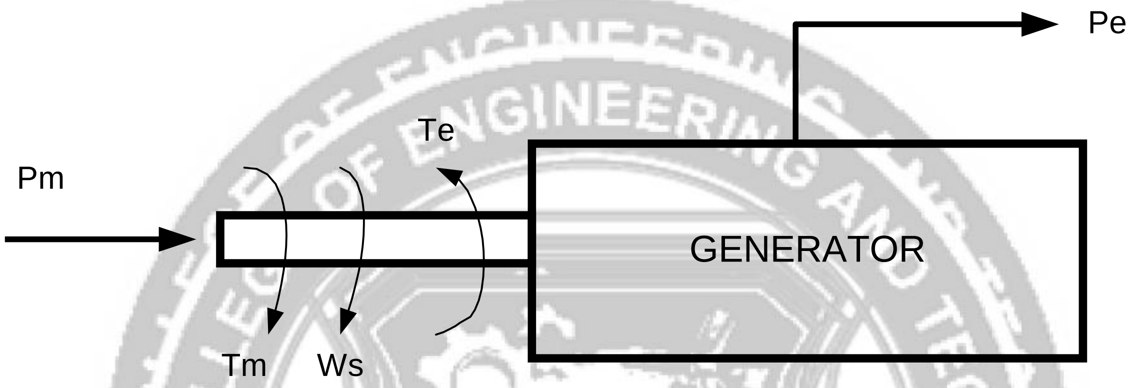
`Swing Equation:-



(Fig.-1 Flow of power in a synchronous generator)

Consider a synchronous generator developing an electromagnetic torque T_e(and a corresponding electromagnetic power P_e) while operating at the synchronous speed $w_{s.}$ If the input torque provided by the prime mover, at the generator shaft is T_i, then under steady state conditions (i.e., without any disturbance).

$$T_e = T_i \quad \text{.......... (10)}$$

Here we have neglected any retarding torque due to rotational losses. Therefore we have

$$T_e w_s = T_i w_s \quad \text{.......... (11)}$$

And $$T_e w_s - T_i w_s = P_i - P_e = 0 \quad \text{.......... (12)}$$

When a change in load or a fault occurs, then input power Pi is not equal to Pe. Therefore left side of equation is not zero and an accelerating torque comes into play. If Pa is the accelerating (or decelerating) power, then

$$P_i - P_e = M.\frac{d^2\theta_e}{dt^2} + D\frac{d\theta_e}{dt} = P_a \quad \text{.......... (13)}$$

Where D = damping coefficient

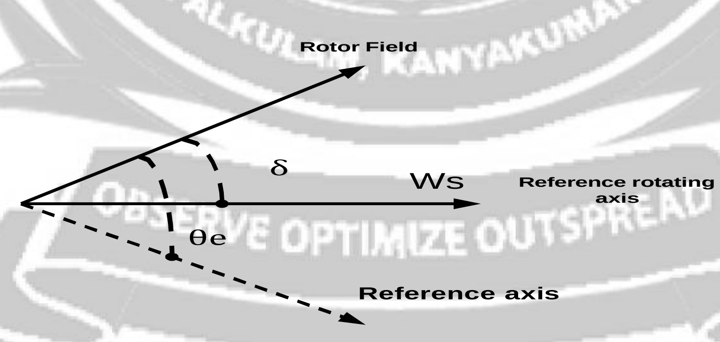
θ_e = electrical angular position of the rotor

It is more convenient to measure the angular position of the rotor with respect to a synchronously rotating frame of reference. Let

$$\delta = \theta_e - w_s.t \quad \text{.......... (14)}$$

So $$\frac{d^2\theta_e}{dt^2} = \frac{d^2\delta}{dt^2} \quad \text{.......... (15)}$$

Where δ is power angle of synchronous machine.



(Fig.2 Angular Position of rotor with respect to reference axis)

Neglecting damping (i.e., D = 0) and substituting equation (15) in equation (13) we get

$$M.\frac{d^2\delta}{dt^2} = P_i - P_e \text{ MW} \ldots \quad (16)$$

Using equation (6) and (16), we get

$$\frac{GH}{\pi f}.\frac{d^2\delta}{dt^2} = P_i - P_e \text{ MW} \quad (17)$$

Dividing throughout by G, the MVA rating of the machine,

$$M_{(pu)}.\frac{d^2\delta}{dt^2} = (P_i - P_e) \text{ pu} \quad (18)$$

Where $$M_{(pu)} = \frac{H}{\pi f} \quad (19)$$

or $$\frac{H}{\pi f}.\frac{d^2\delta}{dt^2} = (P_i - P_e) \text{ pu} \quad (20)$$

Equation (20) is called **Swing Equation.** It describes the rotor dynamics for a synchronous machine. Damping must be considered in dynamic stability study.

