# UNIT IV

INITIAL VALUE PROBLEMS FOR ORDINARY DIFFERENTIAL EQUATIONS

PROBLEMS BASED ON RUNGE KUTTA METHOD METHOD

## $\underline{Runge-Kutta\ Method}$

$$k_1 = hf(x_n, y_n)$$

$$k_2 = hf\left(x_n + \frac{h}{2}, y_n + \frac{k_1}{2}\right)$$

$$k_3 = hf\left(x_n + \frac{h}{2}, y_n + \frac{k_2}{2}\right)$$

$$k_4 = hf(x_n + h, y_n + k_3)$$

$$\Delta y = \frac{1}{6}[k_1 + 2k_2 + 2k_3 + k_4]$$

$$y_1 = y_0 + \Delta y$$

$1.$ *Using Runge – Kutta Method find* $y$ *at* $x = 0.1$

*if* $\frac{dy}{dx} = x + y^2$ *with* $y(0) = 1$

*solution*:

*Given* $y' = f(x, y) = x + y^2$ *and*

$x_0 = 0$ *and* $y_0 = 1$

$x_1 = 0.1$ *and* $y_1 = ?$

$h = x_1 - x_0 = 0.1 - 0 = 0.1$

*By Runge – Kutta Method*

$k_1 = hf(x_n, y_n)$

$k_2 = hf\left(x_n + \frac{h}{2}, y_n + \frac{k_1}{2}\right)$

$k_3 = hf\left(x_n + \frac{h}{2}, y_n + \frac{k_2}{2}\right)$

$k_4 = hf(x_n + h, y_n + k_3)$

$\Delta y = \frac{1}{6}[k_1 + 2k_2 + 2k_3 + k_4]$

$k_1 = hf(x_0, y_0)$

$= (0.1)f(0, 1)$

$= (0.1)[0 + 1^2] = 0.1$

$k_2 = hf\left(x_0 + \frac{h}{2}, y_0 + \frac{k_1}{2}\right)$

$= (0.1)f\left(0 + \frac{0.1}{2}, 1 + \frac{0.1}{2}\right)$

$= (0.1)f(0.05, 1.05)$

$= (0.1)[0.05 + (1.05)^2] = 0.1[0.05 + 1.1025]$

$= 0.1[1.1525]$

$= 0.1153$

$k_3 = hf\left(x_0 + \frac{h}{2}, y_0 + \frac{k_2}{2}\right)$

$= (0.1)f\left(0 + \frac{0.1}{2}, 1 + \frac{0.1153}{2}\right)$

$= (0.1)f(0.05, 1.0577)$

$= (0.1)[0.05 + (1.0577)^2] = 0.1[0.05 + 1.1187]$

$= 0.1[1.1687]$

$= 0.1169$

$k_4 = hf(x_0 + h, y_0 + k_3)$

$= hf(0 + 0.1, 1 + 0.1169)$

$= hf(0.1, 1.1169)$

$= (0.1)[0.1 + (1.1169)^2] = 0.1[0.1 + 1.2475]$

$= 0.1[1.3475]$

$= 0.1348$

$\Delta y = \frac{1}{6}[k_1 + 2k_2 + 2k_3 + k_4]$

$= \frac{1}{6}[0.1 + 2(0.1153) + 2(0.1169) + 0.1348]$

$= \frac{1}{6}[0.4686] = 0.0781$

$y_1 = y_0 + \Delta y = 1 + 0.0781 = 1.0781$

**2.** *Using Runge − Kutta Method find y at* $x = 0.2$

*if* $\frac{dy}{dx} = \frac{y^2 - x^2}{y^2 + x^2}$ *with* $y(0) = 1$

*solution*:

*Given* $y' = f(x, y) = \frac{y^2 - x^2}{y^2 + x^2}$ *and*

$x_0 = 0$ *and* $y_0 = 1$

$x_1 = 0.2$ *and* $y_1 = ?$

$h = x_1 - x_0 = 0.2 - 0 = 0.2$

***By Runge − Kutta Method***

$$k_1 = hf(x_n, y_n)$$

$$k_2 = hf\left(x_n + \frac{h}{2}, y_n + \frac{k_1}{2}\right)$$

$$k_3 = hf\left(x_n + \frac{h}{2}, y_n + \frac{k_2}{2}\right)$$

$$k_4 = hf(x_n + h, y_n + k_3)$$

$$\Delta y = \frac{1}{6}[k_1 + 2k_2 + 2k_3 + k_4]$$

$$k_1 = hf(x_0, y_0)$$

$$= (0.2)f(0, 1)$$

$$= (0.2)\left[\frac{1^2 - 0^2}{1^2 + 0^2}\right] = (0.2)\left[\frac{1}{1}\right] = 0.2$$

