5.5 LTI SYSTEM ANALYSIS USING DTFT

Output of LTI system is given by linear convolution

$$y(n) = \sum_{k=-\infty}^{\infty} h(k)x(n-k)$$

Let the system be excited by the sinusoidal or phaser $e^{j\omega n}$.

$$\therefore x(n) = e^{j\omega n}\ for - \infty < n < \infty$$

Hence the signal is complex in nature .It has unit amplitude and frequency is $'\omega'$.The output is given by

$$y(n) = \sum_{k=-\infty}^{\infty} h(k)e^{j\omega(n-k)}$$

$$== \sum_{k=-\infty}^{\infty} h(k)e^{j\omega n}.e^{j\omega k}$$

$$= \left[\sum_{k=-\infty}^{\infty} h(k)e^{j\omega k}\right]e^{j\omega n}$$

$$= H(\omega)e^{j\omega n}$$

$$H(\omega) = \sum_{k=-\infty}^{\infty} h(k)e^{j\omega k}$$

$H(\omega)$ is the Fourier transform of $h(k)$ and $h(k)$ is the unit sample response. $H(\omega)$ is called the transfer function of the system. $H(\omega)$ is complex valued function of ϖ in the range $-\pi \le \omega \le \pi$.The transfer function of $H(\omega)$ can be expressed in polar form as

$$H(\omega) = |H(\varpi)|e^{j\,\llcorner H(\varpi)}$$

$$|H(\varpi)| is\ the\ magnitude\ of\ H(\varpi)$$

$$\llcorner H(\varpi) is\ the\ angle\ of\ H(\varpi)$$

LTI SYSTEM ANALYSIS USING Z-TRANSFORM

The Z-Transform of impulse response is called transfer or system function H(Z).

$$Y(Z) = X(Z)H(Z)$$

General form of LCCDE

$$\sum_{k=0}^{N} a_k y[n-k] = \sum_{k=0}^{M} b_k x[n-k]$$

$$\sum_{k=0}^{N} a_k z^{-k} Y(z) = \sum_{k=0}^{M} b_k z^{-k} X(z)$$

Computing the Z-Transform

$$H(z) = \frac{Y(z)}{X(z)} = \frac{\sum_{k=0}^{M} b_k z^{-k}}{\sum_{k=0}^{N} a_k z^{-k}}$$

Example 1: Consider the system described by the difference equation.

$$y[n] = x[n] + \frac{1}{3}x[n-1] + \frac{5}{4}y[n-1] - \frac{1}{2}y[n-2] + \frac{1}{16}y[n-3]$$

Solution:

$$y[n] = x[n] + \frac{1}{3}x[n-1] + \frac{5}{4}y[n-1] - \frac{1}{2}y[n-2] + \frac{1}{16}y[n-3]$$

Here $N=3,\ M=1$. Order 3 homogeneous equation:

$$y[n]-\frac{5}{4}y[n-1]+\frac{1}{2}y[n-2]-\frac{1}{16}y[n-3]=0 \qquad n\geq 2$$

The characteristic equation:

$$1-\frac{5}{4}a^{-1}+\frac{1}{2}a^{-2}-\frac{1}{16}a^{-3}=0$$

The roots of this third order polynomial is: $a_1=a_2=1/2 \quad a_3=1/4$ and

$$y_h[n]=h[n]=A_1(\frac{1}{2})^n+A_2 n(\frac{1}{2})^n+A_3(\frac{1}{4})^n, \quad n\geq 2$$

Let us assume $y[-1]=0$ then (3.52) for this case becomes:

$$\begin{bmatrix}a_0 & 0\\ a_1 & a_0\end{bmatrix}.\begin{bmatrix}y[0]\\ y[1]\end{bmatrix}=\begin{bmatrix}b_0\\ b_1\end{bmatrix} \quad \Rightarrow \quad \begin{bmatrix}1 & 0\\ -5/4 & 1\end{bmatrix}.\begin{bmatrix}y[0]\\ y[1]\end{bmatrix}=\begin{bmatrix}1\\ 1/3\end{bmatrix} \quad \Rightarrow y[0]=1;\ y[1]=19/12$$

with these we have the impulse response of this system:

$$h[n]=-\frac{4}{3}(\frac{1}{2})^n+\frac{10}{3}n(\frac{1}{2})^n+\frac{7}{3}(\frac{1}{4})^n, \quad n\geq 0$$