UNIT II

SHAFTS AND COUPLINGS

CHAPTER 1

Introduction of Shafts:

A shaft is a rotating machine element which is used to transmit power from one place to another. The power is delivered to the shaft by some tangential force and the resultant torque (or twisting moment) set up within the shaft permits the power to be transferred to various machines linked up to the shaft. In order to transfer the power from one shaft to another, the various members such as pulleys, gears etc., are mounted on it. These members along with the forces exerted upon them causes the shaft to bending.

In other words, we may say that a shaft is used for the transmission of torque and bending moment. The various members are mounted on the shaft by means of keys or splines. The shafts are usually cylindrical, but may be square or cross-shaped in section. They are solid in cross-section but sometimes hollow shafts are also used. An axle, though similar in shape to the shaft, is a stationary machine element and is used for the transmission of bending moment only. It simply acts as a support for some rotating body such as hoisting drum, a car wheel or a rope sheave. A spindle is a short shaft that imparts motion either to a cutting tool (e.g. drill press spindles) or to a work piece (e.g. lathe spindles).

Material Used for Shafts

The material used for shafts should have the following properties:

1. It should have high strength.
2. It should have good machinability.
3. It should have low notch sensitivity factor.
4. It should have good heat treatment properties.
5. It should have high wear resistant properties.

The material used for ordinary shafts is carbon steel of grades 40 C 8, 45 C 8, 50 C 4 and 50 C 12. The mechanical properties of these grades of carbon steel are given in the following table.

Table 1.1 Mechanical properties of steels used for shafts.

Indian standard designation	Ultimate tensile strength, MPa	Yield strength, MPa
40 C 8	560 - 670	320
45 C 8	610 - 700	350
50 C 4	640 - 760	370
50 C 12	700 Min.	390

[Source: "A Textbook of Machine Design by R.S. Khurmi J.K. Gupta, Page: 510]

Manufacturing of Shafts

Shafts are generally manufactured by hot rolling and finished to size by cold drawing or turning and grinding. The cold rolled shafts are stronger than hot rolled shafts but with higher residual stresses. The residual stresses may cause distortion of the shaft when it is machined, especially when slots or keyways are cut. Shafts of larger diameter are usually forged and turned to size in a lathe.

Types of Shafts

The following two types of shafts are important from the subject point of view:

1.Transmission shafts. These shafts transmit power between the source and the machines absorbing power. The counter shafts, line shafts, overhead shafts and all factory shafts are transmission shafts. Since these shafts carry machine parts such as pulleys, gears etc., therefore they are subjected to bending in addition to twisting.

2.Machine shafts. These shafts form an integral part of the machine itself. The crank shaft is an example of machine shaft.

Stresses in Shafts

The following stresses are induced in the shafts:

1.Shear stresses due to the transmission of torque (i.e. due to torsional load).

2.Bending stresses (tensile or compressive) due to the forces acting upon machine elements like gears, pulleys etc. as well as due to the weight of the shaft itself.

3.Stresses due to combined torsional and bending loads.

Design of Shafts

The shafts may be designed on the basis of

1. Strength, and
2. Rigidity and stiffness

In designing shafts on the basis of strength, the following cases may be considered:

(a) Shafts subjected to twisting moment or torque only,

(b) Shafts subjected to bending moment only,

(c) Shafts subjected to combined twisting and bending moments, and

(d) Shafts subjected to axial loads in addition to combined torsional and bending loads.

Shafts Subjected to Twisting Moment Only a) Solid shaft:

When the shaft is subjected to a twisting moment (or torque) only, then the diameter of the shaft may be obtained by using the torsion equation. We know that

$$\frac{T}{J} = \frac{\tau}{r}$$

where T = Twisting moment (or torque) acting upon the shaft,

J = Polar moment of inertia of the shaft about the axis of rotation,

τ = Torsional shear stress, and

r = Distance from neutral axis to the outer most fibre

= d/2 where d is the diameter of the shaft.

We know that for round solid shaft, polar moment of inertia,

$$J = \frac{\pi}{4} \times d^4$$

The equation (i) may now be written as

$$\frac{T}{\frac{\pi}{4} \times d^4} = \frac{\tau}{\frac{d}{2}}$$

$$T = \frac{\pi}{16} \times \tau \times d^3$$

From this equation, we may determine the diameter of round solid shaft (d).

