Electric Field

The electric field intensity or the electric field strength at a point is defined as the forceper unit charge. That is

$$\vec{E} = \lim_{Q \to 0} \frac{\vec{F}}{Q} \quad \text{or,} \quad \vec{E} = \frac{\vec{F}}{Q} \quad \text{...(2.4)}$$

The electric field intensity *E* at a point *r* (observation point) due a point charge *Q* located at $\vec{r}'$ (source point) is given by:

$$\vec{E} = \frac{Q(\vec{r} - \vec{r}')}{4\pi\varepsilon_0 \left|\vec{r} - \vec{r}'\right|^3} \quad \text{...(2.5)}$$

For a collection of *N* point charges *Q*1 ,*Q*2 ,..........*Q*N located at $\vec{r}_1, \vec{r}_2, \ldots, \vec{r}_N$ electric field intensity at point $\vec{r}$ is obtained as

$$\vec{E} = \frac{1}{4\pi\varepsilon_0} \sum_{i=1}^{N} \frac{Q_k(\vec{r} - \vec{r}_i)}{\left|\vec{r} - \vec{r}_i\right|^3} \quad \text{.......................................(2.6)}$$

The expression *(2.6)* can be modified suitably to compute the Electric filed dueto a continuous distribution of charges.

In figure 2.1 we consider a continuous volume distribution of charge d*(t)* in theregion denoted as the source region.

For an elementary charge $dQ = \rho(\vec{r}')dv'$, i.e. considering this charge as point charge,we can write the field expression as:

$$d\vec{E} = \frac{dQ(\vec{r} - \vec{r}')}{4\pi\varepsilon_0 \left|\vec{r} - \vec{r}'\right|^3} = \frac{\rho(\vec{r}')dv'(\vec{r} - \vec{r}')}{4\pi\varepsilon_0 \left|\vec{r} - \vec{r}'\right|^3} \quad \text{.............(2.7)}$$

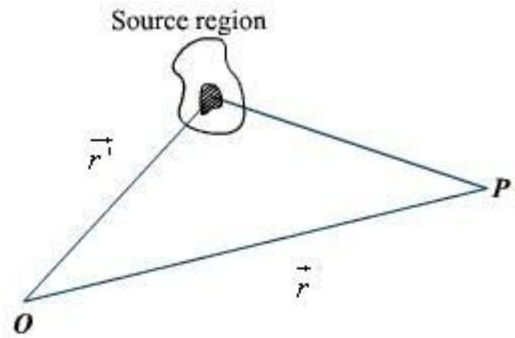

Fig 2.1: Continuous Volume Distribution of Charge
(www.brainkart.com/subject/Electromagnetic-Theory_206/)

When this expression is integrated over the source region, we get the electric field at the point P due to this distribution of charges. Thus the expression for the electric field at P can be written as:

$$\overline{E(r)} = \int_V \frac{\rho(\vec{r}\,')(\vec{r}-\vec{r}\,')}{4\pi\varepsilon_0 \left|\vec{r}-\vec{r}\,'\right|^3} dv' \quad \text{...} \; (2.8)$$

Similar technique can be adopted when the charge distribution is in the form of a line charge density or a surface charge density.

$$\overline{E(r)} = \int_L \frac{\rho_L(\vec{r}\,')(\vec{r}-\vec{r}\,')}{4\pi\varepsilon_0 \left|\vec{r}-\vec{r}\,'\right|^3} dl' \quad \text{..} \; (2.9)$$

$$\overline{E(r)} = \int_S \frac{\rho_s(\vec{r}\,')(\vec{r}-\vec{r}\,')}{4\pi\varepsilon_0 \left|\vec{r}-\vec{r}\,'\right|^3} ds' \quad \text{..} (2.10)$$

Electric flux density:

As stated earlier electric field intensity or simply ‘Electric field' gives the strength of the field at a particular point. The electric field depends on the material media in which the field is being considered. The flux density vector is defined to be independent of the material media (as we'll see that it relates to the charge that is producing it).For a linear isotropic medium under consideration; the flux density vector is defined as:

$$\vec{D} = \varepsilon \vec{E} \quad \text{..} \; \mathit{(2.11)}$$

We define the electric flux as

$$\psi = \int_s \vec{D}.d\vec{s}$$ ....*(2.12)*