

FILTER DESIGN BY THE INSERTION LOSS METHOD

A perfect filter would have zero insertion loss in the passband, infinite attenuation in the stopband, and a linear phase response (to avoid signal distortion) in the passband. Of course, such filters do not exist in practice, so compromises must be made; herein lies the art of filter design. The image parameter method of the previous section may yield a usable filter response for some applications, but there is no methodical way of improving the design. The insertion loss method, however, allows a high degree of control over the passband and stopband amplitude and phase characteristics, with a systematic way to synthesize a desired response. The necessary design trade-offs can be evaluated to best meet the application requirements. If, for example, a minimum insertion loss is most important, a binomial response could be used; a Chebyshev response would satisfy a requirement for the sharpest cutoff. If it is possible to sacrifice the attenuation rate, a better phase response can be obtained by using a linear phase filter design. In addition, in all cases, the insertion loss method allows filter performance to be improved in a straightforward manner, at the expense of a higher order filter. For the filter prototypes to be discussed below, the order of the filter is equal to the number of reactive elements.

Characterization by Power Loss Ratio In the insertion loss method a filter response is defined by its insertion loss, or power loss ratio, PLR:

$$P_{LR} = \frac{\text{Power available from source}}{\text{Power delivered to load}} = \frac{P_{inc}}{|S_{12}|^2}$$

Observe that this quantity is the reciprocal of $|S_{12}|^2$ if both load and source are matched. The insertion loss (IL) in dB is

$$IL = 10 \log P_{LR}$$

we know that $|S_{12}|^2$ is an even function of ω ; therefore it can be expressed as a polynomial in ω^2 . Thus we can write

$$|S_{12}|^2 = \frac{M(\omega^2)}{M(\omega^2) + N(\omega^2)}$$

where M and N are real polynomials in ω^2 . Substituting this form in (8.49) gives the following:

$$P_{LR} = 1 + \frac{M(\omega^2)}{N(\omega^2)}.$$

For a filter to be physically realizable its power loss ratio must be of the form in (8.52). Notice that specifying the power loss ratio simultaneously constrains the magnitude of the reflection coefficient, $|r(\omega)|$. We now discuss some practical filter responses. Maximally flat: This characteristic is also called the binomial or Butterworth response, and is optimum in the sense that it provides the flattest possible passband response for a given filter complexity, or order. For a low-pass filter, it is specified by

$$P_{LR} = 1 + k^2 \left(\frac{\omega}{\omega_c} \right)^{2N},$$

where N is the order of the filter and ω_c is the cutoff frequency. The passband extends from $\omega = 0$ to $\omega = \omega_c$; at the band edge the power loss ratio is $1 + k^2$. If we choose this as the -3 dB point, as is common, we have $k = 1$, which we will assume from now on. For $\omega > \omega_c$, the attenuation increases monotonically with frequency, as shown in Figure 2.12. For $\omega > \omega_c$, $PLR \approx k^2 (\omega/\omega_c)^{2N}$, which shows that the insertion loss increases at the rate of $20N$ dB/decade. Like the binomial response for multisection quarter-wave matching transformers, the first $(2N - 1)$ derivatives of are zero at $\omega = 0$. Equal ripple: If a Chebyshev polynomial is used to specify the insertion loss of an N th order low-pass filter as

$$P_{LR} = 1 + k^2 T_N^2 \left(\frac{\omega}{\omega_c} \right),$$

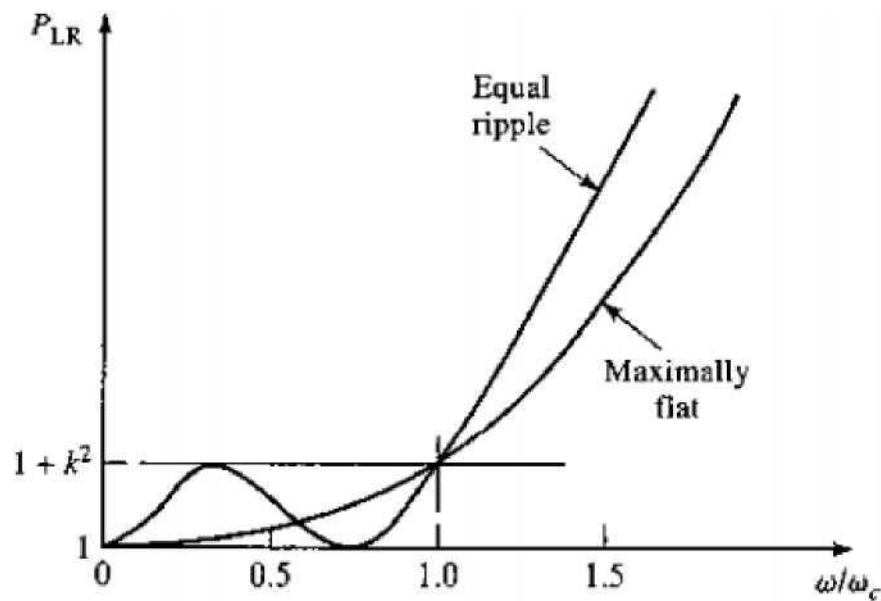


Figure 2.12 Maximally flat and equal-ripple low-pass filter responses (N = 3).