We also know that for hollow shaft, polar moment of inertia,

$$J = \frac{\pi}{32}\,[(d_o)^4 - (d_i)^4]$$

Where, d_o and d_i = Outside and inside diameter of the shaft, and $r = d_o / 2$.

Substituting these values in equation (i), we have,

$$\frac{T}{\frac{\pi}{32}\times[(d_o)^2-(d_i)^2]} = \frac{\tau}{\frac{d_o}{2}}$$

$$T = \frac{\pi}{16}\times\tau\times\frac{[(do)^4 - (di)^4]}{d_o}$$

Let k = Ratio of inside diameter and outside diameter of the shaft = di / do

Now the equation (iii) may be written as

$$T = \frac{\pi}{16}\times\tau\times\frac{(d_o)^4}{d_o}[1- (\frac{d_i}{d_o})^4]$$

$$T = \frac{\pi}{16}\times\tau\times d_o^3\times(1-k^4)$$

From the equations (iii) or (iv), the outside and inside diameter of a hollow shaft may be determined.

It may be noted that

1. The hollow shafts are usually used in marine work. These shafts are stronger per kg of material and they may be forged on a mandrel, thus making the material more homogeneous than would be possible for a solid shaft.

When a hollow shaft is to be made equal in strength to a solid shaft, the twisting moment of both the shafts must be same. In other words, for the same material of both the shafts,

$$T = \frac{\pi}{16}\times\tau\times\frac{[(do)^4 - (di)^4]}{d_o} = \frac{\pi}{16}\times\tau\times d^3$$

$$\frac{[(do)^4 - (di)^4]}{d_o} = d^3 \text{ or } (d_o)^3\,(1 - k^4) = d^3$$

2. The twisting moment (T) may be obtained by using the following relation:

We know that the power transmitted (in watts) by the shaft,

$$P = \frac{2\pi NT}{60}$$

$$T = \frac{P\times 60}{2\pi N}$$

where T = Twisting moment in N-m, and

N = Speed of the shaft in r.p.m.

3. In case of belt drives, the twisting moment (T) is given by

$$T = (T_1 - T_2)\,R$$

where T_1 and T_2 = Tensions in the tight side and slack side of the belt respectively, and

R = Radius of the pulley.

Problem 1.1

Find the diameter of a solid steel shaft to transmit 20 kW at 200 r.p.m. The ultimate shear stress for the steel may be taken as 360 MPa and a factor of safety as 8. If a hollow shaft is to be used in place of the solid shaft, find the inside and outside diameter when the ratio of inside to outside diameters is 0.5.

Given Data:

$P = 20$ kW $= 20 \times 10^3$ W

$N = 200$ r.p.m.

$\tau_u = 360$ MPa $= 360$ N/mm^2

F.S. = 8

$k = d_i / d_o = 0.5$

We know that the allowable shear stress,

$$\tau = \frac{\tau_u}{\text{F.S.}} = \frac{360}{8} = 45 \text{ N/mm}^2$$

Diameter of the solid shaft

Let d = Diameter of the solid shaft.

We know that torque transmitted by the shaft,

$$T = \frac{P \times 60}{2\pi N}$$

$$T = \frac{200 \times 10^3 \times 60}{2\pi \times 200}$$

$$T = 955 \text{ N-m}$$

$$T = 955 \times 10^3 \text{ N-mm}$$

We also know that torque transmitted by the solid shaft (T),

$$955 \times 10^3 = \frac{\pi}{16} \times \tau \times d^3$$

$$= \frac{\pi}{16} \times 45 \times d^3$$

$= 8.84\ d^3$

$\therefore d^3 = 955 \times 10^3 / 8.84 = 108032$

$d = 47.6$ say 50 mm

Diameter of hollow shaft

Let d_i = Inside diameter, and

d_o = Outside diameter.

