STRUCTURES FOR IIR SYSTEMS:

IIR Systems are represented in four different ways

1. Direct Form Structures Form I and Form II
2. Cascade Form Structure
3. Parallel Form Structure
4. Lattice and Lattice-Ladder structure.

DIRECT FORM-I :

Challenge: Obtain the direct form-I, direct form-II,Cascade and parallel form realization of the system $y(n)=-0.1y(n-1)+0.2y(n-2)+3x(n)+3.6x(n-1)+0.6x(n-2)$ [April/May-2015]

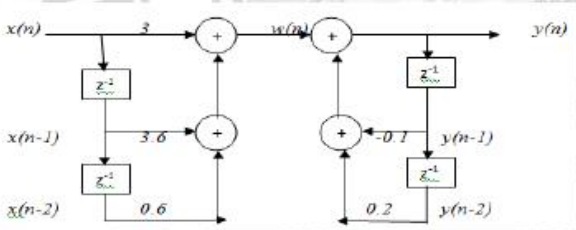
Solution:

Direct Form I:

$Let\ 3x(n)+3.6x(n-1)+0.6x(n-2)=w(n)$

$y(n)=-0.1y(n-1)+0.2y(n-2)+w(n)$

The direct form I realization is

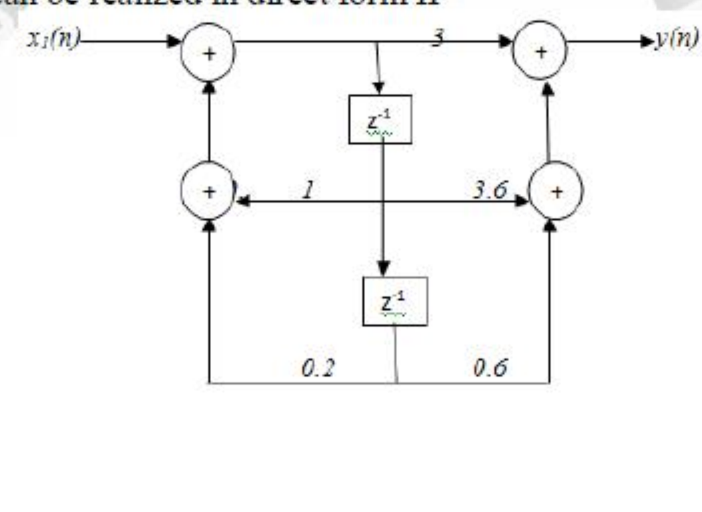


Direct form II:

From the given difference equation we have

$$H(z)=\frac{Y(z)}{X(z)}=\frac{3+3.6z^{-1}+0.6z^{-2}}{1+0.1z^{-1}-0.2z^{-2}}$$

The above system function can be realized in direct form II



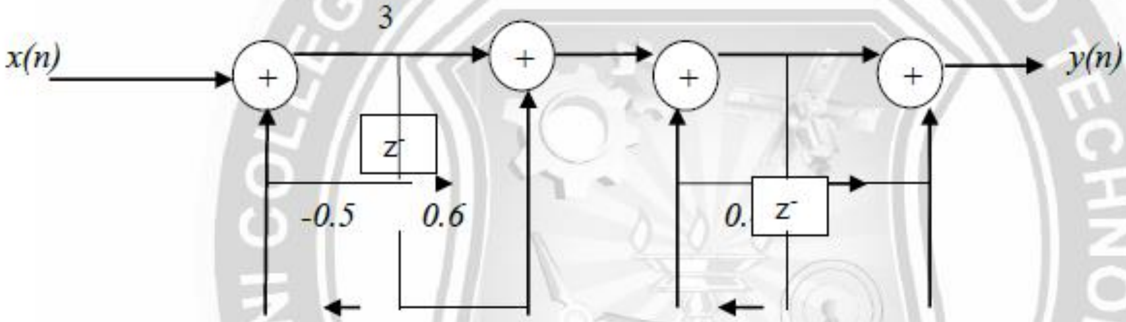
$$\frac{Y(z)}{X(z)} = \frac{3+3.6z^{-}+0.6z^{-}}{1+0.1z^{-}-0.2z^{-}}$$

$$= \frac{(3+0.6z^{-})(1+z^{-})}{(1+0.5z^{-})(1-0.4z^{-})}$$

$$H(z) = \frac{3+0.6z^{-}}{1+0.5z^{-}}$$

$$H(z) = \frac{1+z^{-}}{1-0.4z^{-1}}$$

Now we realize $H_1(z)$ *and* $H_2(z)$ and cascade both to get realization of H(z)

