1.1 GENERAL THEORY OF TRANSMISSION LINES:

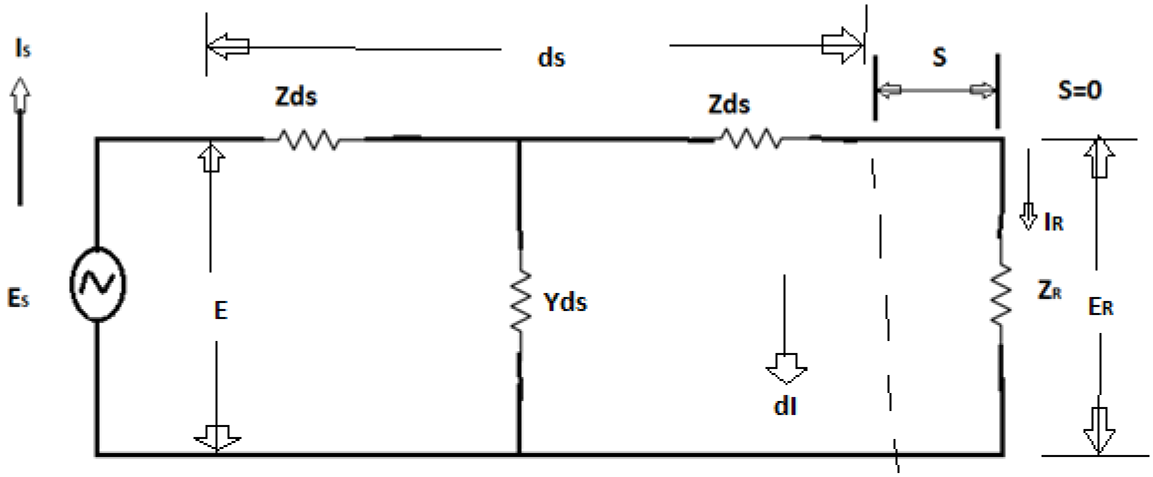

Fig: 1.1.1 A long line, with the elements of one of the infinitesimal sections shown.

From the Fig 1.1.1 consider an infinite length transmission line also an small element ds which located at a distance of s from the receiving end (Z_R). The sending end voltage and current are E_S and I_S. The receiving end voltage and current are E_R and I_R. The current and voltage at any point is E and I

The voltage in the length ds is dE.

dE = I Zds

$\frac{dE}{ds}$ = IZ ………(1)

The current in the length ds is

dI = E Y ds

$\frac{dI}{ds}$ = EY ....(2)

Diff equ (1) and (2)

From equ(1)…… $\frac{d^2E}{ds^2} = Z\frac{dI}{ds}$

From equ(2)…… $\frac{d^2I}{ds^2} = Y\frac{dE}{ds}$

Sub the values of $\frac{dI}{ds}$ and $\frac{dE}{ds}$ in above equ

$\frac{d^2E}{ds^2}$ = Z EY

$\frac{d^2I}{ds^2}$ = Y IZ

$\frac{d^2E}{ds^2}$ - Z EY = 0 (3)

$\frac{d^2I}{ds^2}$ - Y IZ = 0 (4)

These are the differential equations of transmission line.

From equ(3),

$(\frac{d^2}{ds^2}$ - Z Y) E = 0

$(m^2$ - Z Y) E = 0

$(m^2$ - Z Y) = 0

m^2 = Z Y

$m = \pm\sqrt{ZY}$

The general solution can be written as E = A $e^{\sqrt{ZY}s}$ + B $e^{-\sqrt{ZY}s}$ ………(5)

From equ(4),

$(\frac{d^2}{ds^2}$ - Z Y) I = 0

$(m^2$ - Z Y) I = 0

$(m^2$ - Z Y) = 0

m^2 = Z Y

$m = \pm\sqrt{ZY}$

The general solution can be written as

I = C $e^{\sqrt{ZY}s}$ + D$e^{-\sqrt{ZY}s}$ ………(6)

where A, B, C, D are arbitrary constants

We need to find the values of A, B, C, D

By using the condition **s=0**

$I = I_R$

$E = E_R$

Use this condition in equ (5) and (6) From equ (5),

$E_R = A\, e^{\sqrt{ZY}(0)} + B\, e^{-\sqrt{ZY}(0)}$

$E_R = A + B$

From equ (6), IR = C + D …….(7)