We know that the torque transmitted by the hollow shaft (T),

$$955 \times 10^3 = \frac{\pi}{16} \times \tau \times (d_o)^3 (1 - k^4)$$

$$955 \times 10^3 = \frac{\pi}{16} \times 45 \times (d_o)^3 (1 - 0.5^4)$$

$$955 \times 10^3 = 8.3 (d_o)^3$$

$$(d_o)^3 = 955 \times 10^3 / 8.3$$

$$= 115060 \text{ or}$$

$$d_o = 48.6 \text{ say } 50 \text{ mm}$$

and $d_i = 0.5\ d_o = 0.5 \times 50 = 25$ mm

Shafts Subjected to Bending Moment Only

When the shaft is subjected to a bending moment only, then the maximum stress (tensile or compressive) is given by the bending equation. We know that

$$\frac{M}{I} = \frac{\sigma_b}{y}$$

Where M = Bending moment,

I = Moment of inertia of cross-sectional area of the shaft about the axis of rotation,

σ_b = Bending stress, and

y = Distance from neutral axis to the outer-most fibre.

We know that for a round solid shaft, moment of inertia,

$$I = \frac{\pi}{64} \times d^4 \quad \text{..............and } y = \frac{d}{2}$$

Substituting these values in equation (i), we have

$$\frac{M}{\frac{\pi}{64} \times d^4} = \frac{\sigma_b}{\frac{d}{2}}$$

$$M = \frac{\pi}{32} \times \sigma_b \times d^3$$

From this equation, diameter of the solid shaft (d) may be obtained.

We also know that for a hollow shaft, moment of inertia,

$$I = \frac{\pi}{64} \times \; [(d_o)^4 - (d_i)^4]$$

$$= \frac{\pi}{64} \times (d_o)^3 \, (1 - k^4) \qquad \text{...(where } k = d_i / d_o \text{)}$$

$$\text{and } y = \frac{d_o}{2}$$

Again substituting these values in equation (i), we have

$$\frac{M}{\frac{\pi}{64} \times {d_o}^4 (1 - k^4)} = \frac{\sigma_b}{\frac{d_o}{2}}$$

$$M = \frac{\pi}{32} \times \sigma_b \times (d_o)^3 \, (1 - k^4)$$

From this equation, the outside diameter of the shaft (do) may be obtained.

Shafts Subjected to Combined Twisting Moment and Bending Moment

When the shaft is subjected to combined twisting moment and bending moment, then the shaft must be designed on the basis of the two moments simultaneously. Various theories have been suggested to account for the elastic failure of the materials when they are subjected to various types of combined stresses. The following two theories are important from the subject point of view:

1. Maximum shear stress theory or Guest's theory. It is used for ductile materials such as mild steel.
2. Maximum normal stress theory or Rankine's theory. It is used for brittle materials such as cast iron.

Let τ = Shear stress induced due to twisting moment, and

σ_b = Bending stress (tensile or compressive) induced due to bending moment.

According to maximum shear stress theory, the maximum shear stress in the shaft,

$$\tau_{max} = \frac{1}{2}\sqrt{(\sigma_b + 4\tau^2}$$

Substituting the values of τ and σ_b

$$\tau_{max} = \frac{1}{2}\sqrt{(\frac{32M}{\pi d^3})^2 + 4\,(\frac{16T}{\pi d^3})^2}$$

$$= \frac{16}{\pi d^3}\,[\sqrt{M^2 + T^2}]$$

$$\frac{\pi}{16} \times \tau_{max} \times d^3 = \sqrt{M^2 + T^2}$$

The expression $\sqrt{M^2 + T^2}$ is known as equivalent twisting moment and is denoted by T_e. The equivalent twisting moment may be defined as that twisting moment, which when acting alone, produces the same shear stress (τ) as the actual twisting moment. By limiting the maximum shear stress (τ_{max}) equal to the allowable shear stress (τ) for the material, the equation (i) may be written as

$$T_e = \sqrt{M^2 + T^2} = \frac{\pi}{16} \times \tau \times d^3$$

From this expression, diameter of the shaft (d) may be evaluated.

