2.1 JOINT DISTRIBUTION

Two Dimensional Random variables:

Let S be the sample space. Let $X = X(s)$ and $Y = Y(s)$ be two functions each assigning a real number to each outcome $s \in S$. Then (X, Y) is a two dimensional random variable.

Types of Random Variables:

(i) Discrete Random Variables

(ii) Continuous random variables

Discrete random Variables:

Two Dimensional Discrete Random variable

If the possible values (X, Y) are finite, then (X, Y) is called a two – dimensional discrete random variable and it can be represented by $(x_i, y_j), i = 1, 2, \ldots, n; j = 1, 2, \ldots, m$

Two Dimensional Continuous Random variable

If (X, Y) can take all the values in a region R in the XY plane, then (X, Y) is called a two – dimensional continuous random variable.

Discrete Random variables:

In the study of two dimensional discrete random variables we have the following 5 important terms.

(i) Joint Probability function (or) Joint Probability mass function (PMF)

(ii) Joint Probability distribution

(iii) Marginal probability function of X

(iv) Marginal probability function of Y

(v)Conditional Probability function

Joint Probability Function (or) Joint Probability mass function

Let X, Y be a two dimensional discrete random variable for each possible outcome (X_i , Y_j) . We associate a number $P(X_i , Y_j)$ representing $[X = x_i , Y = y_j]$ and satisfies the following conditions

i) $P[x_i, y_j] \geq 0$

ii) $\sum \sum p(x_i, y_j) = 1$

The function $P[x_i, y_j]$ is called joint probability mass function of x , y

Conditional distribution of *X* given *Y*

$$P\left[X = x_i / Y = y_j\right] = \frac{P\left[X = x_i \cap Y = y_j\right]}{P[Y = y_j]}$$

$$= \frac{P\left[X = x_i, Y = y_j\right]}{P[Y = y_j]}$$

$$P\left[Y = y_j / X = x_i\right] = \frac{P\left[Y = y_j \cap X = x_i\right]}{P[X = x_i]}$$

$$= \frac{P\left[X = x_i, Y = y_j\right]}{P[X = x_i]}$$

Test of independent:

$$P\left[X = x_i, Y = y_j\right] = P[X = x_i], P[Y = y_j]$$

Problems on Marginal distribution:

1.The joint probability marginal function of X , Y is given by $P(xy) = K(2x + 3y)$, $x = 0, 1, 2$, $y = 1, 2, 3$ find K. Find all the marginal distribution and conditional probability distribution . Also probability distribution X + Y.

Solution:

	1	2	3	$\sum x$
0	3K	6K	9K	18K
1	5K	8K	11K	24K
2	7K	10K	13K	30K
$\sum y$	15K	24K	33K	72K

