T_2 - T_1)$	$W \propto \frac{\gamma - n}{\gamma - 1}$	$\frac{P_2}{P_1} = \left(\frac{V_1}{V_2}\right)^n$ $\frac{T_2}{T_1} = \left(\frac{V_1}{V_2}\right)^{n-1}$ $\frac{T_2}{T_1} = \left(\frac{P_2}{P_1}\right)^{\frac{n-1}{n}}$

- ① A certain gas of volume  $0.4 \text{ m}^3$ , pressure of  $4.5 \text{ bar}$  and temperature of  $130^\circ\text{C}$ , is heated in a cylinder to  $9 \text{ bar}$ , when the volume remains constant. Calculate  
 (i) temp. at the end of process. (ii) the heat transfer  
 (iii) Change in internal energy (iv) Work done by the gas  
 (v) Change in enthalpy and (vi) mass of the gas.  
 Assume  $C_p = 1.005 \text{ kJ/kgK}$  and  $C_v = 0.718 \text{ kJ/kgK}$ .

Given data:

$$V_1 = 0.4 \text{ m}^3$$

$$C_p = 1.005 \text{ kJ/kgK}$$

$$C_v = 0.718 \text{ kJ/kgK}$$

$$P_1 = 4.5 \text{ bar} = 4.5 \times 10^5 \text{ N/m}^2$$

$$= 450 \times 10^3 \text{ N/m}^2$$

$$T_1 = 130^\circ\text{C} + 273 = 403 \text{ K}$$

$$= 450 \text{ KN/m}^2$$

$$1 \text{ bar} = 10^5 \text{ N/m}^2$$

$$P_2 = 9 \text{ bar} = 900 \text{ KN/m}^2$$

$V_1 = V_2$  (constant volume process)

To find:

- (i)  $T_2$  (ii)  $Q$  (iii)  $\Delta U$  (iv)  $W$  (v)  $\Delta H$  (vi)  $m$ .

Solutions: const. vol. process ( $V_1 = V_2$ )

(i) Temp. at the end of process,  $T_2 = ?$

W.K.T

$$\frac{P_1 V_1}{T_1} = C$$

$$\frac{P_1 V_1}{T_1} = \frac{P_2 V_2}{T_2}$$

$$\frac{P_1}{T_1} = \frac{P_2}{T_2}$$

$$\Rightarrow V_1 = V_2$$

$$T_2 = \frac{P_2}{P_1} \times T_1 = \frac{900}{450} \times 403 = 806 \text{ K}$$

$$T_2 = 806 \text{ K}$$

(ii) Heat transfer  $Q$ :

$$Q = W + \Delta U$$

$$Q = m C_v (T_2 - T_1)$$

$$Q = 1.56 (806 - 403)$$

$$Q = 451.39 \text{ kJ}$$

$$\text{kg} \times \text{kJ} \times \text{K}$$

$$P_1 V_1 = m R T_1$$

$$m = \frac{P_1 V_1}{R T_1}$$

$$R = C_p - C_v$$

$$m = \frac{P_1 V_1}{(C_p - C_v) T_1}$$

$$= 450 \times 0.4$$

$$(1.005 - 0.718) 403$$

$$m = 1.56 \text{ kg}$$

(iii) Change in internal energy  $\Delta U$ :

$$\Delta U = m C_v (T_2 - T_1) \quad (or)$$

$$Q = \Delta U = 451.39 \text{ kJ}$$

$$Q = W + \Delta U$$

$$Q = Q + \Delta U$$

$$Q = \Delta U$$

(iv) Work done:  $w = \int_{V_1}^{V_2} P dV = P(V_2 - V_1)$   
 $= 0$   $V_1 = V_2$

$w = 0$

(v) Change in enthalpy  $\Delta H$ :

$$\begin{aligned}\Delta H &= m C_p (T_2 - T_1) \quad (103) \\ &= 1.56 \times 1.005 (806 - 403) \\ \Delta H &= 631.88 \text{ kJ} = \underline{\underline{450 \text{ kJ}}}\end{aligned}$$

(vi) mass of the gas: (m)

$m = 1.56 \text{ kg}$

8. 5 kg of air at 40°C and 1 bar, is heated in a reversible non-flow constant pressure until the volume is doubled. Find (a) change in volume (b) work done (c) change in internal energy and (d) change in enthalpy.

Given data:

$m = 5 \text{ kg}$

$T_1 = 40^\circ\text{C} + 273 = 313 \text{ K}$

$P_1 = 1 \text{ bar} = 1 \times 10^5 \text{ N/m}^2 = 100 \text{ kN/m}^2$

$P_1 = P_2 \text{ (const. pressure)}$

$V_2 = 2V_1$

To find: (a)  $V_2 - V_1$  (b)  $w$  (c)  $\Delta U$ , (d)  $\Delta H$ .

Solution: Constant pressure process. ( $P_1 = P_2$ )

(a) Change in volume:

From ideal gas eqn.

$P_1 V_1 = m R T_1$

$V_1 = \frac{m R T_1}{P_1} = \frac{5 \times 0.287 \times 313}{100}$

$V_1 = 4.49 \text{ m}^3$

$V_2 = 2V_1 = 2 \times 4.49 = 8.98 \text{ m}^3$

$V_2 = 8.98 \text{ m}^3$

$V_2 - V_1 = 8.98 - 4.49$

$V_2 - V_1 = 4.49 \text{ m}^3$

$C_{P,\text{air}} = 1.005 \text{ kJ/kg K}$

$C_{V,\text{air}} = 0.718 \text{ kJ/kg K}$

$R_{\text{air}} = 0.287 \text{ kJ/kg K}$

(b) Work done: (w):

$w = \int P dV = P [V_2 - V_1]$

$= 100 [8.98 - 4.49]$

$w = 449 \text{ kJ}$

#### (4) Change in Internal energy $\Delta U$ :

$$\Delta U = mC_V(T_2 - T_1)$$

w. n.s.

$$\frac{PV}{T} = C$$

$$\Delta U = 5 \times 0.718 (626 - 313)$$

$$\frac{P_1 V_1}{T_1} = \frac{P_2 V_2}{T_2}$$

$$\therefore [P_1 = P_2]$$

$$\Delta U = 1126.67 \text{ kJ}$$

$$T_2 = \frac{V_2}{V_1} \times T_1$$

$$= \frac{8.98}{4.49} \times 313$$

$$T_2 = 626 \text{ K}$$

#### (5) Change in Enthalpy $\Delta H$ :

$$\Delta H = mC_p(T_2 - T_1)$$

$$= 5 \times 1.005 (626 - 313)$$

$$\Delta H = 1572.825 \text{ kJ}$$

3.

0.25 kg of air at a pressure of 1 bar occupies a volume of 0.3 m<sup>3</sup>. If this air expands isothermally ( $PV = C$ ) to a volume of 0.9 m<sup>3</sup>. Find (i) the initial temperature (ii) the final temp. (iii) External work done (iv) heat absorbed by the air and (v) Change in internal energy. Assume  $R = 0.287 \text{ kJ/kgK}$ .

Given data:

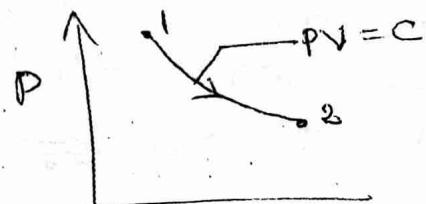
$$m = 0.25 \text{ kg}$$

$$P_1 = 1 \text{ bar} = 100 \text{ kN/m}^2$$

$$V_1 = 0.3 \text{ m}^3$$

$$V_2 = 0.9 \text{ m}^3$$

$$\text{To find: } R = 0.287 \text{ kJ/kgK}$$



$$T_1, T_2, W, Q, \Delta U$$

Solution: Isothermal process ( $PV = C$ ).

$$T = C \quad (T_1 = T_2)$$

#### (i) Initial temp (T<sub>1</sub>)

$$P_1 V_1 = mRT_1$$

$$T_1 = \frac{P_1 V_1}{mR} = \frac{100 \times 0.3}{0.25 \times 0.287}$$

$$T_1 = 418.12 \text{ K}$$

$$T_1 = T_2 = 418.12 \text{ K}$$

(iii) Work done (W)

$$W = P_1 V_1 \ln\left(\frac{V_2}{V_1}\right) \quad (\text{or} \quad mRT_1 \ln\left(\frac{V_2}{V_1}\right))$$
$$= 100 \times 0.3 \ln\left(\frac{0.9}{0.3}\right)$$
$$= 30 \ln(3)$$
$$W = 32.95 \text{ kJ}$$

(iv) Heat absorbed by the air ( $Q$ ) .

$$Q = W + \Delta U$$
$$= W + mC_v(T_2 - T_1)$$
$$= 32.95 + 0$$
$$Q = 32.95 \text{ kJ}$$

$$T_1 = T_2$$

(v) Change in Internal energy ( $\Delta U$ ) :  $mC_v(T_2 - T_1)$

$$\Delta U = 0$$

4. 10 kg of gas at 10 bar and 400°C expands ~~reversibly~~ ~~and~~ adiabatically to 1 bar. Find the work done and change in internal energy.