Diff equ (5) and (6) with.r.to 's'

From equ(5)

$\frac{dE}{ds} = A\sqrt{ZY}e^{\sqrt{ZY}s} + B\,(-\sqrt{ZY})\, e^{-\sqrt{ZY}s}$

[using this formula, $e^{ax} = a\, e^{ax}$]

$\frac{dE}{ds} = A\sqrt{ZY}e^{\sqrt{ZY}s} - B\sqrt{ZY}e^{-\sqrt{ZY}s}$

$\frac{dE}{ds} = IZ$, sub in above equ

$IZ = A\sqrt{ZY}e^{\sqrt{ZY}s} - B\sqrt{ZY}e^{-\sqrt{ZY}s}$

$I = \frac{A\sqrt{ZY}\, e^{\sqrt{ZY}s} - B\sqrt{ZY}\, e^{-\sqrt{ZY}s}}{Z}$

$I = A\sqrt{\frac{Y}{Z}}e^{\sqrt{ZY}s} - B\sqrt{\frac{Y}{Z}}e^{-\sqrt{ZY}s}$ ....(8)

$(\frac{\sqrt{ZY}}{Z} = \sqrt{\frac{Y}{Z}})$

From equ (6),

$\frac{dI}{ds} = C\sqrt{ZY}e^{\sqrt{ZY}s} + D\sqrt{ZY}e^{-\sqrt{ZY}s}$

$E\,Y = C\sqrt{ZY}e^{\sqrt{ZY}s} + D\sqrt{ZY}e^{-\sqrt{ZY}s}$

$(\frac{dI}{ds} = E\,Y)$

$E = \frac{C\sqrt{ZY}\,e^{\sqrt{ZY}s} - D\sqrt{ZY}\,e^{-\sqrt{ZY}s}}{Y}$

$E = C\sqrt{\frac{Z}{Y}}e^{\sqrt{ZY}s} - D\sqrt{\frac{Z}{Y}}e^{-\sqrt{ZY}s}$ ....(9)

$\left(\frac{\sqrt{ZY}}{Y} = \sqrt{\frac{Z}{Y}}\right)$

At, s=0 , $E = E_R$ and $I = I_R$

sub this values in (8) and (9)

from equ (8),

$I_R = A\sqrt{\frac{Y}{Z}} - B\sqrt{\frac{Y}{Z}}$ ....(10)

from equ (9),

$E_R = C\sqrt{\frac{Z}{Y}} - D\sqrt{\frac{Z}{Y}}$ ....(11)

from equ (7),

$B = E_R - A$

To find the value of A sub B value in equ (10)

$I_R = A\sqrt{\frac{Y}{Z}} - (E_R - A)\sqrt{\frac{Y}{Z}}$

$I_R = A\sqrt{\frac{Y}{Z}} - E_R\sqrt{\frac{Y}{Z}} + A\sqrt{\frac{Y}{Z}}$

$I_R + E_R\sqrt{\frac{Y}{Z}} = 2A\sqrt{\frac{Y}{Z}}$

$2A\sqrt{\frac{Y}{Z}} = I_R + E_R\sqrt{\frac{Y}{Z}}$

$A = \frac{I_R + E_R\sqrt{\frac{Y}{Z}}}{2\sqrt{\frac{Y}{Z}}}$

$A = \frac{I_R}{2\sqrt{\frac{Y}{Z}}} + \frac{E_R\sqrt{\frac{Y}{Z}}}{2\sqrt{\frac{Y}{Z}}}$

$A = \frac{I_R\sqrt{\frac{Z}{Y}}}{2} + \frac{E_R}{2}$

$(\sqrt{\frac{Z}{Y}} = Z_o)$

$A = \frac{I_R Z_o}{2} + \frac{E_R}{2}$

$A = \frac{I_R Z_o}{2} + \frac{E_R}{2}$

By using this formula, V = IR

E = IZ

$I = \frac{E}{Z}$

$I_R = \frac{E_R}{Z_R}$

sub I_R value in A,

$A = \frac{E_R}{Z_R}\frac{Z_o}{2} + \frac{E_R}{2}$

$A = \frac{E_R}{2}[1 + \frac{Z_o}{Z_R}]$ ....(12)