Now according to maximum normal stress theory, the maximum normal stress in the shaft,

$$(\sigma_b)_{max} = \frac{1}{2}\,\sigma_b + \frac{1}{2}\sqrt{(\sigma_b)^2 + 4\tau^2}$$

$$= \frac{1}{2} \times \frac{32M}{\pi d^3} + \frac{1}{2}\sqrt{(\frac{32M}{\pi\, d^3})^2 + 4(\frac{16T}{\pi d^3})^2}$$

$$= \frac{32}{\pi d^3}\,[\,\frac{1}{2}\,(\,M + \sqrt{M^2 + T^2})]$$

$$\frac{32}{\pi} \times\ (\sigma_b)_{max} \times d^3\ = \frac{1}{2}\,M + \sqrt{M^2 + T^2}$$

The expression $\frac{1}{2}\,M + \sqrt{M^2 + T^2}$ is known as equivalent bending moment and is denoted by M_e. The equivalent bending moment may be defined as that moment which when acting alone produces the same tensile or compressive stress (σ_b) as the actual bending moment. By limiting the maximum normal stress [$\sigma_{b(max)}$] equal to the allowable bending stress (σ_b), then the equation (iv) may be written as

$$M_e = \frac{1}{2}\,M + \sqrt{M^2 + T^2} = \frac{\pi}{16} \times \sigma_b \times d^3$$

From this expression, diameter of the shaft (d) may be evaluated.

Problem 1.2

A shaft is supported by two bearings placed 1 m apart. A 600 mm diameter pulley is mounted at a distance of 300 mm to the right of left hand bearing and this drives a pulley directly below it with the help of belt having maximum tension of 2.25 kN. Another pulley 400 mm diameter is placed 200 mm to the left of right hand bearing and is driven with the help of electric motor and belt, which is placed horizontally to the right. The angle of contact for both the pulleys is 180° and μ = 0.24. Determine the suitable diameter for a solid shaft, allowing working stress of 63 MPa in tension and 42 MPa in shear for the material of shaft. Assume that the torque on one pulley is equal to that on the other pulley.

Given Data:

AB = 1 m

D_C = 600 mm or R_C = 300 mm = 0.3 m

AC = 300 mm = 0.3 m

T_1 = 2.25 kN = 2250 N

D_D = 400 mm or R_D = 200 mm = 0.2 m

BD = 200 mm = 0.2 m

θ = 180° = π rad

μ = 0.24

σ_b = 63 MPa = 63 N/mm^2

τ = 42 MPa = 42 N/mm^2

The space diagram of the shaft is shown in Fig 1.1 (a).

Let T_1 = Tension in the tight side of the belt on pulley C = 2250 N ...(Given)

T_2 = Tension in the slack side of the belt on pulley C.

We know that

$$2.3 \log\left(\frac{T_1}{T_2}\right) = \mu\theta$$

$$= 0.24 \times \pi$$

$$= 0.754$$

$$\log\left(\frac{T_1}{T_2}\right) = \frac{0.754}{2.3}$$

$\frac{T_1}{T_2} = 2.127$ …………….. (Taking antilog of 0.3278)

$T_2 = \frac{T_1}{2.127}$

$= \frac{2250}{2.127}$

$T_2 = 1058$ N

∴ Vertical load acting on the shaft at C,

$W_C = T_1 + T_2 = 2250 + 1058 = 3308$ N

and vertical load on the shaft at D

$= 0$

The vertical load diagram is shown in Fig. 1.1 (c).

We know that torque acting on the pulley C,

$T = (T_1 – T_2) R_C$

$= (2250 – 1058) 0.3$

$T = 357.6$ N-m

The torque diagram is shown in Fig. 1.1 (b).

Let T_3 = Tension in the tight side of the belt on pulley D, and

T_4 = Tension in the slack side of the belt on pulley D.