then a sharper cutoff will result, although the passband response will have ripples of amplitude $1 + k^2$, as shown in Figure 2.12, since $T_N(x)$ oscillates between ± 1 for $|x| \leq 1$. Thus, k^2 determines the passband ripple level. For large x , $T_N(x) \approx \frac{1}{2} (2x)^N$, so for $\omega \gg \omega_c$ the insertion loss becomes

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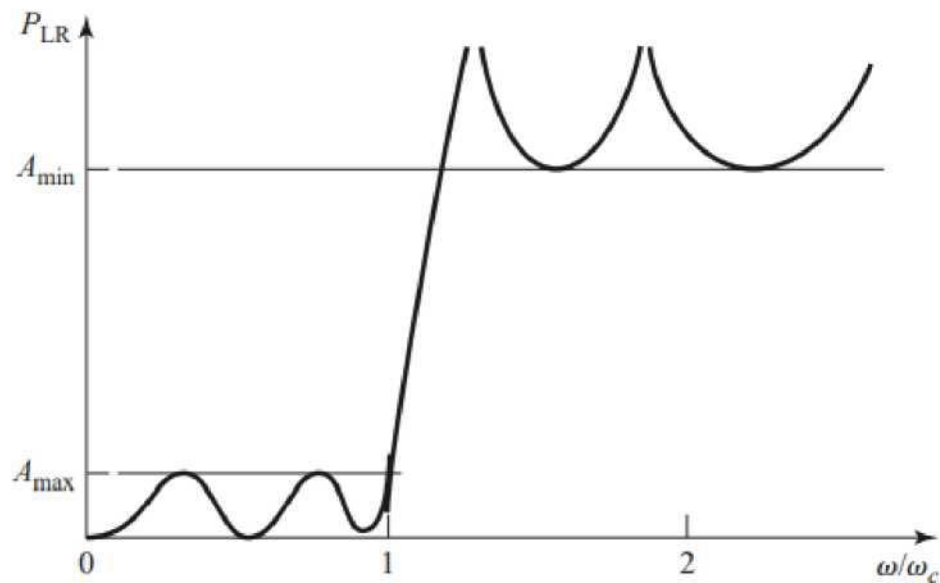


Figure 2.13 Elliptic function low-pass filter response.

which also increases at the rate of $20N$ dB/decade. However, the insertion loss for the Chebyshev case is $(22N)/4$ greater than the binomial response at any given frequency where $w < w_c$. Elliptic function: The maximally flat and equal-ripple responses both have monotonically increasing attenuation in the stopband. In many applications it is adequate to specify a minimum stopband attenuation, in which case a better cutoff rate can be obtained. Such filters are called elliptic function filters, and they have equal-ripple responses in the passband as well as in the stopband, as shown in Figure 2.13. The maximum attenuation in the passband, A_{max} , can be specified, as well as the minimum attenuation in the stopband, A_{min} . Elliptic function filters are difficult to synthesize, so we will not consider them further; the interested reader is referred to reference

Linear phase: The above filters specify the amplitude response, but in some applications (such as multiplexing filters for communication systems) it is important to have a linear phase response in the passband to avoid signal distortion. Since a sharp-cutoff response is generally incompatible with a good phase response, the phase response of a filter must be deliberately synthesized, usually resulting in an inferior attenuation characteristic. A linear phase characteristic can be achieved with the following phase response:

$$\phi(\omega) = A\omega \left[1 + p \left(\frac{\omega}{\omega_c} \right)^{2N} \right],$$

where $\phi(\omega)$ is the phase of the voltage transfer function of the filter, and p is a constant. A related quantity is the group delay, defined as

$$\tau_d = \frac{d\phi}{d\omega} = A \left[1 + p(2N + 1) \left(\frac{\omega}{\omega_c} \right)^{2N} \right],$$

which shows that the group delay for a linear phase filter is a maximally flat function. More general filter specifications can be obtained, but the above cases are the most common. We will next discuss the design of low-pass filter prototypes that are normalized in terms of impedance and frequency; this normalization simplifies the design of filters

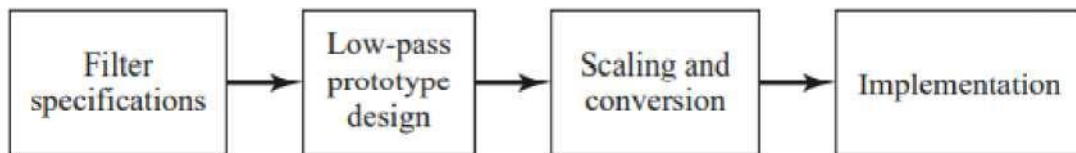


Figure 2.14 The process of filter design by the insertion loss method.

for arbitrary frequency, impedance, and type (low-pass, high-pass, bandpass, or bandstop). The low-pass prototypes are then scaled to the desired frequency and impedance, and the lumped-element components replaced with distributed circuit elements for implementation at microwave frequencies. This design process is illustrated in Figure 2.14.