sub equ (12) in (7),

$E_R = \frac{E_R}{2}[1 + \frac{Z_o}{Z_R}] + B$

$B = E_R - \frac{E_R}{2}[1 + \frac{Z_o}{Z_R}]$

$B = E_R - \frac{E_R}{2} - \frac{Z_o}{Z_R} . \frac{E_R}{2}$

$B = \frac{E_R}{2} - \frac{Z_o}{Z_R} . \frac{E_R}{2}$

$B = \frac{E_R}{2}[1 - \frac{Z_o}{Z_R}]$ ....(13)

from equ (7),

$I_R = C + D$

$D = I_R - C$

sub the above value in (11),

$$E_R = C\sqrt{\frac{Z}{Y}} - (I_R - C)\sqrt{\frac{Z}{Y}}$$

$$E_R = C\sqrt{\frac{Z}{Y}} - I_R\sqrt{\frac{Z}{Y}} + C\sqrt{\frac{Z}{Y}}$$

$$E_R = 2C\sqrt{\frac{Z}{Y}} - I_R\sqrt{\frac{Z}{Y}}$$

$$2C\sqrt{\frac{Z}{Y}} = E_R + I_R\sqrt{\frac{Z}{Y}}$$

$$C = \frac{E_R + I_R\sqrt{\frac{Z}{Y}}}{2\sqrt{\frac{Z}{Y}}}$$

$$C = \frac{E_R}{2\sqrt{\frac{Z}{Y}}} + \frac{I_R\sqrt{\frac{Z}{Y}}}{2\sqrt{\frac{Z}{Y}}}$$

$$C = \frac{E_R}{2\sqrt{\frac{Z}{Y}}} + \frac{I_R}{2}$$

$$C = \frac{E_R}{2Z_o} + \frac{I_R}{2}$$

$$(E_R = I_R Z_R)$$

sub E_R value in C,

$$C = \frac{I_R Z_R}{2Z_o} + \frac{I_R}{2}$$

$$C = \frac{I_R}{2}\left[1 + \frac{Z_R}{Z_O}\right] \quad ....(14)$$

$D = I_R - C$

$$D = I_R - \frac{I_R}{2}\left[1 + \frac{Z_R}{Z_O}\right]$$

$$D = I_R - \frac{I_R}{2} - \frac{I_R}{2}\frac{Z_R}{Z_O}$$

$D = \frac{I_R}{2} - \frac{I_R}{2}\frac{Z_R}{Z_O}$

$D = \frac{I_R}{2}[1 - \frac{Z_R}{Z_O}]$ ....(15)

sub the A, B, C,D values in (5) and (6),

from equ (5), $E = A\, e^{\sqrt{ZY}s} + B\, e^{-\sqrt{ZY}s}$

$E = \frac{E_R}{2}[1 + \frac{Z_o}{Z_R}]\, e^{\sqrt{ZY}s} + \frac{E_R}{2}[1 - \frac{Z_o}{Z_R}]\, e^{-\sqrt{ZY}s}$ ....(16)

$E = \frac{E_R}{2}[\frac{Z_R + Z_o}{Z_R}]\, e^{\sqrt{ZY}s} + \frac{E_R}{2}[\frac{Z_R - Z_o}{Z_R}]\, e^{-\sqrt{ZY}s}$

$E = \frac{E_R}{2}[\frac{Z_R + Z_o}{Z_R}]\left[e^{\sqrt{ZY}S} + [\frac{Z_R - Z_o}{Z_R}][\frac{Z_R}{Z_R + Z_o}]\, e^{-\sqrt{ZY}S}\right]$

$E = \frac{E_R}{2}[\frac{Z_R + Z_o}{Z_R}]\left[e^{\sqrt{ZY}S} + [\frac{Z_R - Z_o}{Z_R + Z_o}]\, e^{-\sqrt{ZY}S}\right]$ ....(17)

from equ (6), $I = C\, e^{\sqrt{ZY}s} + D e^{-\sqrt{ZY}s}$

$I = \frac{I_R}{2}[1 + \frac{Z_R}{Z_O}]\, e^{\sqrt{ZY}s} + \frac{I_R}{2}[1 - \frac{Z_R}{Z_O}]\, e^{-\sqrt{ZY}s}$ ....(18)

$I = \frac{I_R}{2}[\frac{Z_O + Z_R}{Z_O}]\, e^{\sqrt{ZY}s} + \frac{I_R}{2}[\frac{Z_O - Z_R}{Z_O}]\, e^{-\sqrt{ZY}s}$