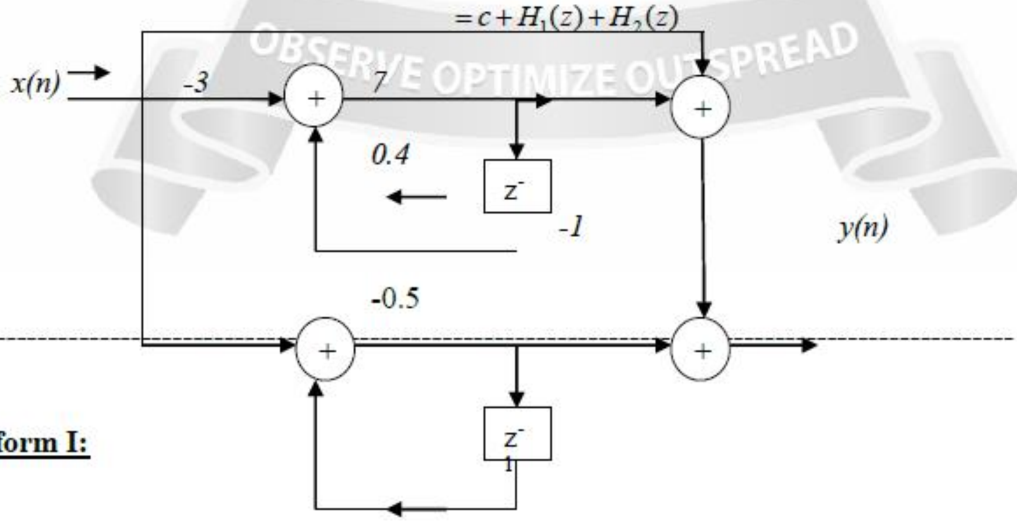


Parallel form:

$$H(z) = \frac{3+3.6z^{-1}+0.6z^{-2}}{1+0.1z^{-1}-0.2z^{-2}}$$

$$= -3 + \frac{7}{1-0.4z^{-1}} - \frac{1}{1+0.5z^{-1}}$$

$$= c + H_1(z) + H_2(z)$$



Direct form I:

Direct Form I Realization

IIR Filter transfer function is,

$$H(Z)=\frac{Y(Z)}{X(Z)}= \frac{\sum_{k=0}^{N} b_k Z^{-K}}{[1+\sum_{k=1}^{N} a_k Z^{-K}]}$$

This rational system function H(z) can be represented as cascade of two systems with system functions $H_1(z)$ and $H_2(z)$

$H(Z)= H_1(z).H_2(z)$

$$\text{where } H_1(z) = \frac{W(z)}{X(z)} = \sum_{k=0}^{M} b_k z^{-k}$$

$$\text{and } H_2(z) = \frac{Y(z)}{W(z)} = \frac{1}{1+\sum_{k=1}^{N} a_k z^{-k}}$$

For example, consider a third order (N=3) filter characterized by the system function,

