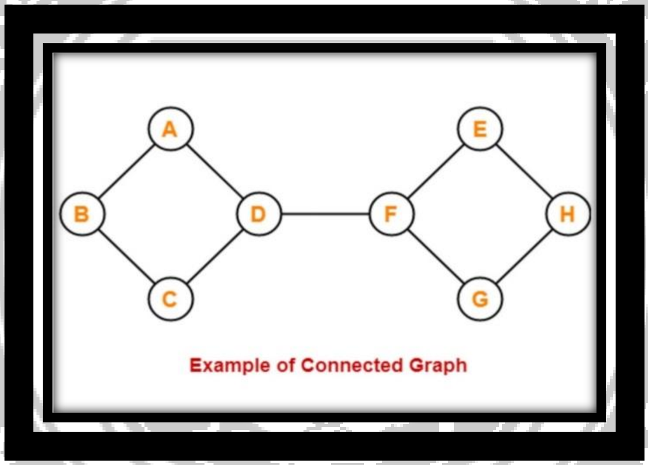
Directed Graphs:

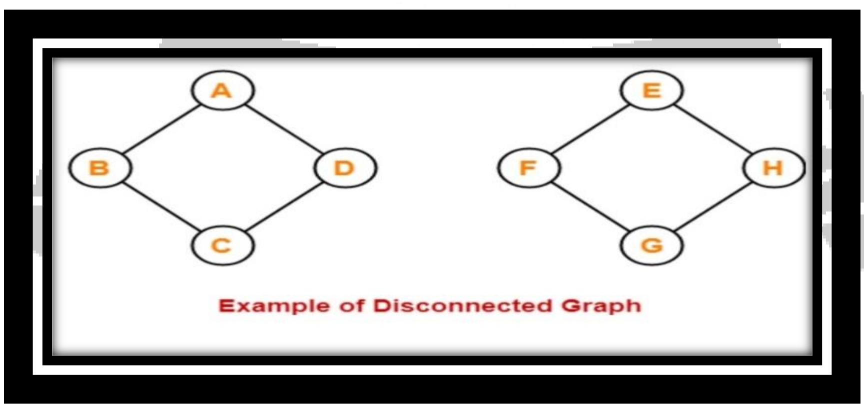
Connected Graph:

An directed graph is said to be connected if any pair of nodes are reachable from one another. That is, there is a path between any pair of nodes.



Disconnected graph:

A graph which is not connected is called disconnected graph.



Unilaterally connected:

A simple digraph is said to be unilaterally connected, if for any pair of nodes of the graph atleast one of the nodes of the pair is reachable from the other node.

Strongly connected:

A simple digraph is said to be strongly connected, if for any pair of nodes of the graph both the nodes of the pair are reachable from one to another.

Weakly connected:

A digraph is weakly connected, if it is connected as an undirected graph in which the direction of the edges is neglected.

Note:

A unilaterally connected digraph is weakly connected, but a weakly connected digraph is not necessarily unilaterally connected.

A strongly connected digraph is both unilaterally and weakly connected.

Theorem: 1

In a simple digraph $G = (V, E)$, every node of the digraph lies in exactly one strong component.

Proof:

Let $v \in V(G)$ and S be the set of all vertices of G which are mutually reachable with v.

Then $v \in S$, and S is a strong component of G. This shows that every vertex of G is contained in a strong component.

Assume that the vertex v is in two strong components S_1 and S_2.

Since $v \in S$, and any pair of vertices are mutually reachable with v, and also any pair of vertices of S_2

Are mutually reachable with v, we get any pair of vertices $S_1 \cup S_2$ are mutually reachable through v.

Therefore, $S_1 \cup S_2$ becomes one strong component of G.

This is impossible.

Therefore every vertex of G lies in exactly one strong component.

Hence the proof.