$I = \frac{I_R}{2}[\frac{Z_O + Z_R}{Z_O}]\left[e^{\sqrt{ZY}S} + \frac{I_R}{2}[\frac{Z_O - Z_R}{Z_O}][\frac{Z_R}{Z_O + Z_R}]\, e^{-\sqrt{ZY}S}\right]$

$I = \frac{I_R}{2}[\frac{Z_O + Z_R}{Z_O}]\left[e^{\sqrt{ZY}S} - \frac{I_R}{2}[\frac{Z_R - Z_O}{Z_R + Z_O}]\, e^{-\sqrt{ZY}S}\right]$ ....(19)

equ (17) and (19) are the first form of voltage and current.

equ (16) may be rearranged as

$E = \frac{E_R}{2}\left[[1 + \frac{Z_o}{Z_R}]\, e^{\sqrt{ZY}S} + [1 - \frac{Z_o}{Z_R}]\, e^{-\sqrt{ZY}S}\right]$

$E = \frac{E_R}{2}\left[e^{\sqrt{ZY}S} + \frac{Z_o}{Z_R} e^{\sqrt{ZY}S} + e^{-\sqrt{ZY}S} - \frac{Z_o}{Z_R} e^{-\sqrt{ZY}S}\right]$

$E = \frac{E_R}{2}\left[e^{\sqrt{ZY}S} + e^{-\sqrt{ZY}S} + \frac{Z_o}{Z_R}[e^{\sqrt{ZY}S} - e^{-\sqrt{ZY}S}]\right]$

for example,

$\cosh\theta = \frac{e^{\theta} + e^{-\theta}}{2}$

$\sinh\theta = \frac{e^{\theta} - e^{-\theta}}{2}$

sub in above equ,

$E = \frac{E_R}{2}\left[2\cosh\sqrt{ZY}S + \frac{2.\ Z_o}{Z_R}\sinh\sqrt{ZY}S\right]$

$E = E_R\cosh\sqrt{ZY}S + \frac{E_R\ Z_o}{Z_R}\sinh\sqrt{ZY}S$

$Z_R = \frac{E_R}{I_R}$

sub the Z_R value in above equ,

$E = E_R\cosh\sqrt{ZY}S + \frac{E_R\ Z_o}{[\frac{E_R}{I_R}]}\sinh\sqrt{ZY}S$

$E = E_R\cosh\sqrt{ZY}S + \frac{I_R\ E_R\ Z_o}{E_R}\sinh\sqrt{ZY}S$

$E = E_R\cosh\sqrt{ZY}S + I_R\ Z_o\sinh\sqrt{ZY}S$ ....(21)

The same procedure will be followed for the current equ,

equ (18) will be,

$I = I_R\cosh\sqrt{ZY}S + \frac{E_R}{Z_O}\sinh\sqrt{ZY}S$ ....(22)

equ (21) and (22) are the second form of voltage and current at any point on a transmission line.

PHYSICAL SIGNIFICANCE OF TRANSMISSION LINE (or) INFINITE LINE (or) THE TWO STANDARD FORM FOR INPUT IMPEDANCE OF THE TRANSMISSION LINE TERMINATED BY AN IMPEDANCE Z_R.

From the Fig 1.1.2 the equation for the current and voltage may be written for the sending end current 'I_S' of a line of length 'l' is,