We know that $\sum \sum P(x, y) = 1$

$$\Rightarrow 72K = 1 \Rightarrow K = \frac{1}{72}$$

Marginal distribution

X	0	1	2
P(X)	$\frac{18}{72}$	$\frac{24}{72}$	$\frac{30}{72}$

Y	1	2	3
P(Y)	$\frac{15}{72}$	$\frac{24}{72}$	$\frac{33}{72}$

Conditional distribution at x given y

$$P[X=0/Y=1]=\frac{P[X=0,Y=1]}{P[Y=1]}=\frac{3/72}{15/72}=\frac{1}{5}$$

$$P[X=2/Y=2]=\frac{P[X=2,Y=2]}{P[Y=2]}=\frac{6/72}{24/72}=\frac{1}{4}$$

$$P[X=0/Y=3]=\frac{P[X=0,Y=3]}{P[Y=3]}=\frac{9/72}{33/72}=\frac{3}{11}$$

$$P[X=1/Y=1]=\frac{P[X=1,Y=2]}{P[Y=1]}=\frac{5/72}{15/72}=\frac{1}{3}$$

$$P[X=1/Y=2]=\frac{P[X=1,Y=2]}{P[Y=2]}=\frac{8/72}{24/72}=\frac{1}{3}$$

$$P[X=1/Y=3]=\frac{P[X=1,Y=3]}{P[Y=3]}=\frac{11/72}{33/72}=\frac{1}{3}$$

$$P[X=2/Y=1]=\frac{P[X=2,Y=1]}{P[Y=1]}=\frac{7/72}{15/72}=\frac{7}{15}$$

$$P[X=2/Y=2]=\frac{P[X=2,Y=2]}{P[Y=2]}=\frac{10/72}{24/72}=\frac{5}{12}$$

$$P[X=2/Y=3]=\frac{P[X=2,Y=3]}{P[Y=3]}=\frac{13/72}{33/72}=\frac{13}{33}$$

Conditional distribution at y given x

$$P[Y=1/X=0]=\frac{P[Y=1,X=0]}{P[X=0]}=\frac{3/72}{18/72}=\frac{1}{6}$$

$$P[Y=1/X=1]=\frac{P[Y=1,X=1]}{P[X=1]}=\frac{5/72}{24/72}=\frac{5}{24}$$

$$P[Y=1/X=2]=\frac{P[Y=1,X=2]}{P[X=2]}=\frac{7/72}{30/72}=\frac{7}{30}$$

$$P[Y=2/X=0]=\frac{P[Y=2,X=0]}{P[X=0]}=\frac{6/72}{18/72}=\frac{1}{3}$$

$$P[Y=2/X=1]=\frac{P[Y=2,X=1]}{P[X=1]}=\frac{8/72}{24/72}=\frac{1}{3}$$

$$P[Y=2/X=2]=\frac{P[Y=2,X=2]}{P[X=2]}=\frac{10/72}{30/72}=\frac{1}{3}$$

$$P[Y=3/X=0]=\frac{P[Y=3,X=0]}{P[X=0]}=\frac{9/72}{18/72}=\frac{1}{2}$$

$$P[Y=3/X=1]=\frac{P[Y=3,X=1]}{P[X=1]}=\frac{11/72}{24/72}=\frac{11}{24}$$

$$P[Y=3/X=2]=\frac{P[Y=3,X=2]}{P[X=2]}=\frac{13/72}{30/72}=\frac{13}{30}$$

Distribution of $x + y$

1	P_{01}	$3/72$
2	$P_{02} + P_{11}$	$11/72$
3	$P_{03} + P_{12} + P_{21}$	$24/72$
4	$P_{13} + P_{22}$	$21/72$

5	P_{23}	$^{13}/_{72}$

2. The joint distribution of X and Y is given by $f(x,y) = \frac{x+y}{21}, x = 1,2,3\ ; y = 1,2$. Find the marginal distributions.

Solution:

Given $f(x,y) = \frac{x+y}{21}, x = 1,2,3\ ; y = 1,2$

$\Rightarrow f(1,1) = \frac{2}{21}$

$\Rightarrow f(1,2) = \frac{3}{21}$

$\Rightarrow f(2,1) = \frac{3}{21},$

$\Rightarrow f(2,2) = \frac{4}{21},$

$\Rightarrow f(3,1) = \frac{4}{21},$

$\Rightarrow f(3,2) = \frac{5}{21}$

The marginal distributions are given in the table.

		X			$P_Y(y) = P(Y = y)$
		1	2	3	
Y	1	$\frac{2}{21}$ P(1,1)	$\frac{3}{21}$ P(2,1)	$\frac{4}{21}$ P(3,1)	$\frac{9}{21}$
	2	$\frac{3}{21}$ P(1,2)	$\frac{4}{21}$ P(2,2)	$\frac{5}{21}$ P(3,2)	$\frac{12}{21}$ P(1,1)
$P_X(x) = P(X = x)$		$\frac{5}{21}$	$\frac{7}{21}$	$\frac{9}{21}$	1

The marginal distribution of X

$P_X(1) = P(X = 1) = \frac{5}{21}$, $P_X(2) = P(X = 2) = \frac{7}{21}$, $P_X(3) = P(X = 3) = \frac{9}{21}$

The marginal distribution of Y

$P_Y(1) = P(Y = 1) = \frac{9}{21}$, $P_Y(2) = P(Y = 2) = \frac{12}{21}$