4.4 Root Locus Concept

The Root locus is the locus of the roots of the characteristic equation by varying system gain K from zero to infinity.

We know that, the characteristic equation of the closed loop control system is

$$1+G(s)\,H(s) = 0$$

We can represent G(s) H(s) as

$$G(s)\,H(s) = K\,[N(s) / D(s)]$$

Where,

- K represents the multiplying factor
- N(s) represents the numerator term having (factored) n^{th} order polynomial of 's'.
- D(s) represents the denominator term having (factored) m^{th} order polynomial of 's'.

Substitute, G(s)H(s) value in the characteristic equation.

$$1+k\,[N(s) / D(s)] = 0$$

$$\Rightarrow D(s) + KN(s) = 0$$

Case 1 − K = 0

If K=0 , then D(s)=0.

That means, the closed loop poles are equal to open loop poles when K is zero.

Case 2 − K = ∞

Re-write the above characteristic equation as

$$K\left(\frac{1}{K}+\frac{N(s)}{D(s)}\right) = 0$$

$$\frac{1}{K}+\frac{N(s)}{D(s)} = 0$$

Substitute, K=∞ in the above equation.

$$\frac{1}{\infty}+\frac{N(s)}{D(s)} = 0$$

$$N(s) = 0$$

If K=∞, then N(s)=0. It means the closed loop poles are equal to the open loop zeros when K is infinity.

From above two cases, we can conclude that the root locus branches start at open loop poles and end at open loop zeros.

Angle Condition and Magnitude Condition

The points on the root locus branches satisfy the angle condition. So, the angle condition is used to know whether the point exist on root locus branch or not. We can find the value of K for the points on the root locus branches by using magnitude condition. So, we can use the magnitude condition for the points, and this satisfies the angle condition.

Characteristic equation of closed loop control system is

$$1+G(s)\,H(s) = 0$$

$$\Rightarrow G(s)\,H(s) = -1+j0$$

The **phase angle** of G(s)H(s) is

$$\angle G(s)H(s)=\tan^{-1}(0 / -1) = (2n+1)\pi$$

The **angle condition** is the point at which the angle of the open loop transfer function is an odd multiple of 180^0.

Magnitude of G(s)H(s) is -

$$|G(s)H(s) = \sqrt{(-1)^2+0^2} = 1$$

The magnitude condition is that the point (which satisfied the angle condition) at which the magnitude of the open loop transfer function is one