Line, surface and volume integrals

In electromagnetic theory, we come across integrals, which contain vector functions. Some representative integrals are listed below:

$$\int_V \vec{F} dv \qquad \int_C \phi d\vec{l} \qquad \int_C \vec{F}.d\vec{l} \qquad \int_S \vec{F}.d\vec{s}$$

In the above, $\vec{F}$ and ϕ respectively represent vector and scalar function of space coordinates. *C*,*S* and *V* represent path, surface and volume of integration. All these integrals are evaluated using extension of the usual one-dimensional integral as the limit of a sum, i.e., if a function *f*(*x*) is defined over arrange *a* to *b* of values of [illegible] l is given by

$$\int_a^b f(x)\mathrm{d}x = \lim_{n\to\infty}\sum_{i=1}^{n} f_i \delta x_i$$

.................................(1.42)

where the interval (*a*,*b*) is subdivided into n continuous interval of lengths $\delta x_1, \ldots \ldots, \delta x_n$.

Line Integral: Line integral $\int_C \vec{E}\cdot\vec{dl}$ is the dot product of a vector with a specified *C*; in other words it is the integral of the tangential component $\vec{E}$ along the curve.

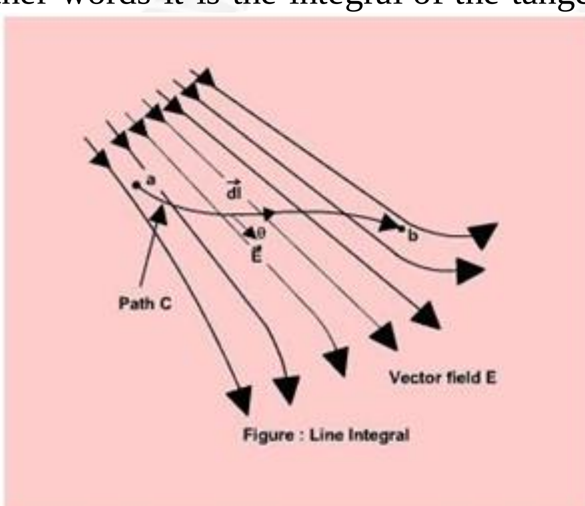

Fig 3.1 : Line Integral

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As shown in the figure 3.1, given a vector $\vec{E}$ around C, we define the integral $\int_C \vec{E}.d\vec{l} = \int_a^b E\cos\theta\, dl$ as the line integral of E along the curve C.

If the path of integration is a closed path as shown in the figure the line integral becomes a closed line integral and is called the circulation of $\vec{E}$ around C and denoted as $\oint_C \vec{E}.d\vec{l}$ as shown in the figure 3.2.

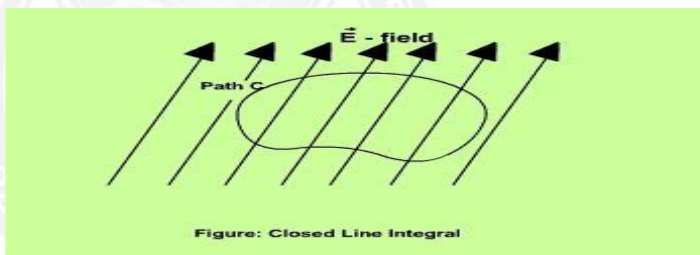

Figure: Closed Line Integral

Fig 3.2: Closed Line Integral

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Surface Integral :

Given a vector field $\vec{A}$, continuous in a region containing the smooth surface S, we define the surface integral or the flux of $\vec{A}$

$$\psi = \int_S A\cos\theta\, dS = \int_S \vec{A}.\hat{a}_n\, dS = \int_S \vec{A}.d\vec{S}$$

as surface integral over surface S as shown in fig 3.3.

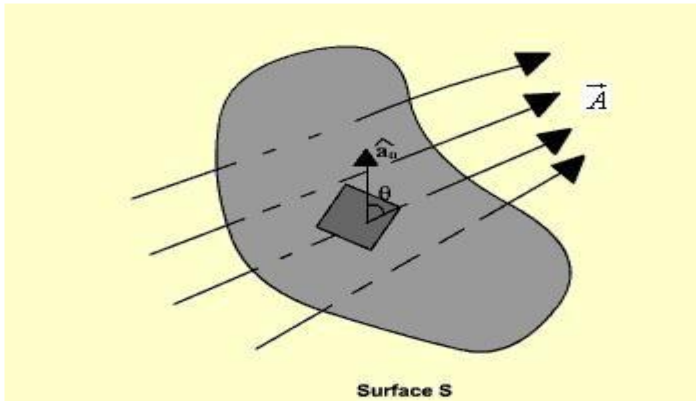

Fig 3.3 : Surface Integral

(www.brainkart.com/subject/Electromagnetic-Theory_206/)

If the surface integral is carried out over a closed surface, then we $\psi = \oint_S \vec{A}\, d\vec{S}$

Volume Integrals:

We define $\int_V f\,dV$ or $\iiint_V f\,dV$ as the volume integral of the scalar function *f*(function spatial coordinates) over the volume *V*. $\int_V \vec{F}\,dV$

Evaluation of $\vec{F}$ integral of the form can be carried out as a sum of three scalar volume integrals, where each scalar volume integral is a component of the vector