Given data:

$$m = 10 \text{ kg}$$

$$P_1 = 10 \text{ bar} = 1000 \text{ kN/m}^2$$

$$T_1 = 400^\circ \text{C} + 273 = 673 \text{ K}$$

$$P_2 = 1 \text{ bar} = 100 \text{ kN/m}^2$$

To find:

W and  $\Delta U$ .

Solution: (adiabatic process)

$$\text{Work done } W = \frac{P_1 V_1 - P_2 V_2}{\gamma - 1} \quad \text{or} \quad \frac{mR(T_1 - T_2)}{\gamma - 1}$$

$$W = \frac{mR(T_1 - T_2)}{\gamma - 1}$$

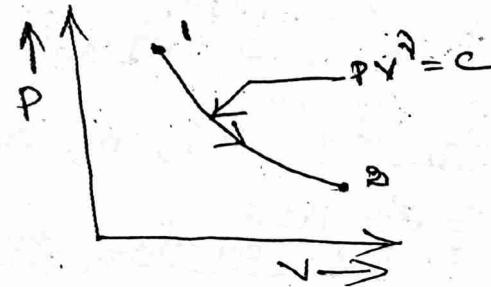
$$\frac{T_2}{T_1} = \left(\frac{P_2}{P_1}\right)^{\frac{1}{\gamma-1}}$$

$$T_2 = 673 \times \left(\frac{100}{1000}\right)^{\frac{1.4-1}{1.4-1}}$$
$$= 673 \times \left(\frac{100}{1000}\right)^{0.2857}$$
$$= 673 \times 0.51796$$

$$T_2 = 348.59 \text{ K}$$

$$W = \frac{10 \times 0.287 (673 - 348.59)}{1.4 - 1}$$

$$W = 2327.64 \text{ kJ}$$



$$\text{Assume } R = 0.287 \text{ kJ/kgK}$$

$$\gamma = 1.4$$

$$\frac{P_2}{P_1} = \left(\frac{V_1}{V_2}\right)^{\frac{1}{\gamma-1}}$$

$$\frac{T_2}{T_1} = \left(\frac{V_1}{V_2}\right)^{\frac{1}{\gamma-1}}$$

$$\frac{T_2}{T_1} = \left(\frac{P_2}{P_1}\right)^{\frac{1}{\gamma-1}}$$

$$\Delta U = ?$$

$$\Delta U = mC_V(T_2 - T_1)$$

$$= 10 \times 0.718 (348.59 - 673)$$

$$\boxed{\Delta U = -2327.74 \text{ kJ}}$$

(Ans)

$$Q = W + \Delta U$$

$$Q = W + \Delta U$$

$$\boxed{\Delta U = -W =}$$

5. A cylinder contains  $1\text{m}^3$  of gas at  $100\text{kPa}$  and  $100^\circ\text{C}$ , the gas is polytropically compressed to a volume of  $0.25\text{m}^3$ . The final pressure is  $600\text{kPa}$ . Determine (a) mass of the gas (b) the value of index "n" (c) change in internal energy (d) heat transferred by the gas during compression.

Assume  $R = 0.287 \text{ kJ/kgK}$ ,  $\gamma = 1.4$ .

Given data:

$$V_1 = 1\text{m}^3$$

$$P_1 = 100\text{kPa} = 100\text{kN/m}^2$$

$$T_1 = 100^\circ\text{C} + 273 = 373\text{K}$$

Polytropic process  $PV^n = c$

$$V_2 = 0.25\text{m}^3$$

$$P_2 = 600\text{kPa}$$

$$R = 0.287 \text{ kJ/kgK}$$

$$\gamma = 1.4$$

To Find:  $m, n, \Delta U, Q$

Solution:

(a) mass of the gas  $m = ?$

$$P_1 V_1 = mRT_1$$

$$m = \frac{P_1 V_1}{RT_1} = \frac{100 \times 1}{0.287 \times 373} = 0.93\text{kg}$$

$$\boxed{m = 0.93\text{kg}}$$

(b) The value of index:

$$n = \frac{\log \left( \frac{P_2}{P_1} \right)}{\log \left( \frac{V_1}{V_2} \right)} = \frac{\log \left( \frac{600}{100} \right)}{\log \left( \frac{1}{0.25} \right)} = \frac{0.77815}{0.60205}$$

$$\boxed{n = 1.29}$$

? also find

$$C_V = ?$$

$$\boxed{C_P - C_V = R}$$

$$\frac{C_V - C_V}{C_V} = \frac{R}{C_V} = \gamma$$

$$C_V(\gamma - 1) = R/C_P = C_V$$

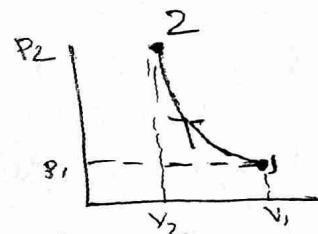
$$C_V = \frac{R}{\gamma - 1}$$

$$C_V = \frac{0.287}{1.4 - 1} = 0.718$$

$$\frac{T_2}{T_1} = \left( \frac{V_1}{V_2} \right)^{\gamma-1}$$

$$T_2 = 373 \times \left( \frac{1}{0.25} \right)^{1.29-1} = 558\text{K}$$

$$\boxed{T_2 = 558\text{K}}$$



$$\begin{aligned}\Delta U &= mC_v(T_2 - T_1) \\ &= 0.93 \times 0.718(558 - 373) \\ \boxed{\Delta U = 123.53 \text{ kJ}}\end{aligned}$$

(d) Heat transfer ( $Q$ ):

$$\begin{aligned}Q &= W \times \frac{\eta - n}{\eta - 1} \\ &= \frac{P_1 V_1 - P_2 V_2}{n-1} \times \frac{\eta - n}{\eta - 1} \quad (\text{or}) \quad \frac{mR(T_1 - T_2)}{n-1} \times \frac{\eta - n}{\eta - 1} \\ &= \frac{100 \times 1 - 600 \times 0.85}{1.29 - 1} \times \frac{1.4 - 1.29}{1.4 - 1} \\ &= -172.413 \times 0.275 \\ \boxed{Q = -47.41 \text{ kJ}}\end{aligned}$$

- ✓
6. Determine the heat transfer and its direction for a system in which a perfect gas, having molecular weight of 17.76 is compressed from 101.3 kPa, 20°C to a pressure of 600 kPa following the law  $PV^{1.3} = \text{constant}$ .   
~~Cp = 1.7 kJ/kgK~~. And also find the  $\Delta H$ .