$$k_2 = hf\left(x_0 + \frac{h}{2}, y_0 + \frac{k_1}{2}\right)$$

$$= (0.2)f\left(0 + \frac{0.2}{2}, 1 + \frac{0.2}{2}\right)$$

$$= (0.2)f(0.1, 1.1)$$

$$= (0.2)\left[\frac{1.1^2 - 0.1^2}{1.1^2 + 0.1^2}\right] = 0.2\left[\frac{1.2}{1.22}\right]$$

$$= 0.2[0.9836]$$

$$= 0.1967$$

$$k_3 = hf\left(x_0 + \frac{h}{2}, y_0 + \frac{k_2}{2}\right)$$

$$= (0.2)f\left(0 + \frac{0.2}{2}, 1 + \frac{0.1967}{2}\right)$$

$$= (0.2)f(0.1, 1.0984)$$

$$= (0.2)\left[\frac{1.0984^2 - 0.1^2}{1.0984^2 + 0.1^2}\right] = 0.1967$$

$$k_4 = hf(x_0 + h, y_0 + k_3)$$

$$= h\,f(0 + 0.2, 1 + 0.1967)$$

$$= (0.2)f(0.2, 1.1967)$$

$$= (0.2)\left[\frac{1.1967^2 - 0.2^2}{1.1967^2 + 0.2^2}\right] = 0.1891$$

$$\Delta y = \frac{1}{6}[k_1 + 2k_2 + 2k_3 + k_4]$$

$$= \frac{1}{6}[0.2 + 2(0.1967) + 2(0.1967) + 0.1891]$$

$$= 0.19598$$

$$y_1 = y_0 + \Delta y$$

$$= 1 + 0.19598 = 1.19598$$

**3. Apply the fourth order Runge kutta method to find an approximate value of y when x=0.2 and x=0.4 given that $y' = x + y, y(0) = 1\ with\ h = 0.2$**