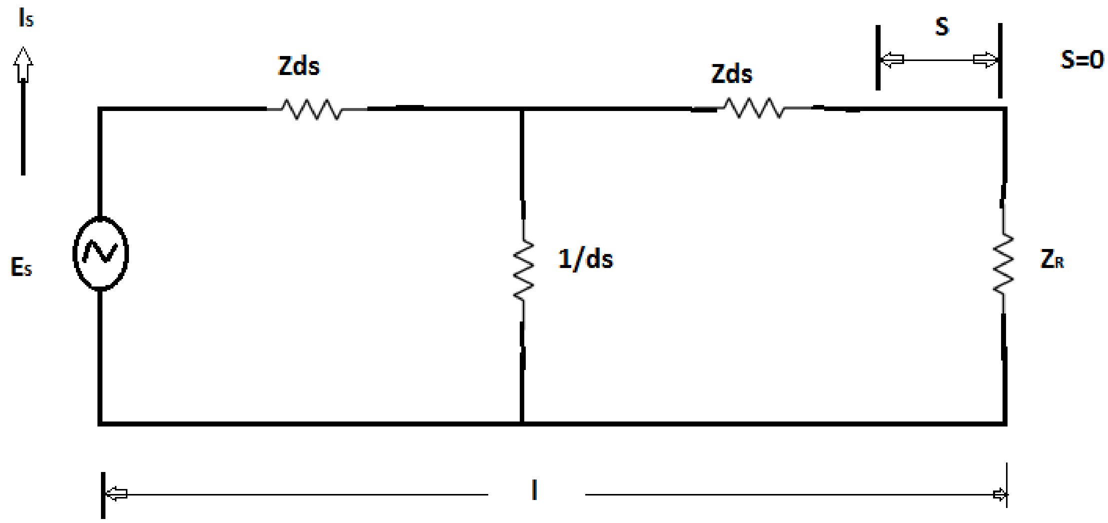

Fig: 1.1.2 A length l taken from an infinite line

The sending current equation is given by,

$I_S = I_R \cosh\sqrt{ZY}.l + \frac{E_R}{Z_O} \sinh\sqrt{ZY}.l$ $[E_R = I_R Z_R]$

Sub E_R value in above equ,

$I_S = I_R \cosh\sqrt{ZY}.l + \frac{I_R Z_R}{Z_O} \sinh\sqrt{ZY}.l$

$I_S = I_R \left[\cosh\sqrt{ZY}.l + \frac{Z_R}{Z_O} \sinh\sqrt{ZY}.l\right]$ ....(1)

The sending voltage equation is given by,

$E_S = E_R \cosh\sqrt{ZY}.l + I_R Z_O \sinh\sqrt{ZY}.l$

$[E_R = I_R Z_R]$

$[I_R = \frac{E_R}{Z_R}]$

Sub I_R value in above equ,

$E_S = E_R \cosh\sqrt{ZY}.l + \frac{E_R Z_O}{Z_R} \sinh\sqrt{ZY}.l$

$E_S = E_R \left[\cosh\sqrt{ZY}.l + \frac{Z_O}{Z_R} \sinh\sqrt{ZY}.l\right]$ ....(2)

Since, We know that,

Propagation Constant $\gamma = \sqrt{ZY}$

Characteristic Impedance $Z_O = \sqrt{\frac{Z}{Y}}$

Sub γ value in equ (1) and (2),

From equ (1),

$I_S = I_R\left[\cosh\gamma l + \frac{Z_R}{Z_O}\sinh\gamma l\right]$ ....(3)

From equ (2),

$E_S = E_R\left[\cosh\gamma l + \frac{Z_O}{Z_R}\sinh\gamma l\right]$ ....(4)

Input Impedance $Z_S = \frac{E_S}{I_S}$ [E = IZ]

$$Z_S = \frac{E_R\left[\cosh\gamma l + \frac{Z_O}{Z_R}\sinh\gamma l\right]}{I_R\left[\cosh\gamma l + \frac{Z_R}{Z_O}\sinh\gamma l\right]}$$

$$Z_S = Z_R\left[\frac{\frac{Z_R\cosh\gamma l + Z_O\sinh\gamma l}{Z_R}}{\frac{Z_O\cosh\gamma l + Z_R\sinh\gamma l}{Z_O}}\right]$$

$E_R = I_R Z_R$

$Z_R = \frac{E_R}{I_R}$

$$Z_S = Z_R\left[\frac{Z_R\cosh\gamma l + Z_O\sinh\gamma l}{Z_R}\ X\ \frac{Z_O}{Z_O\cosh\gamma l + Z_R\sinh\gamma l}\right]$$

$$Z_S = Z_O\left[\frac{Z_R\cosh\gamma l + Z_O\sinh\gamma l}{Z_O\cosh\gamma l + Z_R\sinh\gamma l}\right] \quad ....(5)$$

This is the first standard form of input impedance of the transmission line.