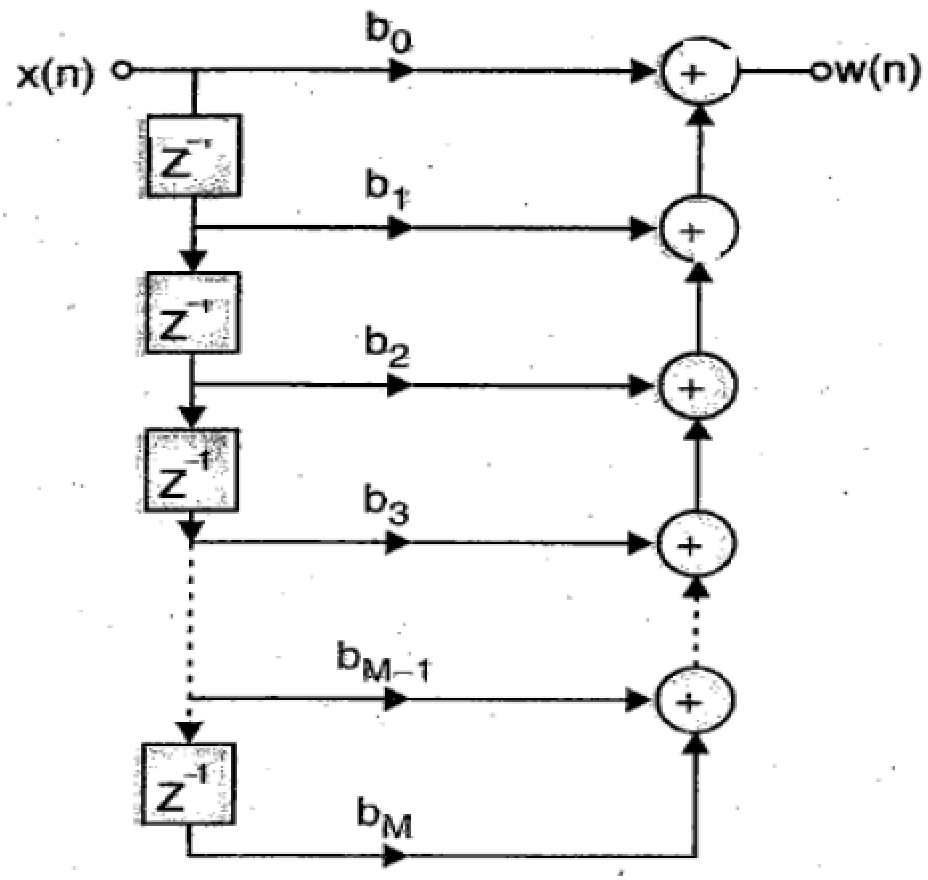
$$H(z) = \frac{Y(z)}{X(z)} = \frac{b_0 + b_1 z^{-1} + b_2 z^{-2} + b_3 z^{-3}}{1 + a_1 z^{-1} + a_2 z^{-2} + a_3 z^{-3}}$$

$$\text{Where } H_1(z) = \frac{W(z)}{X(z)} = b_0 + b_1 z^{-1} + b_2 z^{-2} + b_3 z^{-3}$$

Taking inverse z-transform of equation , we get

$$w(n) = b_0\, x(n) + b_1\, x(n-1) + b_2\, x(n-2) + b_3\, x(n-3)$$

The realization of equation is shown in Fig.2.2

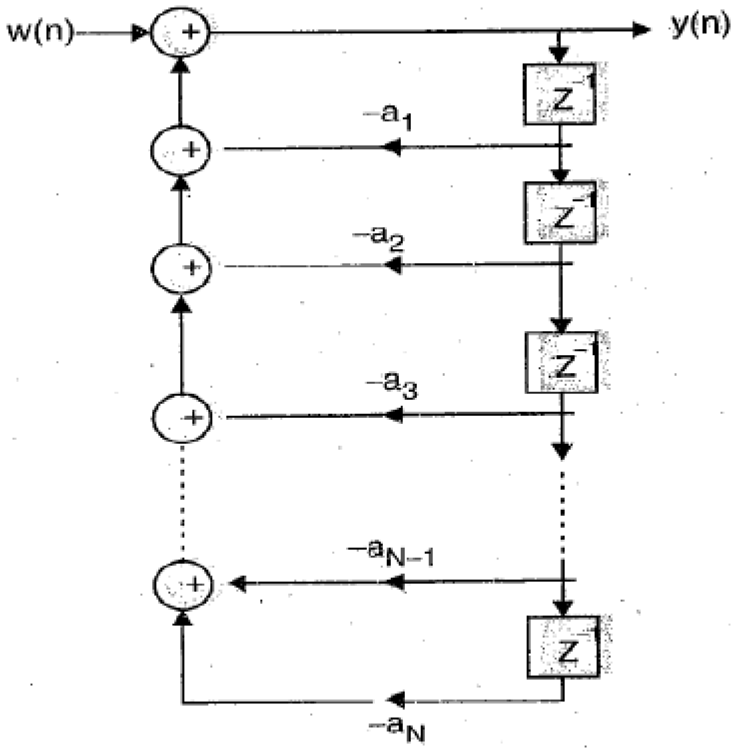


$$H_2(z) = \frac{Y(z)}{W(z)} = \frac{1}{1 + a_1 z^{-1} + a_2 z^{-2} + a_3 z^{-3}}$$

