1.5. SAMPLING TECHNIQUES

A band limited continuous time signal with highest frequency F_m Hertz can be uniquely recovered from its samples provided that the sampling rate F_s is greater than or equal to $2F_m$ samples per second.

The sampling is the process of conversion of a continuous time signal into a discrete time signal. The sampling is performed by taking samples of continuous time signals at definite intervals of time.

The time interval between two successive samples is called sampling time. It is denoted by 'T'.

The inversion of sampling time is called sampling frequency and it is denoted by 'F_s'

Let x (t)= Analog /continuous time signals.

x (n)= Discrete time signal obtained by sampling $x_a(t)$.

Mathematically, the relation between x(n) and x(t) can be expressed as,

$x(n)= x_a(t).|_{t=nT} = x_a (n T)=x_a (n/F_s)$; for n in the range $-\infty < n < \infty$

Where T=Sampling period

$F_s= 1/T$ =Sampling rate or sampling frequency.

While sampling analog signals, the infinite frequency range continuous time signals are mapped to finite frequency range discrete time signals.

The relation between frequency of analog and discrete time signal is,

$$F=F/F_s$$

The range of frequency of discrete time signal is

$$\frac{-1}{2} \leq f \leq \frac{1}{2}$$

$$\frac{-1}{2} \leq \frac{F}{F_s} \leq \frac{1}{2}$$

On multiplying the equation by F_s

$$\frac{-F_s}{2} \leq F \leq \frac{F_s}{2}$$

When an analog signal is sampled at a frequency F_s, the highest analog frequency that can be uniquely represented by a discrete time signal will be $F_s/2$.

Hence the infinite number of high frequency continuous time signals will be represented by a single discrete time signal. Such signals are called alias. The phenomenon of high frequency component getting the identity of low frequency component during sampling is called aliasing.

Sampling an analog signal with frequency F by choosing a sampling frequency F_s such that $\frac{F_s}{2}$>F will not result in alias.but sampling frequency is selected such that Fs/2<F that the frequency above $F_s/2$ will have alias with frequency below $F_s/2$.Hence the point of reflection is $F_s/2$ and the frequency $F_s/2$ is called folding frequency.

Let F_{max} be the maximum frequency of analog signal that can be uniquely represented as discrete time signal when sampled at a frequency F_s.

Now $F_{max}=F_s/2$

$F_s=2F_{max}$

The above equation gives a choice for selecting the sampling frequency. From equation we can say that for unique representation of analog signal with maximum frequency F_{max}, the sampling frequency should be greater than $2F_{max}$.

To avoid aliasing $Fs \geq 2F_{max}$

When sampling frequency $F_s=2F_{max}$, the sampling rate is called Nyquist rate.

It is observed that a non shifted sinusoidal signal when sampled at nyquist rate will produce zero sample sequence .Hence to avoid zero sampling of sinewave, the sampling frequency F_s should be greater than $2F_{ma}$ where F_{max} is the maximum frequency in the analog signal.

A discrete signal obtained by sampling can be reconstructed to an analog signal, only when it is sampled without aliasing.