Since the torque on both the pulleys (i.e. C and D) is same, therefore

$(T_3 – T_4) R_D = T = 357.6$ N-m or

$T_3 – T_4 = \frac{357.6}{R_D}$

$= \frac{357.6}{0.2}$

$T_3 – T_4 = 1788$ N

We know that

$\frac{T_3}{T_4} = \frac{T_1}{T_2} = 2.127$

$T_3 = 2.127\ T_4$

From equations (i) and (ii), we find that

$T_3 = 3376$ N, and $T_4 = 1588$ N

∴ Horizontal load acting on the shaft at D,

$W_D = T_3 + T_4 = 3376 + 1588$

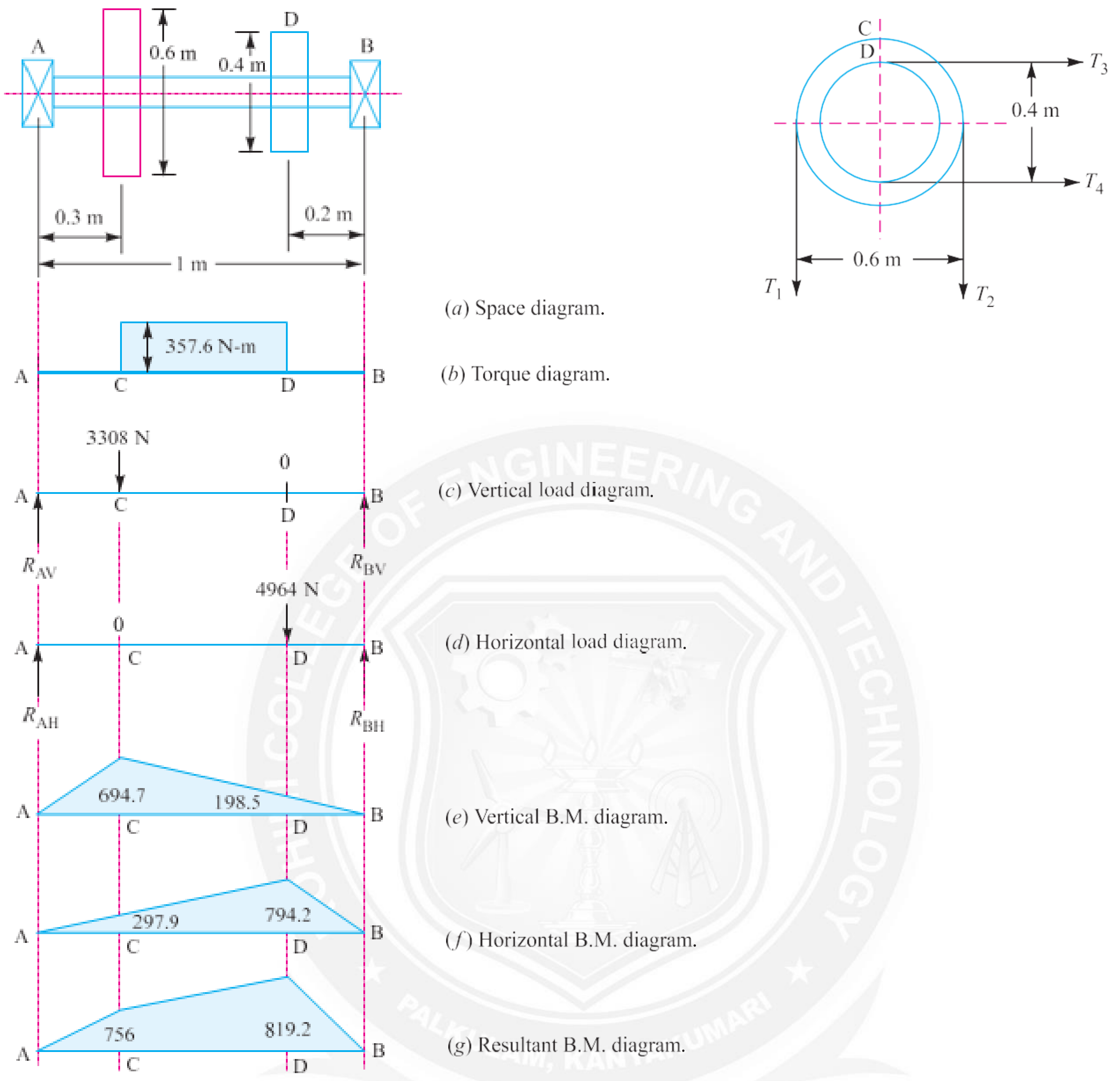

Fig. 1.1

[Source: "A Textbook of Machine Design by R.S. Khurmi J.K. Gupta, Page: 523]

$W_D = 4964$ N

and horizontal load on the shaft at C = 0

The horizontal load diagram is shown in Fig. 1.1(d)

Now let us find the maximum bending moment for vertical and horizontal loading.