Given data:

$$M = 17.76$$

$$P_1 = 101.3 \text{ kPa} = 101.3 \text{ kN/m}^2$$

$$T_1 = 20^\circ\text{C} + 273 = 293 \text{ K}$$

$$P_2 = 600 \text{ kPa} = 600 \text{ kN/m}^2$$

$$PV^{1.3} = C \quad (\text{polytropic flow})$$

$$n = 1.3$$

$$C_p = 1.7 \text{ kJ/kgK}$$

To find:  $Q$  and its direction (+ or -),  $\Delta H$

$$Q = W \times \frac{\eta - n}{\eta - 1}$$

$$W = \frac{P_1 V_1 - P_2 V_2}{n-1} \quad \text{or} \quad \frac{mR(T_1 - T_2)}{n-1}$$

$$W = \frac{mR(T_1 - T_2)}{n-1}$$

$$W = \frac{1 \times 0.268(293 - 441.72)}{1.3 - 1}$$

$$\boxed{W = -2.32 \text{ kJ}}$$

$$Q = -2.32 \times \frac{1.4 - 1.3}{1.4 - 1}$$

$$R = \frac{\bar{R}}{M} = \frac{8.314}{17.76} = 0.468 \text{ kJ/kgK}$$

$$\frac{T_2}{T_1} = \left(\frac{P_2}{P_1}\right)^{\frac{n-1}{n}}$$

$$T_2 = 293 \left(\frac{600}{101.3}\right)^{\frac{1.3-1}{1.3}}$$

$$\boxed{T_2 = 441.72 \text{ K}}$$

$$\frac{P_2}{P_1} \frac{T_2}{T_1}$$

$$Q = w + \frac{C_P}{C_V} \Delta T$$

$$Q = \frac{1.7}{1.232}$$

$$= 1.38$$

$$Q = w \times \frac{T_2 - T_1}{T_2 - T_1}$$

$$= -23.8 \times \frac{1.38 - 1.3}{1.38 - 1}$$

$$Q = -48.8 \text{ kJ}$$

$$R = C_P - C_V$$

$$0.468 = 1.7 - C_V$$

$$C_V = 1.7 - 0.468$$

$$C_V = 1.232 \text{ kJ/kg K}$$

The negative sign indicates that the heat rejection from the system.

$$\Delta H = m C_P (T_2 - T_1)$$

$$= 1 \times 1.7 (441 - 293)$$

$$\Delta H = 251.6 \text{ kJ}$$

7. A mass of air is initially at  $26^\circ\text{C}$  and  $100\text{kPa}$  and occupies  $0.028 \text{ m}^3$ . The air is expanded at a constant pressure to  $0.08 \text{ m}^3$ . A polytropic process with  $n = 1.5$  is then carried out, followed by a constant temperature process. All the processes are reversible.
- (a) Sketch the cycle in  $pV$  and  $Ts$  plane.
  - (b) Find the heat received and heat rejected in the cycle.
  - (c) Find the efficiency of the cycle.

Given data:

$$T_1 = 26^\circ\text{C} + 273 = 533 \text{ K}$$

$$P_1 = 100 \text{ kPa} = 100 \text{ KN/m}^2 = P_2$$

$$V_1 = 0.028 \text{ m}^3$$

$$V_2 = 0.08 \text{ m}^3$$

$$n = 1.5$$

Process 1-2 is const. pressure.

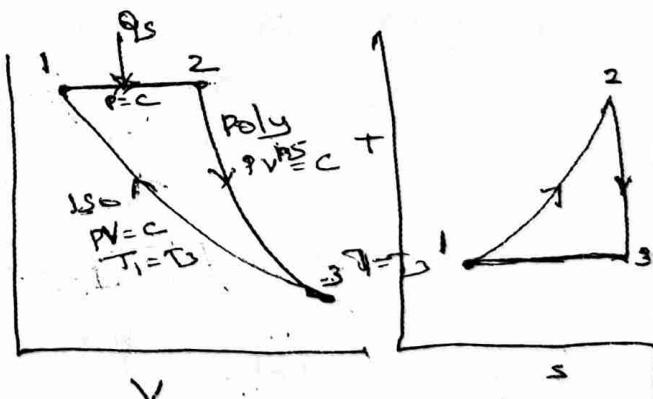
Process 2-3 is polytropic.

Process 3-1 is const. temp.

To find:  $Q$  and  $\eta$ .

$Q_S$  and  $Q_R$

$$\eta = \frac{W}{Q_S} = \frac{W_{1-2} + W_{2-3} + W_{3-1}}{Q_S}$$



Solution

Process 1-2: constant pressure process ( $P_1 = P_2 = P$ )

$$W_{1-2} = P(V_2 - V_1) = 700(0.084 - 0.028) = \underline{39.2 \text{ kJ}}$$

$$Q_{1-2} = mC_p(T_2 - T_1)$$

$$T_1, T_2 = ?$$

$$\text{W.K.J. } PV = mRT$$

$$P_1V_1 = mRT_1$$

$$m = \frac{P_1V_1}{RT_1} = \frac{700 \times 0.028}{0.287 \times 533}$$

$$m = 0.128 \text{ kg}$$

air

$$C_p = 1.005 \text{ kJ/kg}$$
$$C_v = 0.718 \text{ "}$$
$$R = 0.287 \text{ "}$$
$$\gamma = 1.4$$

W.K.T.

$$\frac{PV}{T} = C$$

$$\frac{P_1V_1}{T_1} = \frac{P_2V_2}{T_2}$$

$$\frac{V_1}{T_1} = \frac{V_2}{T_2}$$

$$T_2 = T_1 \times \frac{V_2}{V_1} = 533 \times \frac{0.084}{0.028}$$

$$T_2 = 1599 \text{ K}$$

$$P_1 = P_2$$

$$Q_{1-2} = mC_p(T_2 - T_1) = 0.128 \times 1.005 (1599 - 533)$$

$$Q_{1-2} = 137.13 \text{ kJ}$$

Process 2-3: Polytropic process

$$V_{2-3} = \frac{P_2V_2 - P_3V_3}{n-1}$$

$$P_1 = P_2$$

$$Pv^n = C$$
$$n = 1.5$$

$$P_1 = P_2 = 700 \text{ kN/m}^2$$

$$Q_{2-3} = W_{2-3} \times \frac{\gamma - n}{\gamma - 1}$$

$$P_3, V_3 = ?$$

$$\frac{T_3}{T_2} = \left(\frac{P_3}{P_2}\right)^{\frac{n-1}{n}}$$

$$\frac{T_1}{T_2} = \left(\frac{P_3}{P_2}\right)^{\frac{1.5-1}{1.5}}$$

$$\frac{533}{1599} = \left(\frac{P_3}{700}\right)^{0.333}$$

$$\left(\frac{533}{1599}\right)^{0.333} = \frac{P_3}{700}$$

$$(0.333)^{3.00} = \frac{P_3}{700} \Rightarrow P_3 = 25.76 \text{ kN/m}^2$$

$$\frac{P_3}{P_2} = \left(\frac{V_2}{V_3}\right)^{\frac{1}{n-1}}$$
$$\frac{T_2}{T_1} = \left(\frac{V_2}{V_3}\right)^{\frac{n}{n-1}}$$
$$\frac{T_2}{T_1} = \left(\frac{P_2}{P_1}\right)^{\frac{n}{n-1}}$$

$$T_1 = T$$

$$V_3 = ?$$

W. K. T.

$$\frac{P_3}{P_2} \times \frac{N_2}{N_3}$$

n

1.5

$$P_3 V_3 = m R T_3$$

$$V_3 = \frac{m R T_3}{P_3} = \frac{0.128 \times 0.287 \times 533}{25.76}$$

$$V_3 = 0.76 \text{ m}^3$$

$$P_1 = P_2 = 700 \text{ kPa}$$

$$\frac{25.76}{700} = \frac{N_2}{N_3}$$

$$\frac{25.76}{700} = \frac{0.666}{N_3}$$

$$(0.03678)$$

$$\frac{N_2}{N_3} = 0.11059$$

$$V_3 = 0.76 \text{ m}^3$$

$$V_3 = 0.76 \text{ m}^3$$

$$W_{2-3} = \frac{P_2 V_2 - P_3 V_3}{n-1} = \frac{700 \times 0.084 - 25.76 \times 0.76}{1.5-1}$$

$$= \frac{58.8 - 19.57}{0.5}$$

$$W_{2-3} = 78.46 \text{ kJ}$$

$$Q_{2-3} = W_{1-2} \times \frac{\eta}{\eta-1}$$
  
$$= 78.46 \times \frac{1.4-1.5}{1.4-1}$$

$$Q_{2-3} = -19.615 \text{ kJ}$$

Process 3-1: Constant Temperature process

$$T_3 = T_1 ; P V = C$$

$$W_{3-1} = -PV \ln\left(\frac{P_1}{P_3}\right) \quad (\text{or}) \quad -PV \ln\left(\frac{V_3}{V_1}\right)$$

$$= P_3 V_3 \ln\left(\frac{P_1}{P_3}\right)$$

$$= -25.76 \times 0.76 \ln\left(\frac{700}{25.76}\right)$$

$$W_{3-1} = -64.65 \text{ kJ}$$

By 1<sup>st</sup> Law of thermodynamics:

$$Q = W + \Delta U$$

$$Q = W \quad \Delta U = 0$$

$$Q_{3-1} = W_{3-1} = -64.65 \text{ kJ}$$

Heat Received in the cycle  $Q_s = 137.25 \text{ kJ}$   
(Taking the +ve heat)

Heat Rejected in the cycle  $Q_R = -19.615 - 64.65$

$$Q_R = 84.265 \text{ kJ}$$

(Taking -ve heat)

$$\text{Efficiency} = \frac{W}{Q_s} = \frac{W_{1-2} + W_{2-3} + W_{3-1}}{Q_s}$$

$$= \frac{39.23 + 78.46 + (-64.65)}{137.25}$$

$$= 0.387$$

$$\boxed{= 38.7\%}$$

A three cycle operating with nitrogen as the working substance has constant temperature compression at  $34^\circ\text{C}$ . with initial pressure 100 kPa. Then the gas undergoes a constant volume heating and then polytropic expansion with  $\gamma = 1.33$  as index of compression. The isothermal compression requires  $-67 \text{ kJ/kg}$  of work.