**Solution :**

Given $y' = x + y, y(0) = 1\ with\ h = 0.2$

$$x_0 = 0, y_0 = 1$$

$$f(x, y) = x + y$$

$$By\ Runge-Kutta\ Method$$

$$k_1 = hf(x_n, y_n)$$

$$k_2 = hf\left(x_n + \frac{h}{2}, y_n + \frac{k_1}{2}\right)$$

$$k_3 = hf\left(x_n + \frac{h}{2}, y_n + \frac{k_2}{2}\right)$$

$$k_4 = hf(x_n + h, y_n + k_3)$$

$$\Delta y = \frac{1}{6}[k_1 + 2k_2 + 2k_3 + k_4]$$

$$k_1 = hf(x_0, y_0)$$

$$= 0.2(x_0 + y_0)$$

$$= 0.2(0 + 1)$$

$$= 0.2$$

$$k_2 = hf\left(x_0 + \frac{h}{2}, y_0 + \frac{k_1}{2}\right)$$

$$=0.2\ f\left(0 + \frac{0.2}{2}, 1 + \frac{1}{2}(0.2)\right)$$

$$=0.2\ f(0.1, 1.1)$$

$$=0.2\ (0.1 + 1.1)$$

$$=0.2\ (1.2)$$

$$=0.24$$

$$k_3 = hf\left(x_0 + \frac{h}{2}, y_0 + \frac{k_2}{2}\right)$$

$$=0.2\ f\left(0 + \frac{0.2}{2}, 1 + \frac{1}{2}(0.24)\right)$$

=0.2 $f(0.1, 1.12)$

=0.2 $(0.1 + 1.12)$

=0.2 $(1.22)$

=0.244

$k_4 = hf(x_0 + h, y_0 + k_3)$

$= 0.2\, f(0 + 0.2, 1 + 0.244)$

$= 0.2\, f(0.2, 1.244)$

$= 0.2\,(0.2 + 1.244)$

$= 0.2\,(1.444)$

$= 0.2888$

$\Delta y = \frac{1}{6}[k_1 + 2k_2 + 2k_3 + k_4]$

$= \frac{1}{6}[0.2 + 2(0.24) + 2(0.244) + 0.2888]$

=0.2428

$y_1 = y(0.2) = y_0 + \Delta y$

= 1+0.2428

=1.2428

Now starting from $(x_1, y_1)$ *we get* $(x_1, y_1)$

Here $x_1 = 0.2, y_1 = 1.242$

$k_1 = hf(x_1, y_1)$
$= 0.2(x_1 + y_1)$

$= 0.2(0.2 + 1.2428)$

$= 0.28856$

$$k_2 = hf\left(x_1 + \frac{h}{2}, y_1 + \frac{k_1}{2}\right)$$

$$= 0.2\, f\left(0.2 + \frac{0.2}{2}, 1.2428 + \frac{1}{2}(0.28856\right)$$

$$= 0.2\, f(0.3, 1.38708)$$

$$= 0.2\,(0.3 + 1.38708)$$

$$= 0.337416$$

$$k_3 = hf\left(x_1 + \frac{h}{2}, y_1 + \frac{k_2}{2}\right)$$

$$= 0.2\, f\left(0.2 + \frac{0.2}{2}, 1.2428 + \frac{1}{2}(0.337416\right)$$

$$= 0.2\, f(0.3, 1.411508)$$

$$= 0.2\,(0.3 + 1.411508)$$

$$k_3 = 0.3423$$

$$k_4 = hf(x_1 + h, y_1 + k_3)$$

$$= 0.2\, f(0.2 + 0.2, 1.2428 + 0.3423)$$

$$= 0.2\, f(0.4, 1.5851)$$

$$= 0.2\,(0.4 + 1.5851)$$

$$= 0.39702$$

$$\Delta y = \frac{1}{6}[k_1 + 2k_2 + 2k_3 + k_4]$$

$$\Delta y = \frac{1}{6}[0.28856 + 2(0.337416) + 2(0.3423) + 0.39702]$$

$$= \frac{1}{6}[0.28856 + 0.674832 + 0.6846 + 0.39702]$$

$\Delta y$ =**0.3408**

$$y(0.2) = y_2 = y_1 + \Delta y$$

$$= 1.2428 + 0.3408$$

$$y(0.2) = 1.5836$$

**4. Apply the fourth order Runge kutta method to find an approximate value of y when x=0.2 and x=0.4 given that $y'' + xy' + y = 0,$**

$y(0) = 1 \;\; y'(0) = 0 \; with \; h = 0.2$

**Solution :**

$$y'' = -xy' - y, \qquad y(0) = 1 \;\; y'(0) = 0 \; with \; h = 0.2$$

$$x_0 = 0, y_0 = 1$$

**Setting y'=z**

**The equation becomes**

$$y'' = z' = -xz - y,$$

$$\frac{dy}{dx} = z = f_1(x, y, z)$$

$$\frac{dz}{dx} = -xz - y = f_2(x, y, z)$$

**By Algorithm,**

$$k_1 = hf_1(x_0, y_0, z_0) = (0.1)f_1(0, 1, 0) = (0.1)(0) = 0$$

$$l_1 = hf_2(x_0, y_0, z_0) = (0.1)f_2(0, 1, 0) = (0.1)(-1) = -0.1$$

$$k_2 = hf_1\left(x_0 + \frac{h}{2}, y_0 + \frac{k_1}{2}, z_0 + \frac{l_1}{2}\right)$$

$$k_2 = 0.1f_1\left(x_0 + \frac{h}{2}, y_0 + \frac{k_1}{2}, z_0 + \frac{l_1}{2}\right)$$

$= (0.1)f_1(0.05, 1, -0.05) = (0.1)(-0.05) = -0.005$

$l_2 = (0.1)f_2(0.05, 1 - 0.05) = (0.1)[1 + (0.05)(0.05) - 1]$

$= 0.09975$

$k_3 = hf_1\left(x_0 + \frac{h}{2}, y_0 + \frac{k_2}{2}, z_0 + \frac{l_2}{2}\right)$

$= (0.1)f_1(0.05, 0.9975, -0.0499) = (0.1)(-0.0499) = -0.00499$

$l_3 = (0.1)f_2(0.05, 0.9975, -0.0499)$

$= (0.1)(0.05 + 0.9975 - 0.0499)$

=-0.09950

$k_3 = hf_1(x_0 + h, y_0 + k_3, z_0 + l_3)$

$= (0.1)f_1(0.1, 0.99511, -0.0995) = (0.1)(-0.0995) = -0.00995$

$l_4 = (0.1)f_2(0.1, 0.99511, -0.0995)$

$= (0.1)(-(0.1 - 0.0995 + 0.99511)$

=-0.0985

$y_1 = y(0.1) = y_0 + \Delta y$=$1 + \frac{1}{6}[0 + 2(-0.005) + 2(-0.00499) - 0.00995]$