$\text{Cosh}\theta = \frac{e^{\theta} + e^{-\theta}}{2}$

$\text{Sinh}\theta = \frac{e^{\theta} - e^{-\theta}}{2}$

Sub the above formula in equ (5),

$$Z_S = Z_O\left[\frac{Z_R\left(\frac{e^{\gamma l}+e^{-\gamma l}}{2}\right)+Z_O\left(\frac{e^{\gamma l}-e^{-\gamma l}}{2}\right)}{Z_O\left(\frac{e^{\gamma l}+e^{-\gamma l}}{2}\right)+Z_R\left(\frac{e^{\gamma l}-e^{-\gamma l}}{2}\right)}\right]$$

$$Z_S = \frac{2Z_O}{2}\left[\frac{Z_R(e^{\gamma l}+e^{-\gamma l})+Z_O(e^{\gamma l}-e^{-\gamma l})}{Z_O(e^{\gamma l}+e^{-\gamma l})+Z_R(e^{\gamma l}-e^{-\gamma l})}\right]$$

$$Z_S = Z_O\left[\frac{Z_R e^{\gamma l}+Z_R e^{-\gamma l}+Z_O e^{\gamma l}-Z_O e^{-\gamma l}}{Z_O e^{\gamma l}+Z_O e^{-\gamma l}+Z_R e^{\gamma l}-Z_R e^{-\gamma l}}\right]$$

$$Z_S = Z_O\left[\frac{e^{\gamma l}[Z_R+Z_O]+e^{-\gamma l}[Z_R-Z_O]}{e^{\gamma l}[Z_R+Z_O]-e^{-\gamma l}[Z_R-Z_O]}\right]$$

$$Z_S = Z_O\frac{[Z_R+Z_O]}{[Z_R+Z_O]}\left[\frac{e^{\gamma l}+e^{-\gamma l}\left[\frac{Z_R-Z_O}{Z_R+Z_O}\right]}{e^{\gamma l}-e^{-\gamma l}\left[\frac{Z_R-Z_O}{Z_R+Z_O}\right]}\right]$$

$$Z_S = Z_O\left[\frac{e^{\gamma l}+e^{-\gamma l}\left[\frac{Z_R-Z_O}{Z_R+Z_O}\right]}{e^{\gamma l}-e^{-\gamma l}\left[\frac{Z_R-Z_O}{Z_R+Z_O}\right]}\right] \quad \ldots\ldots\ldots(6)$$

This is the second standard form of input impedance of the transmission line.

WAVELENGTH AND VELOCITY OF PROPAGATION

WAVELENGTH:

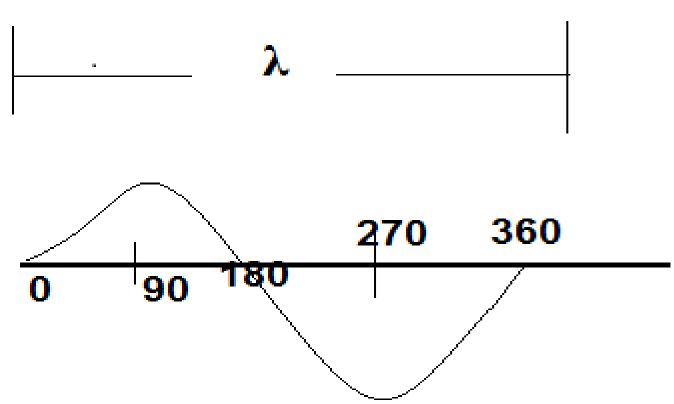

Fig: 1.1.3 Propagation of current from sending and towards receiving end

The distance with which the wave changes its phase by 2π radians is known as wavelength.

In Fig 1.1.3 the distance from the sending end to point 8 is thus one wavelength. The distance the wave travels along the line where the phase angle is changing to 2π radians is known as wavelength.

It is denoted by λ,

$\lambda = \frac{2\pi}{\beta}$ ……(1)

and also we know,

$\lambda = \frac{v}{f}$

v = λ.f

v- velocity

f- frequency

$v = \frac{2\pi}{\beta} . f$

$v = \frac{\omega}{\beta}$ ………(2)

$[\ \lambda = \frac{2\pi}{\beta}\ ]$

[ω = 2πf]

VELOCITY OF PROPAGATION:

The velocity of propagation along the line depends on the change in the phase along the line. Therefore, this velocity is called phase velocity or wave velocity.