Determine:

- $P, V$  and  $T$  around the cycle.
- Heat in and out.
- Net work.

T & S

For Nitrogen gas  $C_v = 0.731 \text{ kJ/kg}\cdot\text{K}$ .

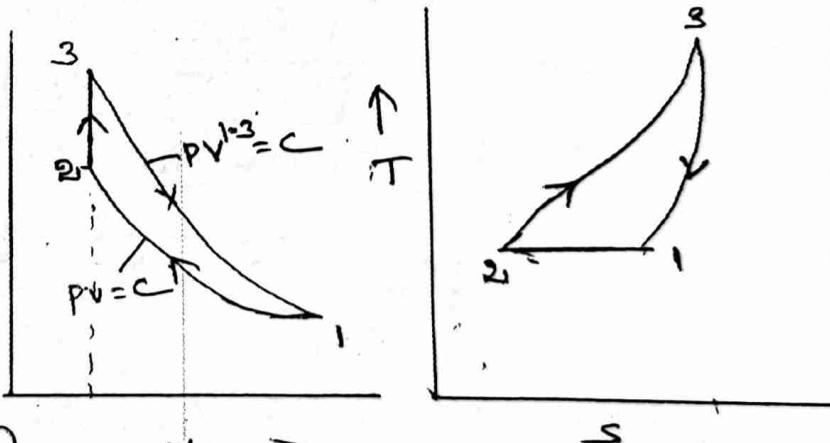
Given data:

$$T_1 = 34^\circ\text{C} = T_2 = 34 + 273 = 307 \text{ K}$$

$$P_1 = 100 \text{ kPa} = 100 \text{ kN/m}^2$$

$$\gamma = 1.33$$

$$W_{1-2} = -67 \text{ kJ/kg}$$



Process:

1-2 - constant temp. process.  
(compression)

2-3 - constant vol. (heat addition)

3-1 - polytropic process.

To find:

pressure and Temp. at all points.

Heat in and Heat out and net work.

$$Q_s = ?$$

$$Q_R = ?$$

Net  $W_{\text{net}} = Q_s - Q_R$

Solution:

Molecular weight of nitrogen

$$N_2 = 2 \times 14 = 28$$

Gas constant for Nitrogen  $R = \frac{R}{M} = \frac{8.314}{28}$

$$R_{N_2} = 0.297 \text{ kJ/kgK}$$

deg

process 1-2: constant temperature process.

$$W_{1-2} = PV \ln \left( \frac{V_1}{V_2} \right) \quad \begin{matrix} \text{large vol} \\ \text{small vol} \end{matrix} \quad P V \ln \left( \frac{P_2}{P_1} \right)$$

$$W_{1-2} = mRT \ln \left( \frac{P_1}{P_2} \right) \quad \text{or} \quad mRT \ln \left( \frac{P_2}{P_1} \right)$$

$$W_{1-2} = mRT_1 \ln \left( \frac{P_2}{P_1} \right)$$

Work input

Assume  $m = 1 \text{ kg}$

$$-67 = -0.297 \times 307 \ln \left( \frac{P_2}{100} \right)$$

$$\ln \left( \frac{P_2}{100} \right) = +0.7348$$

$$\frac{P_2}{100} = e^{+0.7348}$$

$$P_2 =$$

$$P_2 = 208.51 \text{ kN/m}^2$$

$$T_1 = T_2 = 307 \text{ K}$$

Heat transfer  $Q_{1-2} = W + \Delta U$

$$Q_{1-2} = W_{1-2} = -67 \text{ kJ} \quad \Delta U = 0 \text{ for isothermal process.}$$

$$mC_v(T_2 - T_1)$$

Process 2-3: Constant volume process:

$$V_2 = V_3$$

$W_{2-3} = 0$  for constant vol. process.

From general gas eqn.

$$\frac{P_2 V_2}{T_2} = \frac{P_3 V_3}{T_3}$$

$$T_3 = T_2 \times \frac{P_3}{P_2} = 307 \times \frac{P_3}{208.51}$$

$$T_3 = 1.472 P_3$$

$$Q_{2-3} = mC_v(T_3 - T_2)$$

$$Q = W + \Delta U$$

$$Q = 0 + \Delta U$$

### Process 3-1: Polytropic Process.

$$\frac{T_3}{T_1} = \left(\frac{P_3}{P_1}\right)^{\frac{n-1}{n}}$$

$$\frac{1.472 P_3}{307} = \left(\frac{P_3}{100}\right)^{\frac{1.3-1}{1.3}}$$

$$\frac{307}{1.472 P_3} = \left(\frac{100}{P_3}\right)^{\frac{1.3-1}{1.3}}$$

$$\frac{208.56}{P_3} = \left(\frac{100}{P_3}\right)^{0.231}$$

$$\frac{208.56}{P_3} = \left(\frac{100}{(P_3)^{0.231}}\right)$$

$$71.9831 = \frac{P_3}{P_3^{0.231}} = P_3 \times P_3^{-0.231} = P_3^{1-0.231}$$

$$71.9831 = P_3^{0.769}$$

$$P_3 = (71.9831)^{\frac{1}{0.769}} =$$

$$P_3 = 260 \text{ kN/m}^2$$

$$T_3 = 1.472 \times 260$$

$$T_3 = 382.72 \text{ K}$$

$$W_{3-1} = \frac{P_3 V_3 - P_1 V_1}{n-1} \quad (\text{or}) \quad m R \frac{(T_3 - T_1)}{n-1}$$

$$= 0.297 \frac{(382.72 - 307)}{1.3-1}$$

$$W_{3-1} = 74.963 \text{ kJ/kg}$$

$$Q_{3-1} = W \times \frac{\eta - 1}{\eta - 1} = 74.963 \times \frac{1.4 - 1.3}{1.4 - 1}$$

$$Q_{3-1} = 18.741 \text{ kJ/kg}$$

At constant vol. process

Process : 2-3

$$Q_{2-3} = 1 \times 0.931 (382.72 - 307)$$

$$Q_{2-3} = 55.35 \text{ kJ/kg}$$

Heat Supplied  $Q_s = 18 + 55.35 = 73.35$   
 (consider only +ve heat)

Heat Rejected  $Q_R = -67$  (consider only -ve heat)  
 $\text{kg}^{-1}$

$$\text{Net work, } W_{\text{net}} = W_{1-2} + W_{2-3} + W_{3-1} \\ = -67 + 0 + 74.963$$

$$W_{\text{net}} = 7.963 \text{ kJ/kg}$$

$$\frac{PV}{TV}$$

$$\frac{TV}{TP}$$

$$\frac{T_3}{T_1} = \left(\frac{P_3}{P_1}\right)^{\frac{n-1}{n}}$$

9. A gas of mass 0.35 kg pressure 1535 kN/m<sup>2</sup> and temperature of 335°C is expanded adiabatically to a pressure of 126 kN/m<sup>2</sup>. The gas is then heated at constant volume until it reaches 335°C; when its pressure is found to be 275 kN/m<sup>2</sup>. Finally the gas is compressed isothermally until the original pressure of 1535 kN/m<sup>2</sup> obtained. Draw P-V diagram and find out the following.
- (i) The value of adiabatic index
  - (ii) Change in internal energy during adiabatic process.
  - (iii) Heat transfer during Constant volume process. Take  $C_p = 1.005 \text{ kJ/kg K}$ .