Taking inverse z-transform of equation , we get

$$y(n) + a_1 x(n-1) + a_2 x(n-2) + a_3 x(n-3) = w(n)$$

$$\therefore\ y(n) = w(n) - a_1 y(n-1) - a_2 y(n-2) - a_3 y(n-3)\ ..$$



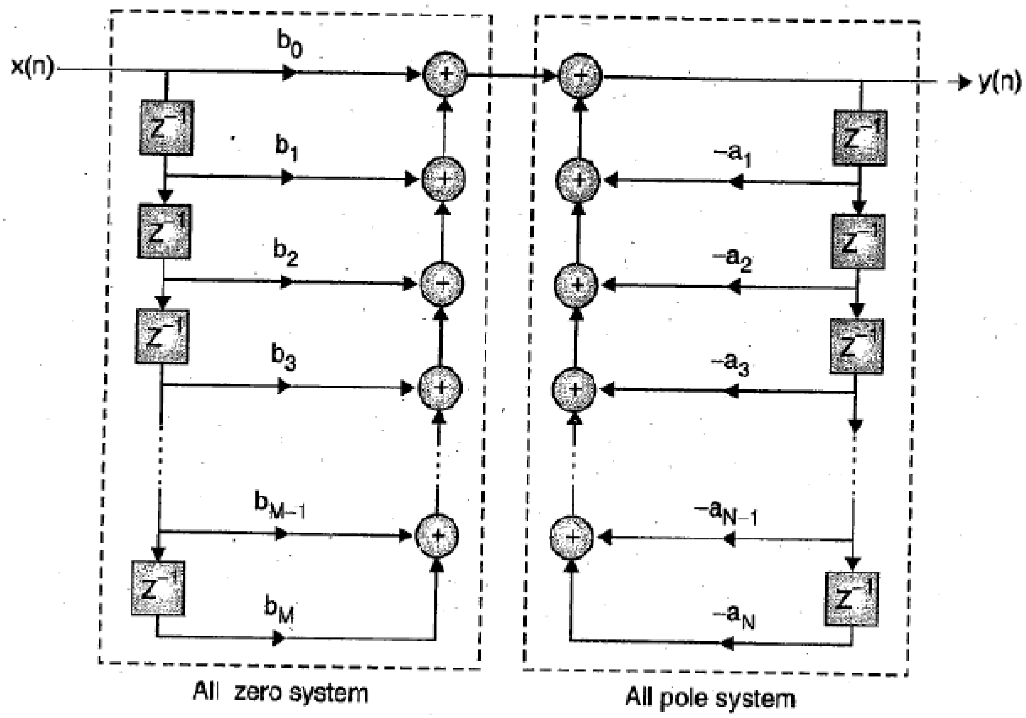


Fig.2.4 Direct form - I structure

The resulting structure is called *direct form I* structure. We observe that the direct form I structure is non canonic as it employs 6 delays for third order system.

Limitations of direct form I

- Since the number of delay elements used in direct form-I is more than the order of the difference equation, it is not effective.
- It lacks hardware flexibility.
- There are chances of instability due to the quantization noise.

Direct Form II Realization

The direct form II structure is an alternative to direct form I structure .It is more advantages to use direct form II technique than direct form I, because it uses less number of delay elements than direct form I structure.

The transfer function of IIR is H(z) and its value as

$$H(Z)=\frac{Y(Z)}{X(Z)}=\frac{\sum_{k=0}^{N} b_k Z^{-K}}{[1+\sum_{k=1}^{N} a_k Z^{-K}]}$$

