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DEPARTMENT OF MATHEMATICS

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**UNIT – IV : INTERPOLATION, NUMERICAL
DIFFERENTIATION & INTEGRATION**

UNIT IV- INTERPOLATION , NUMERICAL DIFFERENTIATION AND INTEGRATION

Class Notes

Interpolation and Extrapolation:

Interpolation is the process of finding the values of $f(x)$ for intermediate values of x in the given interval. Extrapolation is the process of finding the values of $f(x)$ for extreme values of x .

Lagrange's Interpolation formula

$$f(x) = \frac{(x-x_1)(x-x_2)\dots(x-x_n)}{(x_0-x_1)(x_0-x_2)\dots(x_0-x_n)} y_0 + \frac{(x-x_0)(x-x_2)\dots(x-x_n)}{(x_1-x_0)(x_1-x_2)\dots(x_1-x_n)} y_1 + \frac{(x-x_0)(x-x_1)\dots(x-x_n)}{(x_n-x_0)(x_n-x_1)\dots(x_n-x_{n-1})} y_n$$

Inverse Lagrange's Interpolation

$$x = \frac{(y-y_1)(y-y_2)\dots(y-y_n)}{(y_0-y_1)(y_0-y_2)\dots(y_0-y_n)} x_0 + \frac{(y-y_0)(y-y_2)\dots(y-y_n)}{(y_1-y_0)(y_1-y_2)\dots(y_1-y_n)} x_1 + \dots + \frac{(y-y_0)(y-y_1)\dots(y-y_{n-1})}{(y_n-y_0)(y_n-y_1)\dots(y_n-y_{n-1})} x_n$$

Uses of Newton's forward formula and Newton's Backward formula

Newton's forward formula is used to interpolate value of y nearer to the beginning value of the table. Newton's Backward formula is used to interpolate value of y nearer to the end of set of tabular values. This may also be used to extrapolate closure to right of y_n .

Newton's divided difference formula

$$f(x) = f(x_0) + (x-x_0)f(x_0, x_1) + (x-x_0)(x-x_1)f(x_0, x_1, x_2) + \dots + (x-x_0)(x-x_1)\dots(x-x_{n-1})f(x_0, x_1, x_2, \dots, x_n)$$

Properties of Divided difference.

- (i) The divided differences are symmetrical in all their arguments. i.e. the value of any difference is independent of the order of the arguments.
- (ii) The divided differences of the sum or difference of two functions is equal to the sum or difference of the corresponding separate divided differences.

Numerical Differentiation:

Numerical differentiation is the process of computing the values of \dots . For some particular values of x from the given data (x_i, y_i) where y is not known explicitly.

Numerical Integration:

The process of computing the values of a definite integrals $\int_a^b y dx$ from a set of values

(x_i, y_i) , $i = 0, 1, 2, \dots, n$, where $x_0 = a$, $x_n = b$ of the function $y = f(x)$ is called Numerical integration

Problems:

1. From the following table, estimate the number of students who obtained marks between 40 and 45.

Marks	30-40	40-50	50-60	60-70	70-80
No.of Students	31	42	51	35	31

x y Δ Δ^2 Δ^3 Δ^4

Below 40	31				
		42			
Below 50	73		9		
		51		-25	
Below 60	124		-16		37
		35		12	
Below 70	159		-4		
		31			
Below 80	190				

$$u = \frac{x-x_0}{h} = \frac{45-40}{10} = 0.5$$

By Newton's forward interpolation formula

No.of students with the marks less than 45 = 48

No.of students with the marks less than 40 = 31

Therefore No.of students who obtained marks between 40 and 45 = 48 - 31 = 17

2. Given the tables

x	5	7	11	13	17
y	150	392	1452	2366	5202

Evaluate f(9) using Newton's divided difference formula.

Solution:

The divided difference table is given as follows:

x	y	Δ	Δ^2	Δ^3	Δ^4
5	150				
		121			
7	392		24		
		265		1	
11	1452		32		0
		457		1	
13	2366		42		
		709			
17	5202				

$$f(9) = 150 + 484 + 192 - 16 = 810$$

3. Find the missing values from the following table

X	0	5	10	15	20	25
Y	6	10	-	17	-	31

Solution:

The finite difference table is

x	y	Δ	Δ^2	Δ^3	Δ^4
0	6				
		4			
5	10		a-14		
		a-4		41-3a	
10	a		27-2a		-102+6a+b
		17-a		-61+3a+b	
15	17		-34+b+a		143-4b-4a
		b-17		82-3b-a	
20	b		48-2b		
		31-b			
25	31				

Since only four values are given, we have fourth difference is zero

Therefore $\Delta^4 y = 0$

6a+b=102, 4a+b=143
Solving this we get a=13.25, b= 22.5

4 Find the expression f(x) using Lagrange's formula for the following data

x	0	1	4	5
f(x)	4	3	24	39

Solution:

The Lagrange's interpolation formula

$$y = f(x) = \frac{(x-x_1)(x-x_2)(x-x_3)}{(x_0-x_1)(x_0-x_2)(x_0-x_3)}y_0 + \frac{(x-x_0)(x-x_2)(x-x_3)}{(x_1-x_0)(x_1-x_2)(x_1-x_3)}y_1 + \frac{(x-x_0)(x-x_1)(x-x_3)}{(x_2-x_0)(x_2-x_1)(x_2-x_3)}y_2 + \frac{(x-x_0)(x-x_1)(x-x_2)}{(x_3-x_0)(x_3-x_1)(x_3-x_2)}y_3$$

$x_0=0, x_1=1, x_2=4, x_3=5, y_0=4, y_1=3, y_2=24, y_3=39$
 $f(x) = 2x^2 - 3x + 4$

5 Find an approximate polynomial for f(x) using Lagrange's interpolation for the following data

x	0	1	2	5
f(x)	2	3	12	147

Solution:

The Lagrange's interpolation formula

$$y = f(x) = \frac{(x-x_1)(x-x_2)(x-x_3)}{(x_0-x_1)(x_0-x_2)(x_0-x_3)}y_0 + \frac{(x-x_0)(x-x_2)(x-x_3)}{(x_1-x_0)(x_1-x_2)(x_1-x_3)}y_1 + \frac{(x-x_0)(x-x_1)(x-x_3)}{(x_2-x_0)(x_2-x_1)(x_2-x_3)}y_2 + \frac{(x-x_0)(x-x_1)(x-x_2)}{(x_3-x_0)(x_3-x_1)(x_3-x_2)}y_3$$

$$= \frac{(x-1)(x-2)(x-5)}{(0-1)(0-2)(0-5)}(2) + \frac{(x-0)(x-2)(x-5)}{(1-0)(1-2)(1-5)}(3) + \frac{(x-0)(x-1)(x-5)}{(2-0)(2-1)(2-5)}(12) + \frac{(x-0)(x-1)(x-2)}{(5-0)(5-1)(5-2)}(147)$$

$$= \frac{1}{20}[-x^3 + 166x^2 - 184x - 40]$$

6 Find y'(1) if

x	-1	0	2	3
y	-8	3	1	12

Solution:

The Lagrange's interpolation formula is,

$$y = f(x) = \frac{(x-x_1)(x-x_2)(x-x_3)}{(x_0-x_1)(x_0-x_2)(x_0-x_3)}y_0 + \frac{(x-x_0)(x-x_2)(x-x_3)}{(x_1-x_0)(x_1-x_2)(x_1-x_3)}y_1 + \frac{(x-x_0)(x-x_1)(x-x_3)}{(x_2-x_0)(x_2-x_1)(x_2-x_3)}y_2 + \frac{(x-x_0)(x-x_1)(x-x_2)}{(x_3-x_0)(x_3-x_1)(x_3-x_2)}y_3$$

$$= 2x^3 - 6x^2 + 3x + 3$$

$$y'(x) = 6x^2 - 12x + 3$$

$$y'(1) = -3$$

7 **Using Lagrange's formula, fit a polynomial to the data**

x	-1	0	2	3
y	-8	3	1	12

Hence find y at x = 1.5 and x = 1

Solution:

The Lagrange's interpolation formula is,

$$y = f(x) = \frac{(x-x_1)(x-x_2)(x-x_3)}{(x_0-x_1)(x_0-x_2)(x_0-x_3)} y_0 + \frac{(x-x_0)(x-x_2)(x-x_3)}{(x_1-x_0)(x_1-x_2)(x_1-x_3)} y_1 +$$

$$\frac{(x-x_0)(x-x_1)(x-x_3)}{(x_2-x_0)(x_2-x_1)(x_2-x_3)} y_2 + \frac{(x-x_0)(x-x_1)(x-x_2)}{(x_3-x_0)(x_3-x_1)(x_3-x_2)} y_3$$

$$x = 1.5$$

$$= \frac{(1.5-0)(1.5-2)(1.5-3)}{(-1-0)(-1-2)(-1-3)} (-8) + \frac{(1.5+1)(1.5-2)(1.5-3)}{(0+1)(0-2)(0-3)} (3) + \frac{(1.5+1)(1.5-0)(1.5-3)}{(2+1)(2-0)(2-3)} (1)$$

$$+ \frac{(1.5-0)(1.5+1)(1.5-2)}{(3+1)(3-0)(3-2)} (12)$$

$$= 0.75$$

$$x = 1$$

$$= \frac{(1-0)(1-2)(1-3)}{(-1-0)(-1-2)(-1-3)} (-8) + \frac{(1+1)(1-2)(1-3)}{(0+1)(0-2)(0-3)} (3) + \frac{(1+1)(1-0)(1-3)}{(2+1)(2-0)(2-3)} (1)$$

$$+ \frac{(1-0)(1+1)(1-2)}{(3+1)(3-0)(3-2)} (12)$$

$$= 2$$

8 **Using Lagrange's inverse interpolation formula, find the value of x when y=20 from the given data**