Given data

$$m = 0.35 \text{ kg}$$

$$P_1 = 1535 \text{ kN/m}^2$$

$$T_1 = 335^\circ\text{C} + 273 = 608 \text{ K}$$

$$P_2 = 126 \text{ kN/m}^2$$

$$P_3 = 275 \text{ kN/m}^2$$

$$T_3 = T_1 = 335^\circ\text{C} = 608 \text{ K}$$

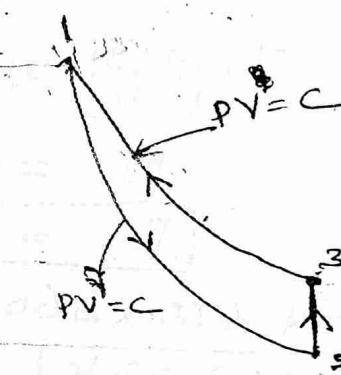
$$C_p = 1.005 \text{ kJ/kg K}$$

To find

- (i)  $\dot{Q}$  (ii)  $\Delta U$  (iii)  $Q$ .

Solutions:

Consider Process 2-3



const. vol. process.

$$\frac{P_2 V_2}{T_2} = \frac{P_3 V_3}{T_3} \quad \therefore V_2 = V_3$$

$$\frac{P_2}{T_2} = \frac{P_3}{T_3}$$

$$\frac{126}{T_2} = \frac{275}{608}$$

$$T_2 = 278.57 \text{ K}$$

Consider process 1-2: Adiabatic process.

$$\frac{T_2}{T_1} = \left(\frac{P_2}{P_1}\right)^{\frac{1}{\gamma}}$$

Taking log on both side.

$$\log\left(\frac{T_2}{T_1}\right) = \frac{1}{\gamma} \log\left(\frac{P_2}{P_1}\right)$$

$$\log\left(\frac{278.57}{608}\right) = \frac{1}{\gamma} \log\left(\frac{126}{1535}\right)$$

$$3398 = \frac{\gamma - 1}{\gamma} \times -1.0857$$

$$\frac{\gamma - 1}{\gamma} = 0.3129$$

$$\gamma - 1 = 0.3129 \gamma$$

$$\gamma - 0.3129 \gamma = 1$$

$$\gamma (1 - 0.3129) = 1$$

$$\gamma = 1.454$$

$$V_1 = \frac{mRT_1}{P_1}$$

$$V_2 = \frac{mRT_2}{P_2}$$

### Change in internal energy ( $\Delta U$ )

$$\Delta U = mC_V(T_2 - T_1)$$

$$\Delta U = 0.35 \times 0.691 (278.57 - 608)$$

$$\boxed{\Delta U = -79.69 \text{ kJ}}$$

$$\gamma = \frac{C_P}{C_V}$$

$$C_V = \frac{C_P}{\gamma} = \frac{1.005}{1.454}$$

$$\boxed{C_V = 0.691}$$

### Heat transfer during Constant volume process

$$Q = W + \Delta U$$

$$q = 0 + \Delta U$$

$$Q = \Delta U = m C_V (T_3 - T_2)$$

$$= 0.35 \times 0.691 (608 - 278.57)$$

$$\boxed{Q = 79.69 \text{ kJ}}$$

✓

AU.9.  
B1104  
11(a)

A fluid is confined in a cylinder by a spring-less frictionless piston so that the pressure in the fluid is a linear function of the volume ( $P = a + bV$ ). The internal energy of the fluid is given by the following equation  $U = 34 + 3.15PV$ . If the fluid changes from an initial state of 170kPa,  $0.03\text{m}^3$  to a final state of 400kPa,  $0.06\text{m}^3$ , find the direction and ~~magnitude~~ magnitude of the work and heat transfer. (Nov-01 & Nov-12)

G.I.D

$$P = a + bV \quad \text{--- ①}$$

$$U = 34 + 3.15PV \quad \text{--- ②}$$

$$P_1 = 170\text{kPa}$$

$$V_1 = 0.03\text{m}^3$$

$$P_2 = 400\text{kPa}$$

$$V_2 = 0.06\text{m}^3$$

To find:  $W, Q = ?$

Solution:

$$\textcircled{1} \quad P = a + bV \quad \text{--- ①}$$

$$P_1 = a + bV_1$$

$$170 = a + b(0.03)$$

$$a + 0.03b = 170 \quad \text{--- ③}$$

Equation ④ - ③

$$P = -60 + 7666.67V$$

$$P_2 = a + bV_2$$

$$400 = a + b(0.06)$$

$$a + 0.06b = 400 \quad \text{--- ④}$$

$$a + 0.06b = 400$$

$$\begin{array}{r} \textcircled{1} \\ \textcircled{2} \\ \textcircled{3} \end{array} \quad a + 0.03b = 170$$

$$0.03b = 230$$

$$b = 7666.67$$

$$a + 0.06(7666.67) = 400$$

$$a = -60$$

⑩

Work done (W):

$$W = \int_{V_1}^{V_2} P dV = \int_{V_1}^{V_2} (-60 + 7666.67V) dV$$

$$= \int_{V_1}^{V_2} -60 dV + \int_{V_1}^{V_2} 7666.67V dV$$

$$= -60 \int_{V_1}^{V_2} dV + 7666.67 \int_{V_1}^{V_2} V dV$$

$$= -60 [V]_{V_1}^{V_2} + 7666.67 \left[ \frac{V^2}{2} \right]_{V_1}^{V_2}$$

$$= -60 [V_2 - V_1] + \frac{7666.67}{2} [V_2^2 - V_1^2]$$

$$W = -60[0.06-0.03] + \frac{7666.67}{2} [(0.06)^2 - (0.03)^2]$$

$$W = +8.55 \text{ kJ}$$

+ve sign indicates work is done by the system.

Heat transfer ( $Q$ ):

$$Q = W + \Delta U$$

$$\Delta U = ? \quad U_2 - U_1 = ?$$

$$U = 34 + 3.15 PV$$

(2)

$$U_1 = 34 + 3.15 P_1 V_1$$

$$U_2 = 34 + 3.15 P_2 V_2$$

$$\Delta U = U_2 - U_1 = 34 + 3.15 P_2 V_2 - 34 - 3.15 P_1 V_1$$

$$= 3.15 P_2 V_2 - 3.15 P_1 V_1$$

$$= 3.15 [P_2 V_2 - P_1 V_1]$$

$$= 3.15 [400 \times 0.06 - 170(0.03)]$$

$$\Delta U = 59.535 \text{ kJ}$$

$$Q = W + \Delta U$$

$$= 8.55 + 59.535$$

$$Q = 68.085 \text{ kJ}$$

Q. The internal energy of the certain substance is given by the following equation  $U = 3.5 \beta PV + 84$ , where  $U$  is given in kJ/kg,  $P$  is in kPa and  $V$  is in  $\text{m}^3/\text{kg}$ . A system compressed of 3kg of this substance expands from an initial pressure of 500kPa and a volume of  $0.022 \text{ m}^3$  to a final pressure of 100kPa in a process in which pressure and volume are related by  $PV^{1.2} = C$ .

Find: (i) work (W)

(ii) Change in internal energy ( $\Delta U$ )

(iii) Heat ( $Q$ )

G.R.:

$$U = 3.5 \beta PV + 84$$

$$m = 3 \text{ kg}$$

$$P_1 = 500 \text{ kPa} = 500 \text{ KN/m}^2$$

$$V_1 = 0.022 \text{ m}^3$$

$$P_2 = 100 \text{ kPa} = 100 \text{ KN/m}^2$$

$$PV^{1.2} = C \quad (\text{Polytropic})$$

$$n = 1.2$$

To find:

(i)  $W$

(ii)  $\Delta U = U_2 - U_1$

(iii)  $Q$

Solution: (Polytropic process)

(i) Work done ( $W$ )

$$W = \frac{P_1 V_1 - P_2 V_2}{n-1}$$

$$W = \frac{500 \times 0.22 - 100 \times 0.84}{1.2 - 1}$$

$$W = 180 \text{ kJ}$$

(ii) Change in Internal energy

$$\Delta U = U_2 - U_1$$

$$U = 3.5 PV + 84$$

$$U_1 = 3.5 P_1 V_1 + 84$$

$$U_2 = 3.5 P_2 V_2 + 84$$

$$U_2 - U_1 = 3.5 [P_2 V_2 - P_1 V_1] + 84 - 84 \\ = 3.5 [100 \times 0.84 - 500 \times 0.22]$$

$$\Delta U = -92.56 \text{ kJ}$$

(iii) Heat transfer:  $Q$

$$Q = W + \Delta U$$

$$= 130 + (-92.56)$$

$$= 130 - 92.56$$

$$Q = 37.44 \text{ kJ}$$

Ans. 8.