$\gamma = \ln\left(\frac{V_1}{V_2}\right) = \ln\left(\frac{I_1}{I_2}\right)$

In general,

γ = α+jβ

$\gamma = \sqrt{ZY}$ …………(1)

where,

Z = R+jωL

Y = G+jωC ………(2)

Sub equ (2) in equ (1)

$\gamma = \sqrt{(R + j\omega L)(G + j\omega C)}$

$\gamma = \sqrt{RG + j\omega RC + j\omega LG - \omega^2 LC}$

$\alpha + j\beta = \sqrt{RG - \omega^2 LC + j(\omega RC + \omega LG)}$

Squaring on both sides,

$(\alpha + j\beta)^2 = RG - \omega^2 LC + j(\omega RC + \omega LG)$

$\alpha^2 + \beta^2 - 2j\alpha\beta = RG - \omega^2 LC + j(\omega RC + \omega LG)$

Equating real and imaginary parts,

$\alpha^2 + \beta^2 = RC - \omega^2 LG$ ………(3)

$2\alpha\beta = \omega RC + \omega LG$

$2\alpha\beta = \omega(RC + LG)$ ……….(4)

From equ (3),

$\alpha^2 = RG - \omega^2 LC + \beta^2$ ………(5)

Squaring equ (4),

$4\alpha^2\beta^2 = \omega^2 (RC + LG)^2$ ………(6)

Sub equ (5) in equ (6)

$4(RG - \omega^2 LC + \beta^2)\beta^2 = \omega^2 (RC + LG)^2$

$4(RG\beta^2 - \omega^2 LC\,\beta^2 + \beta^4) = \omega^2 (RC + LG)^2$

$RG\beta^2 - \omega^2 LC\,\beta^2 + \beta^4 = \frac{\omega^2}{4} (RC + LG)^2$

$RG\beta^2 - \omega^2 LC\,\beta^2 + \beta^4 - \frac{\omega^2}{4} (RC + LG)^2 = 0$

$\beta^4 + \beta^2(RG - \omega^2 LC) - \frac{\omega^2}{4} (RC + LG)^2 = 0$ …….(7)

The above equation os of the form of $ax^4+bx^2+c = 0$

$$x^2 = \frac{-b \pm \sqrt{b^2-4ac}}{2a}$$

a=1, b= $RG - \omega^2 LC$, c= $-\frac{\omega^2}{4}(RC+LG)^2$

$$\beta^2 = \frac{-(RG-\omega^2 LC) \pm \sqrt{(RG-\omega^2 LC)^2 - 4\left(-\frac{\omega^2}{4}(RC+LG)^2\right)}}{2}$$

Neglect the negative value,

$$\beta^2 = \frac{(\omega^2 LC - RG) + \sqrt{(RG-\omega^2 LC)^2 + (\omega^2 (RC+LG)^2)}}{2}$$

$$\beta = \sqrt{\frac{(\omega^2 LC - RG) + \sqrt{(RG-\omega^2 LC)^2 + (\omega^2 (RC+LG)^2)}}{2}} \qquad \text{........(8)}$$

Sub β^2 value in equ (5),

$$\alpha^2 = RG - \omega^2 LC + \left(\frac{(\omega^2 LC - RG) + \sqrt{(RG-\omega^2 LC)^2 + (\omega^2 (RC+LG)^2)}}{2}\right)$$

$$\alpha^2 = \frac{2\,(RG-\omega^2 LC) + (\omega^2 LC - RG) + \sqrt{(RG-\omega^2 LC)^2 + (\omega^2 (RC+LG)^2)}}{2}$$

$$\alpha^2 = \frac{(RG-\omega^2 LC) + \sqrt{(RG-\omega^2 LC)^2 + (\omega^2 (RC+LG)^2)}}{2}$$

$$\alpha = \sqrt{\frac{(RG-\omega^2 LC) + \sqrt{(RG-\omega^2 LC)^2 + (\omega^2 (RC+LG)^2)}}{2}} \qquad \text{......(9)}$$

In a perfectly matched line R=0 and G=0,

Sub the above condition in β.

From equ (8),

$$\beta = \sqrt{\frac{(\omega^2 LC) + \sqrt{(-\omega^2 LC)^2}}{2}}$$

$$\beta = \omega\sqrt{LC}$$

The velocity of propagation of a ideal line is,

$$v = \frac{\omega}{\beta}$$

$$v = \frac{\omega}{\omega\sqrt{LC}}$$

$$v = \frac{1}{\sqrt{LC}}$$

The velocity of propagation is constant for a given L and C.