First of all, considering the vertical loading at C. Let R_{AV} and R_{BV} be the reactions at the bearings A and B respectively. We know that

$R_{AV} + R_{BV} = 3308$ N

Taking moments about A,

$R_{BV} \times 1 = 3308 \times 0.3$ or

$R_{BV} = 992.4$ N

and $R_{AV} = 3308 – 992.4 = 2315.6$ N

We know that B.M. at A and B,

$M_{AV} = M_{BV} = 0$

B.M. at C, $M_{CV} = R_{AV} \times 0.3 = 2315.6 \times 0.3 = 694.7$ N-m

B.M. at D, $M_{DV} = R_{BV} \times 0.2 = 992.4 \times 0.2 = 198.5$ N-m

The bending moment diagram for vertical loading in shown in Fig. 1.1 (e).

Now considering horizontal loading at D. Let R_{AH} and R_{BH} be the reactions at the bearings A and B respectively. We know that

$R_{AH} + R_{BH} = 4964$ N

Taking moments about A,

$R_{BH} \times 1 = 4964 \times 0.8$ or

$R_{BH} = 3971$ N

and $R_{AH} = 4964 – 3971 = 993$ N

We know that B.M. at A and B,

$M_{AH} = M_{BH} = 0$

B.M. at C, $M_{CH} = R_{AH} \times 0.3 = 993 \times 0.3 = 297.9$ N-m

B.M. at D, $M_{DH} = R_{BH} \times 0.2 = 3971 \times 0.2 = 794.2$ N-m

The bending moment diagram for horizontal loading is shown in Fig. 1.1 (f).

Resultant B.M. at C,

$$M_C = \sqrt{(M_{CV})^2 + (M_{CH})^2}$$

$$= \sqrt{(694.7)^2 + (297.9)^2}$$

$M_C = 756$ N-m

and resultant B.M. at D,

$M_D = \sqrt{(M_{DV})^2 + (M_{DH})^2}$

$= \sqrt{(198.5)^2 + (794.2)^2}$

$M_C = 819.2$ N-m

The resultant bending moment diagram is shown in Fig. 1.1 (g).

We see that bending moment is maximum at D.

∴Maximum bending moment,

$M = M_D = 819.2$ N-m

Let d = Diameter of the shaft.

We know that equivalent twisting moment,

$T_e = \sqrt{M^2 + T^2}$

$= \sqrt{(819.2)^2 + (357.6)^2}$

$T_e = 894$ N-m $= 894 \times 10^3$ N-mm

We also know that equivalent twisting moment (T_e),

$894 \times 10^3 = \frac{\pi}{16} \times \tau \times d^3$

$= \frac{\pi}{16} \times \tau \times d^3 = 8.25d^3$

$d^3 = 894 \times 10^3 / 8.25$

$= 108 \times 103$ or

$d = 47.6$ mm

Again we know that equivalent bending moment,

$M_e = \frac{1}{2} M + \sqrt{M^2 + T^2}$

$= \frac{1}{2} [M + T_e]$

$= \frac{1}{2} (819.2 + 894) = 856.6$ N-m $= 856.6 \times 10^3$ N-mm

We also know that equivalent bending moment (M_e),

$856.6 \times 10^3 = \frac{\pi}{32} \times \sigma_b \times d^3$

$$856.6 \times 10^3 = \frac{\pi}{32} \times 63 \times d^3$$

$$= 6.2\ d^3$$

$$\therefore d^3 = 856.6 \times 10^3/6.2 = 138.2 \times 10^3 \text{ or}$$

$$d = 51.7 \text{ mm}$$

Taking larger of the two values, we have

$$d = 51.7 \text{ say } 55 \text{ mm}$$

Shafts Subjected to Fluctuating Loads

In the previous articles we have assumed that the shaft is subjected to constant torque and bending moment. But in actual practice, the shafts are subjected to fluctuating torque and bending moments. In order to design such shafts like line shaft and counter shafts, the combined shock and fatigue factors must be taken into account for the computed twisting moment (T) and bending moment (M). Thus for a shaft subjected to combined bending and torsion, the equivalent twisting moment,

$$T_e = \sqrt{(K_m + M)^2 + (K_t + T)^2}$$

and equivalent bending moment,

$$M_e = \frac{1}{2}\left[K_m \times M + \sqrt{(K_m + M)^2 + (K_t + T)^2}\right]$$

where K_m = Combined shock and fatigue factor for bending, and

K_t = Combined shock and fatigue factor for torsion.