Pg 1.12

$$m = 1.5 \text{ kg}$$

$$P = a + bV$$

$$P_1 = 1000 \text{ kPa}$$

$$P_2 = 200 \text{ kPa}$$

$$V_1 = 0.2 \text{ m}^3$$

$$V_2 = 1.2 \text{ m}^3$$

$$U = 1.5 PV - 85 \text{ kJ/kg}$$

~~Given:~~ Solution:

$$P = a + bV$$

$$P_1 = a + bV_1$$

$$1000 = a + 0.2b$$

Subtract eqn ① by eqn ②

To find:  $Q$  and  $\Delta U, V_{final}$

$$W = \int_a^b P dV$$

$$P_2 = a + b V_2$$

$$200 = a + 1.2b \quad \textcircled{B}$$

$$1000 = a + 0.2b \quad \textcircled{A}$$

$$200 = a + 1.2b \quad \textcircled{C}$$

$$800 = -1b$$

$$b = -800$$

Energy: It is defined as the capacity of the substance to do work.

### (1) Potential energy (P.E):

Energy passed due to its position is called potential energy.

$$\text{Potential energy, } P.E = m \times g \times z$$

m - mass

g - Acceleration due to gravity

z - height from datum.

### (2) Kinetic energy (K.E)

Energy passed due to its motion is called kinetic energy.

$$K.E = \frac{1}{2} m c^2$$

where  
m - mass

c - velocity of system m/s

### (3) Flow energy (F.E)

is defined as the flow of mass across the boundaries of the system.

$$F.E = P \times V$$

P - Pressure

V - Volume.

$$1000 = a + 0.2(-800)$$

$$1000 = a - 160$$

$$a = 1160$$

Sub a & b value in eqn no. 1.

$$P = 1160 - 800V$$

$$W = \int_{V_1}^{V_2} (1160 - 800V) dV$$

$$W = \int_{V_1}^{V_2} 1160 dV - 800 \int_{V_1}^{V_2} V dV$$

$$= 1160 \left[ V \right]_{V_1}^{V_2} - 800 \left[ \frac{V^2}{2} \right]_{V_1}^{V_2}$$

$$= 1160 [V_2 - V_1] - \frac{800}{2} [V_2^2 - V_1^2]$$

$$= 1160 [1.2 - 0.2] - 400 [(1.2)^2 - (0.2)^2]$$

$$= 1160 - 560$$

$$W = 600 \text{ kJ}$$

$$\Delta U = ? \quad \Delta U = U_2 - U_1$$

~~$$U = 1.5PV - 85$$~~

$$U_1 = 1.5P_1 V_1 - 85$$

$$U_2 = 1.5P_2 V_2 - 85$$

$$\Delta U = U_2 - U_1 = 1.5P_2 V_2 - 85 - (1.5P_1 V_1 - 85)$$

$$= 1.5 P_2 V_2 - 85 - 1.5 P_1 V_1 + 85$$

$$\Delta U = 1.5 (P_2 V_2 - P_1 V_1)$$

$$\Delta U = 1.5 (200 \times 1.2 - 1000 \times 0.2)$$

$$\Delta U = 60 \text{ kJ}$$

$$Q = W + \Delta U = 600 + 60 = 660 \text{ kJ}$$

$$\text{Max. Internal Energy} \quad \frac{dU}{dV} = 0$$

$$1.5PV - 85$$

~~$$\frac{dU}{dV} = 1.5PV$$~~

$$U = 1.5 (1160 - 800V)V - 85$$

$$U = 1740V - 1200V^2 - 85$$

$$\frac{dU}{dV} = 1740 - 2 \times 1200V$$

$$= 1740 - 2400V = 0$$

$$V = \frac{1740}{2400} = 0.725 \text{ m}^3$$

$$U_{\max} = 1740V - 1200V^2 - 85$$

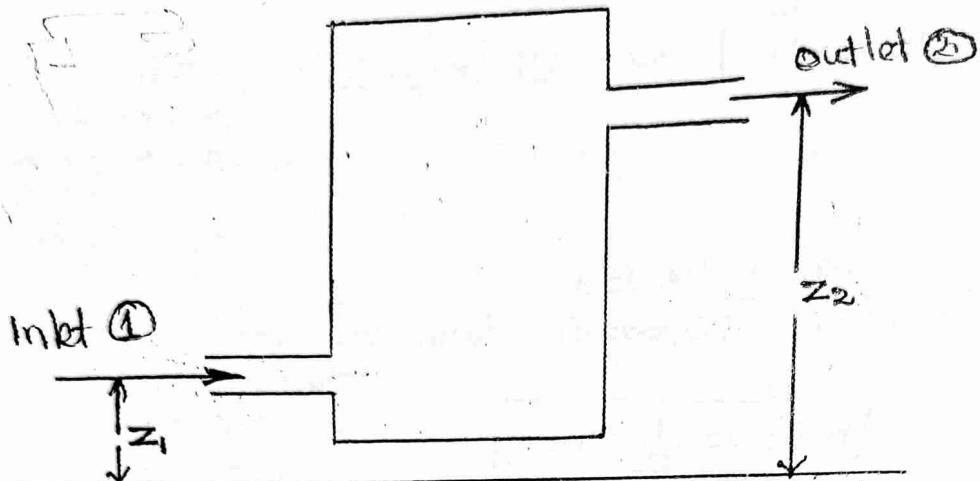
$$= 1740 \times 0.725 - 1200 (0.725)^2$$

$$= 1261.5 - 630.75 - 85$$

$$U_{\max} = 345.75 \text{ kJ}$$

## Steady flow energy Equation.

In any system, the rate of flow of working fluid is constant with respect to time, then the system is known as steady flow system.



Consider an open system through which the working substance flows as a steady rate. The working fluid entering the system at ① and leaves the system at ②

Let  $P_1$  - Pressure of the working substance entering the system ( $N/m^2$ )

$v_1$  - sp. Volume of " "  $(m^3/kg)$

$c_1$  - Velocity of " "  $(m/s)$ .

$u_1$  - internal energy of " "  $(J/kg)$ .

$z_1$  - height above the datum level.  $(m)$ .

$P_2, v_2, c_2, u_2, z_2$  are the corresponding values for the working substance leaving the system.

$Q$  - Heat supplied to the system ( $J/kg$ ).

$W$  - Work delivered by the system ( $J/kg$ ).

Total energy entering the system = Total energy leaving the system.

$$P.E + K.E + I.E + F.E + \text{Heat energy} = P.E + K.E + I.E + F.E + \text{Work}$$

$\therefore m=1$

$$gz_1 + \frac{1}{2}c_1^2 + u_1 + P_1v_1 + Q = gz_2 + \frac{1}{2}c_2^2 + u_2 + P_2v_2 + W$$

$\therefore h = u + Pv$

$$gz_1 + \frac{c_1^2}{2} + h_1 + Q = gz_2 + \frac{c_2^2}{2} + h_2 + W$$

This equation is called steady flow energy eqn.

Suppose, If we consider the mass,

$$m \left[ g z_1 + \frac{C_1^2}{2} + h_1 + Q \right] = m \left[ g z_2 + \frac{C_2^2}{2} + h_2 + W \right]$$

Note

If the values of  $Q$  and  $W$  in KJ/kg and  $h_1$ ,  $h_2$  are in KJ/kg then the above eqn. becomes.

$$m \left( \frac{g z_1}{1000} + \frac{C_1^2}{2000} + h_1 + Q \right) = m \left[ \frac{g z_2}{1000} + \frac{C_2^2}{2000} + h_2 + W \right]$$

If  $Q$  and  $W$  are KW or KJ/sec and  $h_1$ ,  $h_2$  are KJ/kg then the eqn. becomes,

$$m \left[ \frac{g z_1}{1000} + \frac{C_1^2}{2000} + h_1 \right] + Q = m \left[ \frac{g z_2}{1000} + \frac{C_2^2}{2000} + h_2 \right] + W.$$

Note: In a steady flow system

$$m = \frac{A_1 C_1}{V_1} = \frac{A_2 C_2}{V_2}$$

$A_1$  - Area of cross section at entry

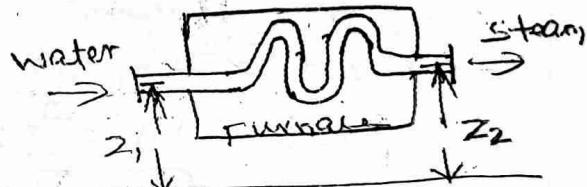
$A_2$  - Area of cross section at exit.