$$H(Z)=\frac{Y(Z)}{X(Z)}=\frac{Y(Z)}{W(Z)}\frac{W(Z)}{X(Z)}$$

By rearranging the terms,

$$H(z)=\frac{W(Z)}{X(Z)}\frac{Y(Z)}{W(Z)}=H_1(z)\,.\,H_2(z)$$

$$H_1(z)=\frac{W(Z)}{X(Z)}=\frac{1}{1+\sum_{k=1}^{N} a_k Z^{-K}}$$

$$H_2(z)=\frac{Y(Z)}{W(Z)}=\sum_{k=0}^{N} b_k Z^{-K}$$

From the above equations, we can get X(Z) as,

$$X(Z) = W(Z)[\,1 + \sum_{k=1}^{N} a_k Z^{-K}]$$

$$X(Z) = W(Z) + \sum_{k=1}^{N} a_k Z^{-K} W(Z)$$

$$X(Z) - \sum_{k=1}^{N} a_k Z^{-K} W(Z) = W(Z)$$

$$W(z) = X(z) - a_1 z^{-1} W(z) - a_2 z^{-2} W(z) \ldots\ a_N z^{-N} W(z)$$

Taking inverse Z transform on both sides,

$$W(n) = x(n) - a_1 W(n-1) - a_2 W(n-N)$$

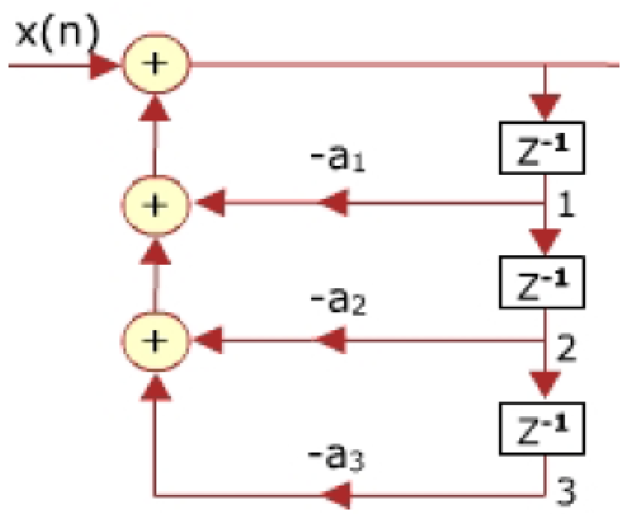


Fig.2.5 Realisation structure of $H_1(z)$

$$H_2(z) = \frac{Y(Z)}{W(Z)} = \sum_{k=0}^{N} b_k Z^{-K}$$

$$Y(z) = \sum_{k=0}^{N} b_k Z^{-K} \, W(z)$$

$$Y(z) = b_0 W(z) + b_1 z^{-1} W(z) + b_2 z^{-2} W(z) + \ldots b_M z^{-M} W(z)$$

Taking inverse Z transform on both sides,

$$Y(n) = b_0 W(n) + b_1 W(n-1) + b_2 W(n-2) + \ldots b_M W(n-M)$$

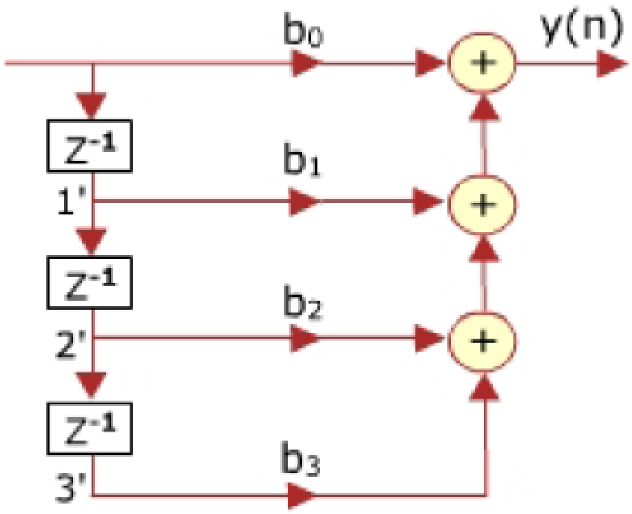


Fig.2.6 Realisation structure of $H_2(z)$

Combine equation $H_1(z)$ and $H_2(z)$ realization, we get direct form II

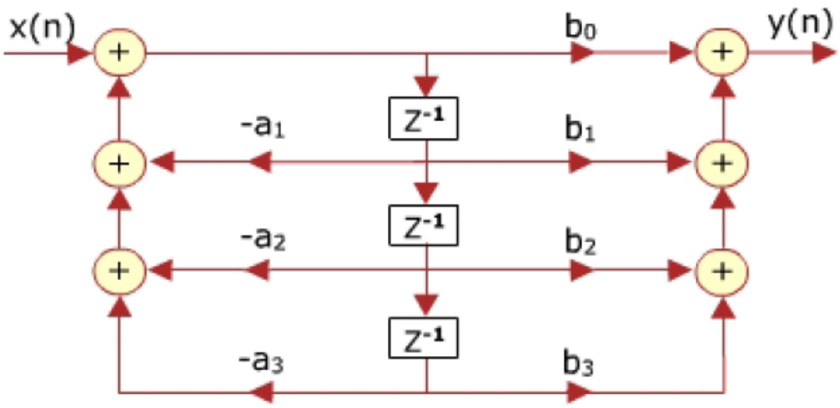


Fig.2.8 Direct form - II structure