The following table shows the recommended values for K_m and K_t.

Table 14.2. Recommended values for K_m and K_t.

Nature of load	K_m	K_t
1. Stationary shafts		
(a) Gradually applied load	1.0	1.0
(b) Suddenly applied load	1.5 to 2.0	1.5 to 2.0
2. Rotating shafts		
(a) Gradually applied or steady load	1.5	1.0
(b) Suddenly applied load with minor shocks only	1.5 to 2.0	1.5 to 2.0
(c) Suddenly applied load with heavy shocks	2.0 to 3.0	1.5 to 3.0

[Source: "A Textbook of Machine Design by R.S. Khurmi J.K. Gupta, Page: 531]

Shafts Subjected to Axial Load in addition to Combined Torsion and Bending Loads

When the shaft is subjected to an axial load (F) in addition to torsion and bending loads as in propeller shafts of ships and shafts for driving worm gears, then the stress due to axial load must be added to the bending stress (σ_b). We know that bending equation is

$$\frac{M}{I} = \frac{\sigma_b}{y}$$

$$\sigma_b = \frac{M \times y}{I}$$

$$= \frac{M \times \frac{d}{2}}{\frac{\pi}{64} \times d^4}$$

$$= \frac{32 \times M}{\pi \times d^3}$$

and stress due to axial load,

$$= \frac{F}{\frac{\pi}{4} \times d^2}$$

$$= \frac{4F}{\pi d^2} \quad \text{...(For round solid shaft)}$$

$$= \frac{4F}{\pi[(do)^2 - (di)^2]} \quad \text{...(For hollow shaft)}$$

$$= \frac{4F}{\pi[{d_o}^2(1 - k^2)]} \quad \text{... } (k = d_i/d_o)$$

∴ Resultant stress (tensile or compressive) for solid shaft,

$$\sigma_1 = \frac{32M}{\pi d^3} + \frac{4F}{\pi d^3}$$

$$= \frac{32}{\pi d^3}\left(M + \frac{F \times d}{8}\right)$$

$$= \frac{32M_1}{\pi d^3}$$

In case of a hollow shaft, the resultant stress,

$$\sigma_1 = \frac{32M}{\pi {d_o}^4(1 - k^4)} + \frac{4F}{\pi {d_o}^2(1 - k^2)}$$

$$\sigma_1 = \frac{32}{\pi {d_o}^4(1 - k^4)}\left[M + \frac{Fd_o(1+k^2)}{8}\right]$$

$$\sigma_1 = \frac{32M_1}{\pi {d_o}^4(1 - k^4)}$$

... Substituting for hollow shaft, $M_1 = M + \frac{Fd_o(1+k^2)}{8}$

In case of long shafts (slender shafts) subjected to compressive loads, a factor known as column factor (α) must be introduced to take the column effect into account.

∴ Stress due to the compressive load,

$$\sigma_c = \frac{\alpha \times 4F}{\pi \times d^4}$$

$$\sigma_c = \frac{\alpha \times 4F}{\pi {d_o}^2(1 - k^2)}$$

The value of column factor (α) for compressive loads* may be obtained from the following relation:

$$\text{Column factor, } \alpha = \frac{1}{1 - 0.0044\left(\frac{L}{K}\right)}$$

This expression is used when the slenderness ratio (L / K) is less than 115. When the slenderness ratio (L / K) is more than 115, then the value of column factor may be obtained from the following relation:

$$\text{Column factor, } \alpha = \frac{\sigma_y (\frac{L}{K})^2}{C\pi^2 E}$$

where L = Length of shaft between the bearings,

K = Least radius of gyration,

σ_y = Compressive yield point stress of shaft material, and

C = Coefficient in Euler's formula depending upon the end conditions.

The following are the different values of C depending upon the end conditions.

C =1, for hinged ends,

= 2.25, for fixed ends,

= 1.6, for ends that are partly restrained as in bearings.

Design of Shafts on the basis of Rigidity

Sometimes the shafts are to be designed on the basis of rigidity. We shall consider the following two types of rigidity.