Multi Machine System:-

In a multi machine system a common base must be selected. Let

$G_{machine}$ = machine rating (base)

G_{system} = system base

Equation (20) can be written as:

$$\frac{G_{machine}}{G_{system}}\left(\frac{H_{machine}}{f}\right)\frac{d^2\delta}{dt^2} = (P_i - P_e).\frac{G_{machine}}{G_{system}} \quad (21)$$

So $$\left(\frac{H_{system}}{f}\right)\frac{d^2\delta}{dt^2} = (P_i - P_e) \text{ pu on system base} \ldots \quad (22)$$

Where $$H_{system} = \frac{G_{machine}}{G_{system}}.H_{machine} \quad (23)$$

= machine inertia constant in system base

Machines Swinging in Unison (Coherently) :-

Let us consider the swing equations of two machines on a common system base, i.e.,

$$\frac{H_1}{\pi f}.\frac{d^2\delta_1}{dt^2} = (P_{i1} - P_{e1}) \quad (24)$$

$$\frac{H_2}{\pi f} \cdot \frac{d^2\delta_2}{dt^2} = (P_{i2} - P_{e2}) \text{..} (25)$$

Since the machines rotor swing in unison,

$$\delta_1 = \delta_2 = \delta \text{..} (26)$$

Adding equations (24) and (25) and substituting equation (26), we get

$$\frac{H_{eq}}{f} \cdot \frac{d^2\delta}{dt^2} = (P_i - P_e) \text{..} (27)$$

Where $P_i = P_{i1} + P_{i2}$

$P_e = P_{e1} + P_{e2}$

$H_{eq} = H_1 + H_2$

Equivalent inertia H_{eq} can be expressed as:

$$H_{eq} = \left(\frac{G_{1,machine}}{G_{System}}\right) . H_{1,machine} + \left(\frac{G_{2,machine}}{G_{system}}\right) . H_{2,machine} \text{................} (28)$$

<u>Example1</u>:-

A 60 Hz, 4 pole turbo-generator rated 100MVA, 13.8 KV has inertia constant of 10 MJ/MVA.

(a) Find stored energy in the rotor at synchronous speed.
(b) If the input to the generator is suddenly raised to 60 MW for an electrical load of 50 MW, find rotor acceleration.
(c) If the rotor acceleration calculated in part (b) is maintained for 12 cycles, find the change in torque angle and rotor speed in rpm at the end of this period.
(d) Another generator 150 MVA, having inertia constant 4 MJ/MVA is put in parallel with above generator. Find the inertia constant for the equivalent generator on a base 50 MVA.

<u>Solution</u>:-

(a) Stored energy = GH
= 100MVA x 10MJ/MVA
= 1000MJ

(b) $P_a = P_i - P_e = 60\text{-}50 = 10\text{MW}$

We know, $M = \frac{GH}{180f} = \frac{100X10}{180X60} = \frac{5}{54}$ MJ.sec/elect.deg.\

Now $M.\frac{d^2\delta}{dt^2} = P_i - P_e = P_a$

⇨ $\frac{5}{54}\frac{d^2\delta}{dt^2} = 10$

⇨ $\frac{d^2\delta}{dt^2} = \frac{10X54}{5}$ = 108 elect.deg./sec^2

So, α = acceleration = 108 elect.deg./sec^2

(c) 12 cycles = 12/60 = 0.2sec.

Change in δ = ½ α.(Δt)^2 = ½.108.(0.2)^2 =2.16 elect.deg

Now α = 108 elect.deg./sec^2

= 60 x (108/360^o) rpm/sec

= 18 rpm/sec

Hence rotor speed at the end of 12 cycles

$= \frac{120f}{P} + \alpha.\Delta t$

$= (\frac{120K60}{4} + 18X0.2)$ rpm

= 1803.6 rpm.

(d) $H_{eq} = \frac{H1G1}{G_b} + \frac{H_2G_2}{G_b} = \frac{10x100}{50} + \frac{4x150}{50}$ =32MJ/MVA