## 3.1. DISCRETE FOURIER TRANSFORM

The discrete Fourier transform converts a finite sequence of equally-spaced samples of a function into a same-length sequence of equally-spaced samples of the discrete-time Fourier transform, which is a complex-valued function of frequency.

The N point DFT of x (n) can be expressed as

$$X(K) = \sum_{n=0}^{N-1} x(n) e^{\frac{-j2\pi kn}{N}}$$

For k=0, 1, 2 ….N-1

**Example-1**

Compute the DFT of the sequence is given by

x (n)= {0,1,2,1}

**Soln:**

$$X(K) = \sum_{n=0}^{N-1} x(n) e^{\frac{-j2\pi kn}{N}}$$

The given signal x (n) is 4 point signal. Let us compute 4 point DFT.

$X(K) = \sum_{n=0}^{4-1} x(n) e^{\frac{-j2\pi kn}{4}}$

$= \sum_{n=0}^{3} x(n) e^{\frac{-j\pi kn}{2}}$

$= x(0)e^{0} + x(1)\, e^{\frac{-j\pi k}{2}} + x(2)\, e^{-j\pi k} + x(3)\, e^{\frac{-j3\pi k}{2}}$

$= 0 + e^{\frac{-j\pi k}{2}} + 2\, e^{-j\pi k} + e^{\frac{-j3\pi k}{2}}$

$= \cos\frac{\pi k}{2} - j \sin\frac{\pi k}{2} + 2(\cos \pi k - j \sin \pi k) + \cos\frac{3\pi k}{2} - j \sin\frac{3\pi k}{2}$

$= (\cos\frac{\pi k}{2} + 2\cos \pi k + \cos\frac{3\pi k}{2}) - j(\sin\frac{\pi k}{2} + \sin\frac{3\pi k}{2})$

K=0, 1, 2, 3

When k=0;

X (0) = (cos 0+2 cos 0+cos 0)-j(sin 0+sin 0)

= (1+2+1)-j (0+0) =4

When k=1

$X(1) = (\cos\frac{\pi}{2} + 2\cos\pi + \cos\frac{3\pi}{2}) - j(\sin\frac{\pi}{2} + \sin\frac{3\pi}{2})$

= (0-2+0)-j (1-1) = -2

When k=2

$X(2) = (\cos\pi + 2\cos 2\pi + \cos 3\pi) - j(\sin\pi + \sin 3\pi)$

= (-1+2-1)-j(0+0)

=0

When k=3

$X(3) = (\cos\frac{3\pi}{2} + 2\cos 3\pi + \cos\frac{9\pi}{2}) - j(\sin\frac{3\pi}{2} + \sin\frac{9\pi}{2})$

= (0-2+0)-j (-1+1) =-2

**Answer:**

X(0)=4,X(1)=2,X(2)=0,X(3)=-2

### 3.1.1. MAGNITUDE AND PHASE REPRESENTATION:

The X (k) is a discrete function of frequency of discrete time signal ω, and so it is also called discrete frequency spectrum of the discrete time signal x (n).

The X (k) is a complex valued function of k and so it can be expressed in rectangular form as,

$X(k) = X_r(k) + j X_i(k)$

Where

$X_r(k)$ = Real part of X (k)

$X_i(k)$ = Imaginary part of X (k)

Now the magnitude function (or) Magnitude spectrum $|X(k)|$ can be defined as

$|X(k)|^2 = X(k) X^*(k)$ (or) $|X(k)| = \sqrt{X(k) X*(k)}$

Where $X^*(k)$ is a complex conjugate of X (k)

The Phase function (or phase spectrum) $\angle X(k)$ is defined as

$\angle X(k) = \text{Arg}[X(k)] = \tan^{-1}\left[\frac{Xi(k)}{Xr(k)}\right]$

Since X (k) is a sequence consisting of N-complex numbers for k=0, 1, 2,.N-1,the magnitude and phase spectrum of X(k) can be expressed as a sequence as shown below.

Magnitude sequence, $|X(k)| = \{|X(0)|, |X(1)|, |X(2)|, \ldots\ldots |X(N-1)|\}$

Phase sequence, $\angle X(k) = \{\angle X(0), \angle X(1), \angle X(2), \ldots\ldots\ldots, \angle X(N-1)\}$

The plot of samples of magnitude sequence versus k is called magnitude spectrum and the plot of samples of phase sequence versus k is called phase spectrum. In general, these plots are called frequency spectrum.