1. Torsional rigidity. The torsional rigidity is important in the case of camshaft of an I.C. engine where the timing of the valves would be effected. The permissible amount of twist should not exceed 0.25° per metre length of such shafts. For line shafts or transmission shafts, deflections 2.5 to 3 degree per metre length may be used as limiting value. The widely used deflection for the shafts is limited to 1 degree in a length equal to twenty times the diameter of the shaft. The torsional deflection may be obtained by using the torsion equation,

$$\frac{T}{J} = \frac{G.\theta}{L}$$

$$\theta = \frac{T.L}{J.G}$$

where θ = Torsional deflection or angle of twist in radians,

T = Twisting moment or torque on the shaft,

J = Polar moment of inertia of the cross-sectional area about the axis of rotation,

$$= \frac{\pi}{32} \times d^4$$

$$= \frac{\pi}{32} \times [(d_o)^4 - (d_i)^4]$$

G = Modulus of rigidity for the shaft material, and

L = Length of the shaft.

2. Lateral rigidity. It is important in case of transmission shafting and shafts running at high speed, where small lateral deflection would cause huge out-of-balance forces. The lateral rigidity is also important for maintaining proper bearing clearances and for correct gear teeth alignment. If the shaft is of uniform cross-section, then the lateral deflection of a shaft may be obtained by using the deflection formulae as in Strength of Materials. But when the shaft is of variable cross-section, then the lateral deflection may be determined from the fundamental equation for the elastic curve of a beam, i.e.

$$\frac{d^2y}{dx^2} = \frac{M}{EI}$$

Problem 1.3

Compare the weight, strength and stiffness of a hollow shaft of the same external diameter as that of solid shaft. The inside diameter of the hollow shaft being half the external diameter. Both the shafts have the same material and length.

Given Data:

$d_o = d$

$d_i = d_o / 2$ or

$k = d_i / d_o = 1 / 2 = 0.5$

Comparison of weight

We know that weight of a hollow shaft,

W_H = Cross-sectional area × Length × Density

$$= \frac{\pi}{4} \times [(d_o)^2 - (d_i)^2] \times \text{Length} \times \text{Density}$$

and weight of the solid shaft,

$$W_S = \frac{\pi}{4} \times d^2 \times \text{Length} \times \text{Density}$$

Since both the shafts have the same material and length, therefore by dividing equation (i) by equation (ii), we get,

$$\frac{W_H}{W_S} = \frac{[(do)^2 - (di)^2]}{d^2}$$

$$= \frac{[(do)^2 - (di)^2]}{{d_o}^2}$$

$$\frac{W_H}{W_S} = 1 - \frac{(d_o)^2}{(d_i)^2}$$

Comparison of strength

We know that strength of the hollow shaft,

$$T_H = \frac{\pi}{16} \times \tau \times (d_o)^3(1-k^4)$$

and strength of the solid shaft,

$$T_S = \frac{\pi}{16} \times \tau \times d^3$$

Dividing equation (iii) by equation (iv), we get

$$\frac{T_H}{T_S} = \frac{{d_o}^3(1-k^4)}{d^3}$$

$$\frac{T_H}{T_S} = \frac{{d_o}^3(1-k^4)}{{d_o}^3} = 1 - k^4$$

$$= 1 - (0.5)^4 = 0.9375$$

Comparison of stiffness

We know that stiffness,

$$\frac{T}{\theta} = \frac{G.J}{L}$$

∴ Stiffness of a hollow shaft,

$$S_H = \frac{G}{L} \times \frac{\pi}{32} \times [(d_o)^4 - (d_i)^4]$$

and stiffness of a solid shaft,

$$S_S = \frac{G}{L} \times \frac{\pi}{32} \times d^4$$

Dividing equation (v) by equation (vi), we get

$$\frac{S_H}{S_S} = \frac{(d_o)^4 - (d_i)^4}{d^4} = \frac{(d_o)^4 - (d_i)^4}{{d_o}^2}$$

$$\frac{S_H}{S_S} = 1 - \frac{(d_o)^4}{(d_i)^4}$$

$= 1 - k^4$

$= 1 - (0.5)^4$

$\frac{S_H}{S_S} = 0.9375$