Application of steady flow energy equation

① Boiler (or) steam generator: is a steam generating device.

S.F.E.E

$$g z_1 + \frac{C_1^2}{2} + h_1 + Q = g z_2 + \frac{C_2^2}{2} + h_2 + W$$



Assumes

Change in P.E & K.E are negligible.

$$z_1 = z_2$$

$$C_1 = C_2$$

$$W = 0 \text{ (No shaft work)}$$

$$h_1 + Q = h_2$$

$$Q = h_2 - h_1 \text{ in KJ.}$$

② Condenser: is a device (or) H.E. which is used to convert the steam into water.

S.F.E.E

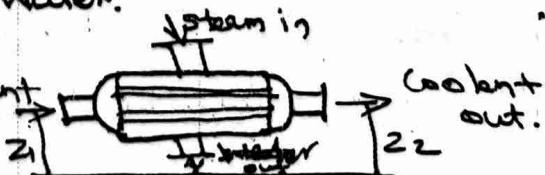
$$g z_1 + \frac{C_1^2}{2} + h_1 + Q = g z_2 + \frac{C_2^2}{2} + h_2 + W$$

Assume Change in P.E & K.E are negligible.

$$z_1 = z_2$$

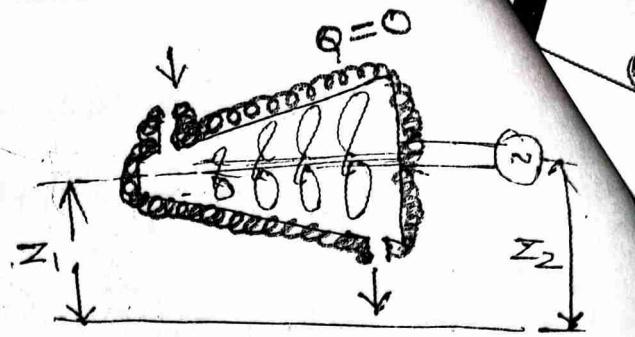
$$C_1 = C_2$$

$$W = 0 \text{ (No shaft work)}$$



$$Q = h_2 - h_1 \text{ in KJ.}$$

③ Turbine: is a device, which converts K.E of Working fluid into mechanical work. The turbine is fully insulated.



$$z_1 = z_2$$

$$C_1 = C_2$$

$$Q = 0$$

$$W = +W$$

S.F.E.E.

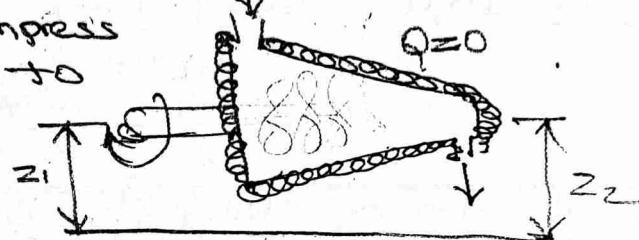
$$g z_1 + \frac{C_1^2}{2} + h_1 + Q = g z_2 + \frac{C_2^2}{2} + h_2 + W$$

$$h_1 + 0 = h_2 + W$$

$$W = h_1 - h_2 \text{ in kJ.}$$

④ Compressor: (if Rotary compressor) is a device

which is used to compress air from low pressure to high pressure.



$$z_1 = z_2$$

$$C_1 = C_2$$

$$Q = 0$$

$$W = -W \text{ (Work is done on the system).}$$

S.F.E.E

$$g z_2 + \frac{C_2^2}{2} + h_1 + Q = g z_1 + \frac{C_1^2}{2} + h_2 + W$$

$$h_1 = h_2 - W$$

$$W = h_2 - h_1$$

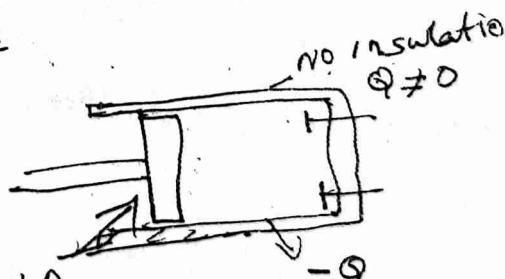
⑤ Reciprocating Compressor:

$$z_1 = z_2$$

$$C_1 = C_2$$

$$Q = -Q \text{ (Heat loss)}$$

$$W = -W \text{ (Work is done on the system) } (-W)$$



$$h_1 - Q = h_2 - W$$

$$W = Q + (h_2 - h_1)$$

in kJ.

⑥ Nozzle: is a device which increases the velocity at outlet.

$$Z_1 = Z_2$$

$$C_1 \neq C_2$$

$$Q = 0 \checkmark$$

$$W = 0 \checkmark$$

S.F.E.E.:

$$gZ_1 + \frac{C_1^2}{2} + h_1 + Q = gZ_2 + \frac{C_2^2}{2} + h_2 + W$$

$$\frac{C_1^2}{2} + h_1 = \frac{C_2^2}{2} + h_2$$

$$h_1 - h_2 = \frac{C_2^2}{2} - \frac{C_1^2}{2}$$

$$\frac{C_2^2 - C_1^2}{2 \times 1000} = h_1 - h_2$$

$$C_2^2 - C_1^2 = 2^{1000} (h_1 - h_2)$$

$$C_2^2 = \frac{1000}{2} (h_1 - h_2) + C_1^2$$

$$C_2 = \sqrt{2(h_1 - h_2) + C_1^2}$$

If initial velocity is very small

$$C_2 = \sqrt{2(h_1 - h_2)}$$

(a)

$$C_2 = \sqrt{2mC_p(T_1 - T_2)}$$

$$h_1 - h_2 = mC_p(T_1 - T_2)$$

$$h_1 = mC_pT_1$$

$$h_2 = mC_pT_2$$

=

Note: Mass flow rate of the steady flow system.

$$m = \frac{A_1 C_1}{V_1} = \frac{A_2 C_2}{V_2}$$

thus eqn is

a known as continuity eqn::

$A_1$  and  $A_2$  are the Area of cross section at inlet and exit.

— — — — —

— — — — —

1. A Boiler produces steam from water at  $35^{\circ}\text{C}$ . The enthalpy of a steam is  $2675 \text{ kJ/kg}$ . Calculate the heat transferred per kg. The sp. heat capacity of water is  $4.19 \text{ kJ/kg}$ . Neglect the potential and kinetic energies.

G.I.D

$$T_w = 35^{\circ}\text{C} + 273 = 308 \text{ K}$$

$$h_2 = 2675 \text{ kJ/kg}$$

$$C_{pw} = 4.19 \text{ kJ/kg}$$

$$m = 1$$

To find: Heat transfer "Q"

S.F.E.E.

$$gz_1 + \frac{C_1^2}{2} + h_1 + Q = gz_2 + \frac{C_2^2}{2} + h_2 + w$$

$$z_1 = z_2$$

$$C_1 = C_2$$

w = 0 (No shaft work)

$$h_1 + Q = h_2$$

$$Q = h_2 - h_1$$

$$h_1 = m C_{pw} T_w$$

$$= 1 \times 4.19 \times 308$$

$$h_1 = 1290.52 \text{ kJ/kg}$$

$$\Delta H = m C_p (\Delta T)$$

$$Q = h_2 - h_1 = 2675 - 1290.52$$

$$Q = 1384.48 \text{ kJ/kg}$$

2. A turbine operates under steady flow condition receiving steam at the following state pressure =  $1.2 \text{ MPa}$ , temp. =  $188^{\circ}\text{C}$ , enthalpy =  $2785 \text{ kJ/kg}$ , velocity =  $33.3 \text{ m/s}$  and elevation =  $3 \text{ m}$ . The steam leaves the turbine at the following state. Pressure =  $20 \text{ kPa}$ , enthalpy =  $2512 \text{ kJ/kg}$ , velocity =  $100 \text{ m/s}$  and elevation =  $0 \text{ m}$ . Heat is lost to the surroundings at the rate of  $0.29 \text{ kJ/sec}$ . If the rate of steam flow through the turbine is  $0.42 \text{ kg/s}$ , what is the power output of the turbine in  $\text{kW}$ ?