=0.9950

.5. $Cosider\ the\ initial\ value\ problem\ \frac{dy}{dx} = y - x^2 + 1$

$$with\ y(0) = 0.5$$

$(i) Using\ the$ modified Euler method ,find y(0.2)

$(ii) Using\ the\ 4th$ order Runge kutta method ,find y(0.4) and y(0.6)

**Solution :**

(i) $(x, y) = y - x^2 + 1\ ,\ x_0 = 0$ $\quad y_0 = 0.5$ , h=0.2 $x_1 = 0.2$

***Modified Euler Method***

$$y_{n+1} = y_0 + h\left[f\left(x_n + \frac{h}{2}, y_n + \frac{h}{2} f(x_n, y_n)\right)\right]$$

$$y_1 = y_0 + h\left[f\left(x_0 + \frac{h}{2}, y_0 + \frac{h}{2} f(x_0, y_0)\right)\right]$$

$$y_1 = 0.5 + 0.2\left[f\left(0 + \frac{0.2}{2}, 0.5 + \frac{0.2}{2} f(0, 0.5)\right)\right]$$

$$y_1 = 0.5 + 0.2[f(0.1, 0.5 + (0.1)(0.5 + 1))]$$

$$y_1 = 0.5 + 0.2[f(0.1, 0.65)]$$

$$y_1 = 0.5 + 0.2[0.65 - (0.1)^2 + 1\ ]$$

$$y_1 = 0.5 + 0.328$$

$$y_1 = 0.828$$

*(ii)* $(x, y) = y - x^2 + 1\ ,\ x_0 = 0$ $\quad y_0 = 0.5$ , h=0.2 $x_1 = 0.2$

$y_1 = 0.828,\quad x_2 = 0.4$

To find $\quad y_2 = y(0.4)$

## By Runge Kutta 4th order method

***Runge – Kutta Method***

$k_1 = hf(x_n, y_n)$

$k_2 = hf\left(x_n + \frac{h}{2}, y_n + \frac{k_1}{2}\right)$

$k_3 = hf\left(x_n + \frac{h}{2}, y_n + \frac{k_2}{2}\right)$

$k_4 = hf(x_n + h, y_n + k_3)$

$\Delta y = \frac{1}{6}[k_1 + 2k_2 + 2k_3 + k_4]$

$$y_1 = y_0 + \Delta y$$

$k_1 = hf(x_1, y_1)$

$=(0.2)[y_1 - x_1^2 + 1]$

$=(0.2)[0.828 - (0.2)^2 + 1]$

$=0.3576$

$k_2 = hf\left(x_1 + \frac{h}{2}, y_1 + \frac{k_1}{2}\right)$

$$= (0.2)f\left((0.2) + \frac{(0.2)}{2}, 0.828 + \frac{0.3576}{2}\right)$$

$$= (0.2)f(0.3, 1.0068)$$

$=(0.2)[1.0068 - (0.3)^2 + 1]$

=(0.2)[1.9168]

=0.3834

$$k_3 = hf\left(x_1 + \frac{h}{2}, y_1 + \frac{k_2}{2}\right)$$

$$= (0.2)f\left((0.2) + \frac{(0.2)}{2}, 0.828 + \frac{0.3834}{2}\right)$$

$$= (0.2)f(0.3, 1.0197)$$

=(0.2)$[1.0197 - (0.3)^2 + 1]$

=(0.2)[1.9297]

=0.3859

$$k_4 = hf(x_1 + h, y_1 + k_3)$$

$$k_4 = (0.2)f(0.2 + 0.2, 0.828 + 0.3859)$$

$$= (0.2)f(0.4, 1.2139)$$

= (0.2)$[1.2139 - (0.4)^2 + 1]$

= (0.2)[2.0539]

=0.4108

$$\Delta y = \frac{1}{6}[k_1 + 2k_2 + 2k_3 + k_4]$$

$$= \frac{1}{6}[0.3576 + 2(0.3834) + 2(0.3859) + 0.4108]$$

**=0.3845**

$y(0.4) = y_1 + \Delta y$

$= 0.828 + 0.3845$

$y(0.4) = y_2 = 1.2125$

**To find** $y_3 = y(0.6)$

**By Runge Kutta 4th order method**

$k_1 = hf(x_2, y_2)$

$= (0.2)f(0.4, 1.2125)$

$= (0.2)[1.2125 - (0.4)^2 + 1] = 0.4105$

$$k_2 = hf\left(x_2 + \frac{h}{2}, y_2 + \frac{k_1}{2}\right)$$

$$k_2 = (0.2)f\left(0.4 + \frac{0.2}{2}, 1.2125 + \frac{0.4105}{2}\right)$$

$$k_2 = (0.2)f(0.5, 1.41775)$$

$= (0.2)[1.41775 - (0.5)^2 + 1] = 0.4336$

$$k_3 = hf\left(x_3 + \frac{h}{2}, y_3 + \frac{k_2}{2}\right)$$

$$= (0.2)f\left(0.4 + \frac{0.2}{2}, 1.2125 + \frac{0.4336}{2}\right)$$

$$= (0.2)f(0.5, 1.4293)$$

$$= (0.2)[1.4293 - (0.5)^2 + 1] = 0.4359$$

$$k_4 = hf(x_2 + h, y_2 + k_3)$$

$$k_4 = (0.2)f(0.4 + 0.2, 1.2125 + 0.4359)$$

$$k_4 = (0.2)f(0.6, 1.6484)$$

$$k_4 = (0.2)[1.6484 - (0.6)^2 + 1] = 0.4577$$

$$\Delta y = \frac{1}{6}[0.4105 + 2(0.4336) + 2(0.4359) + 0.4577]$$

$$= 0.4345$$

$$y(0.6) = y_3 = y_2 + \Delta y = 1.2125 + 0.4345 = 1.647$$

$$6.\ Using\ Runge - Kutta\ \ fourth\ order\ Method$$

$find\ y\ at\ x = 0.1, 0.2, 0.3\ if\ \frac{dy}{dx} = xy + y^2\ \ with\ y(0) = 1$and alsofind the solution at $x = 0.4$ using Milne's method -

$solution$:

$Given\ y' = f(x, y) = xy + y^2\ \ and$

$$x_0 = 0\ and\ y_0 = 1$$

$$h = x_1 - x_0 = 0.1 - 0 = 0.$$

$By\ Runge - Kutta\ Method$

$k_1 = hf(x_n, y_n)$

$k_2 = hf\left(x_n + \frac{h}{2}, y_n + \frac{k_1}{2}\right)$

$k_3 = hf\left(x_n + \frac{h}{2}, y_n + \frac{k_2}{2}\right)$

$k_4 = hf(x_n + h, y_n + k_3)$

$$\Delta y = \frac{1}{6}[k_1 + 2k_2 + 2k_3 + k_4]$$

$k_1 = hf(x_0, y_0)$

$= (0.1)f(0, 1)$

$= (0.1)[0 + 1^2\ ] = 0.1$

$k_2 = hf\left(x_0 + \frac{h}{2}, y_0 + \frac{k_1}{2}\right)$

$= (0.1)f\left(0 + \frac{0.1}{2}, 1 + \frac{0.1}{2}\right)$

$= (0.1)f(0.05, 1.05)$

$= (0.1)[(0.05)(1.05) + (1.05)^2\ ] = 0.1[0.0525 + 1.1025]$

$= 0.1[1.155]$

$= 0.1155$

$k_3 = hf\left(x_0 + \frac{h}{2}, y_0 + \frac{k_2}{2}\right)$

$= (0.1)f\left(0 + \frac{0.1}{2}, 1 + \frac{0.1155}{2}\right)$

$= (0.1)f(0.05, 1.05775)$

$= (0.1)[(0.05)(1.05775) + (1.05775)^2 ] = 0.1[0.0528875 + 1.118835]$

$= 0.1[1.1717225]$

$= 0.11717$

$k_4 = hf(x_0 + h, y_0 + k_3)$

$= 0.1f(0 + 0.1, 1 + 0.11717)$

$= hf(0.1, 1.11717)$

$= (0.1)[(0.1)(1.11717) + (1.11717)^2 ] = 0.1[0.111717 + 1.24807]$

$= 0.13598$

$$\Delta y = \frac{1}{6}[k_1 + 2k_2 + 2k_3 + k_4]$$

$$= \frac{1}{6}[0.1 + 2(0.1155) + 2(0.11717) + 0.13598]$$

$= \frac{1}{6}[0.70132] = 0.11689$

$y_1 = y_0 + \Delta y = 1 + 0.11689 = 1.11689$

**To find $y(0.2)$**

$$x_1 = 0.1 \qquad y_1 = 1.11689$$

$$k_1 = hf(x_1, y_1) = (0.1)f(0.1, 1.11689)$$

$$= (0.1)[(0.1)(1.11689) + (1.11689)^2]$$

$$= (0.1)[0.111689 + 1.24744]$$

$$= 0.1359$$

$$k_2 = hf\left(x_1 + \frac{h}{2}, y_1 + \frac{k_1}{2}\right)$$

$$= (0.1)f\left(0.1 + \frac{0.1}{2}, 1.11689 + \frac{0.1359}{2}\right)$$

$$= (0.1)f(0.15, 1.18484)$$

$$= (0.1)[(0.15)(1.18484) + (1.18484)^2]$$

$$= (0.1)[0.177726 + 1.403846]$$

$$= 0.1582$$

$$k_3 = hf\left(x_1 + \frac{h}{2}, y_1 + \frac{k_2}{2}\right)$$

$$= 0.1f\left(0.1 + \frac{0.1}{2}, 1.1689 + \frac{0.1582}{2}\right)$$

$$= 0.1f(0.15, 1.19599)$$

$$= (0.1)[(0.15)(1.19599) + (1.19599)^2]$$

$$= (0.1)[0.1793985 + 1.43039208]$$

$$=0.16098$$

$k_4 = hf(x_1 + h, y_1 + k_3)$

$= 0.1f(0.1 + 0.1, 1.11689 + 0.16098)$

$= 0.1f(0.2, 1.27787)$

$$=(0.1)[(0.2)(1.27787) + (1.27787)^2]$$

$$=(0.1)[0.255574 + 1.63295]$$

$$=0.1889$$

$$\Delta y = \frac{1}{6}[k_1 + 2k_2 + 2k_3 + k_4]$$

$$= \frac{1}{6}[0.1359 + 2(0.1582) + 2(0.16098) + 0.1889]$$

$$= 0.16053$$

$y_2 = y(0.2) = y_1 + \Delta y = 1.11689 + 0.16053 = 1.2774$

**To find $y(0.3)$**

$$x_2 = 0.2 \qquad y_2 = 1.2774$$

$$k_1 = hf(x_2, y_2) = (0.1)f(0.2, 1.2774)$$

$$= (0.1)[(0.2)(1.2774) + (1.2774)^2]$$

$$= (0.1)[(0.25548) + (1.63175)]$$

$$= 0.1887$$

$$k_2 = hf\left(x_2 + \frac{h}{2}, y_2 + \frac{k_1}{2}\right)$$

$$= (0.1)f\left(0.2 + \frac{0.1}{2}, 1.2774 + \frac{0.1887}{2}\right)$$

$$= (0.1)f(0.25, 1.37175)$$

$$= (0.1)[(0.25)(1.37175) + (1.37175)^2]$$

$$=(0.1)[(0.3429375) + (1.881698)^2]$$

$$=0.22246$$

$$k_3 = hf\left(x_2 + \frac{h}{2}, y_2 + \frac{k_2}{2}\right)$$

$$= (0.1)f\left(0.2 + \frac{0.1}{2}, 1.2774 + \frac{0.22246}{2}\right)$$

$$= (0.1)f(0.25, 1.38863)$$

$$= (0.1)[(0.25)(1.38863) + (1.38863)^2]$$

$$= 0.2275$$

$$k_4 = hf(x_2 + h, y_2 + k_3)$$

$$= 0.1f(0.2 + 0.1, 1.2774 + 0.2275)$$

$$= 0.1f(0.3, 1.5049)$$

$$=(0.1)[(0.3)(1.5049) + (1.5049)^2]$$

$$=0.2716$$

$$\Delta y = \frac{1}{6}[k_1 + 2k_2 + 2k_3 + k_4]$$

$$= \frac{1}{6}[0.1887 + 2(0.22246) + 2(0.2275) + 0.2716]$$

**=0.2267**

$$y(0.3) = y_2 + \Delta y$$

**=1.2774+0.2267**

**= 1.5041**