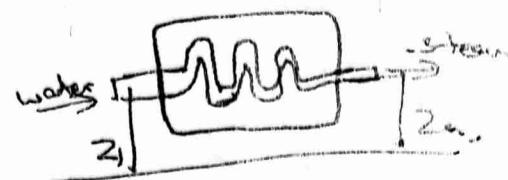
G.I.D:  $P_1 = 1.2 \text{ MPa} = 1.2 \times 10^6 \text{ N/m}^2$   
 $= 1200 \text{ kN/m}^2$

$$T_1 = 188 + 273 = 461 \text{ K}$$

$$h_1 = 2785 \text{ kJ/kg}$$

$$C_1 = 33.3 \text{ m/s}$$

$$z_1 = 3 \text{ m}$$



$$P_1$$

$$T_1$$

$$h_1$$

$$C_1$$

$$z_1$$

$$P_2$$

$$T_2$$

$$h_2$$

$$C_2$$

$$z_2$$

$$P_2 = 20 \text{ kPa} = 20 \text{ kN/m}^2$$

$$h_2 = 2512 \text{ kJ/kg}$$

$$C_2 = 100 \text{ m/s}$$

$$Z_2 = 0$$

$$Q = -0.29 \text{ kJ/s}$$

$$\therefore = 0.42 \text{ kJ/sec.}$$

$$h_1, h_2 = \text{kJ/kg}$$

$$Q = \text{kJ/s}$$

To find:  $W = ? \text{ kJ/s}$

Solution:

$\sum F \perp \Sigma E$

$$h_1, h_2 = \text{kJ/kg}$$

$$Q = \text{kJ/s}$$

$$m \left( \frac{g z_1}{1000} + \frac{C_1^2}{2 \times 1000} + h_1 \right) + Q = m \left( \frac{g z_2}{1000} + \frac{C_2^2}{2 \times 1000} + h_2 \right) + W$$

$$0.42 \left( \frac{9.81 \times 3}{1000} + \frac{(33.3)^2}{2 \times 1000} + 2725 \right) - 0.29 = 0.42 \left( 9.81(0) + \frac{(0)^2}{2 \times 1000} + 2512 \right) + W$$

$$W = 112.58 \text{ kJ/s} (\approx) \text{ kW}$$

3. Air flows steadily at the rate of  $0.5 \text{ kg/s}$  through an air compressor (entering at  $7 \text{ m/s}$  velocity,  $100 \text{ kPa}$  pressure and  $0.95 \text{ m}^3/\text{kg}$ , volume and leaving at  $5 \text{ m/s}$ ,  $700 \text{ kPa}$  and  $0.19 \text{ m}^3/\text{kg}$ ). The internal energy of the air leaving is  $90 \text{ kJ/kg}$  greater than that of the air entering. The cooling water in the compressor jackets absorbs heat from the air at the rate of  $58 \text{ kW}$ .

Calculate (a) Rate of shaft work in  $\text{kW}$ .  
(b) Find the ratio of the inlet pipe diameter to the outlet pipe diameter.

Given:

$$m = 0.5 \text{ kg/s}$$

$$C_1 = 7 \text{ m/s}$$

$$P_1 = 100 \text{ kPa}$$

$$V_1 = 0.95 \text{ m}^3/\text{kg}$$

$$C_2 = 5 \text{ m/s}$$

$$P_2 = 700 \text{ kPa}$$

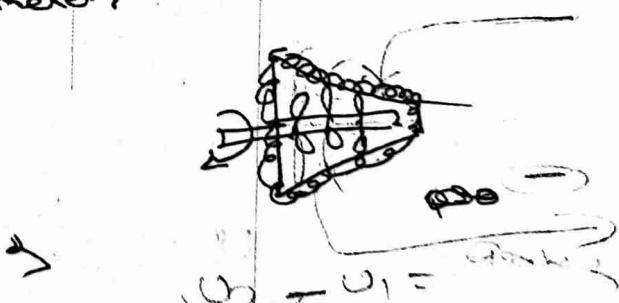
$$V_2 = 0.19 \text{ m}^3/\text{kg}$$

$$U_2 - U_1 = 90 \text{ kJ/kg}$$

$$Q = -58 \text{ kW}$$

$\sum F \perp \Sigma E$

$C_1 = P_1$



To find:  $W = ?$

$$\frac{P_1}{D_2} = ?$$

Solution:

S.F.E.L

$$\cancel{m(2v_1g + \frac{1}{2}C_1^2 + v_1 + P_1v_1)} + Q = m(2v_2g + \frac{1}{2}C_2^2 + v_2 + P_2v_2) + w$$

Assume  $v_1 = 22$

$$m\left(\frac{1}{2}C_1^2 + P_1v_1\right) + Q = m(v_2 - v_1) + m\left(\frac{1}{2}C_2^2 + P_2v_2\right) + w$$

$$0.95\left(\frac{(7)^2}{2 \times 1000} + (100 \times 0.05)\right) - 58 = 0.5(90) + 0.5\left(\frac{B^2}{2 \times 1000} + (100 \times 0.19)\right) + w$$

$$W = -121.99 \text{ kN}$$

-ve sign indicates work is done  
on the system.

$$\frac{P_1}{P_2} = ?$$

W.K.T. continuity eqn.

$$\frac{A_1 C_1}{v_1} = \frac{A_2 C_2}{v_2}$$

$$\frac{A_1}{A_2} = \frac{v_1}{v_2} \times \frac{C_2}{C_1}$$

$$= \cancel{\frac{7}{19} \times \frac{2.95}{1}}$$

$$\frac{\cancel{A_1} D_1^2}{\cancel{A_2} D_2^2} = \frac{0.95}{0.19} \times \frac{5}{7} = 3.57$$

$$\cancel{A_1} D_2^2$$

$$\left(\frac{D_1}{D_2}\right)^2 = 3.57$$

$$\frac{D_1}{D_2} = (3.57)^{1/2} = \sqrt{3.57}$$

$$\boxed{\frac{D_1}{D_2} = 1.89}$$

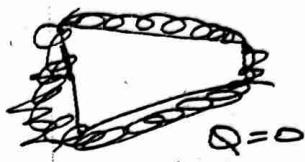
At the inlet of the nozzle, the enthalpy and velocity of the fluid are 3000 kJ/kg and 50 m/s respectively. And, at the outlet of the nozzle enthalpy is 2450 kJ/kg if the nozzle is horizontal. Find the velocity of the fluid at exit.

G.I.D.:

$$h_1 = 3000 \text{ kJ/kg}$$

$$C_1 = 50 \text{ m/s}$$

$$h_2 = 2450 \text{ kJ/kg}$$



To find:  $C_2$

S.F.E.E.

$$32_1 + \frac{C_1^2}{2} + h_1 + Q = 32_2 + \frac{C_2^2}{2} + h_2 + w$$

Assume

$$z_1 = z_2$$

$$Q = 0$$

$$w = 0 \quad \frac{C_1^2}{2} + h_1 = \frac{C_2^2}{2} + h_2$$

$$\frac{C_2^2 - C_1^2}{2} = h_1 - h_2$$

~~$$\frac{C_2^2 - C_1^2}{2 \times 1000} = 3000 - 2450 \approx 550$$~~

~~$$C_2^2 - C_1^2 = 2000 \times 550 = 1100000$$~~

~~$$C_2^2 = 1100000 + C_1^2 = 1100000 + (50)^2 = 1105000$$~~

~~$$C_2^2 = 3600$$~~

$$C_2 = \sqrt{3600}$$

$$\frac{C_2^2 - C_1^2}{2 \times 1000} = h_1 - h_2$$

$$C_2^2 - C_1^2 = 2000(h_1 - h_2)$$

$$C_2^2 = 2000(h_1 - h_2) + C_1^2$$

$$C_2 = \sqrt{2000(h_1 - h_2) + C_1^2}$$

$$C_2 = \sqrt{2000(3000 - 2450) + (50)^2}$$

$$C_2 = 1050 \text{ m/s}$$