x	1	2	3	4
y	1	8	27	64

Solution:

$$x = \frac{(y-y_1)(y-y_2)(y-y_3)}{(y_0-y_1)(y_0-y_2)(y_0-y_3)} x_0 + \frac{(y-y_0)(y-y_2)(y-y_3)}{(y_1-y_0)(y_1-y_2)(y_1-y_3)} x_1 +$$

$$+ \frac{(y-y_0)(y-y_1)(y-y_3)}{(y_2-y_0)(y_2-y_1)(y_2-y_3)} x_2 + \frac{(y-y_0)(y-y_1)(y-y_2)}{(y_3-y_0)(y_3-y_1)(y_3-y_2)} x_3$$

$$y = 20 \quad y_0 = 1 \quad y_1 = 8 \quad y_2 = 27 \quad y_3 = 64$$

$$x_0 = 1 \quad x_1 = 2 \quad x_2 = 3 \quad x_3 = 4$$

$$x = 2.8468$$

10

Given the tables

X	0	2	3	4	7	9
Y	4	26	58	112	466	922

Evaluate f(10) and f'(6) using Newton's divided difference formula.

Solution: The divided difference table is given as follows:

x	y	Δ	Δ^2	Δ^3	Δ^4
0	4				
		11			
2	26		7		
		32		1	
3	58		11		0
		54		1	
4	112		16		0
		118		1	
7	446		22		
		228			
9	922				

Newton's divided difference formula.

$$f(x) = f(x_0) + (x - x_0)f(x_0, x_1) + (x - x_0)(x - x_1)f(x_0, x_1, x_2) + \dots$$

$$+ (x - x_0)(x - x_1)\dots(x - x_{n-1})f(x_0, x_1, x_2, \dots, x_n)$$

$$f(x) = x^3 + 2x^2 + 3x + 4$$

$$f'(x) = 3x^2 + 4x + 3$$

$$f(10) = 1234$$

$$f'(6) = 135$$

11

Find f(x) as a Polynomial in x for the following data by Newton's divided difference formula and hence find f(8).

$$\begin{array}{cccccc} x : & 4 & 5 & 7 & 10 & 11 & 13 \\ f(x) : & 48 & 100 & 294 & 900 & 1210 & 2028 \end{array}$$

Solution:

x	f(x)	$\Delta f(x)$	$\Delta^2 f(x)$	$\Delta^3 f(x)$	$\Delta^4 f(x)$

4	48				
		52			
5	100		15		
		97		1	0
7	294		21		
		202		1	0
10	900		27		
		310		1	
11	1210		33		
		409			
13	2028				

$$f(x) = f(x_0) + (x - x_0)f(x_0, x_1) + (x - x_0)(x - x_1)f(x_0, x_1, x_2) + (x - x_0)(x - x_1)(x - x_2)f(x_0, x_1, x_2, x_3) + (x - x_0)(x - x_1)(x - x_2)(x - x_3)f(x_0, x_1, x_2, x_3, x_4)$$

$$f(x) = 48 + (x - 4)52 + (x - 4)(x - 5)15 + (x - 4)(x - 5)(x - 7)(1)$$

$$\text{hence } f(8) = 448$$

12 Using Newton's divided difference formula compute f(2) from the data

x	-4	-1	0	2	5
y	1245	33	5	9	1335

Solution:

$$f(x) = f(x_0) + (x - x_0)f(x_0, x_1) + (x - x_0)(x - x_1)f(x_0, x_1, x_2) + (x - x_0)(x - x_1)(x - x_2)f(x_0, x_1, x_2, x_3) + (x - x_0)(x - x_1)(x - x_2)(x - x_3)f(x_0, x_1, x_2, x_3, x_4)$$

x	f(x)	$\Delta f(x)$	$\Delta^2 f(x)$	$\Delta^3 f(x)$	$\Delta^4 f(x)$
-4	1245				
		-404			
-1	33		94		
		-28		-14	
0	5		10		3
		2		13	
2	9		88		
		442			
5	1335				

here $x_0 = -4, x_1 = -1, x_2 = 0, x_3 = 2, x_4 = 5$

$$f(x_0) = 1245$$

$$f(x_0, x_1) = -404$$

$$f(x_0, x_1, x_2) = 94$$

$$f(x_0, x_1, x_2, x_3) = -14$$

$$f(x_0, x_1, x_2, x_3, x_4) = 3$$

$$f(x) = 3x^4 - 5x^3 + 6x^2 - 14x + 5$$

$$f(2) = 9$$

- 13 Using Newton's forward interpolation formula, find the polynomial $f(x)$ satisfying the following data and hence find the value of y for $x = 5$.

X	4	6	8	10
Y	1	3	8	16

Solution:

x	y	Δ	Δ^2	Δ^3
4	1			
		2		
6	3		3	

		5		0
8	8		3	
		8		
10	16			

$$u = \frac{x - x_0}{h} = \frac{12 - 10}{5} = 0.4$$

$$y(x) = y_0 + u \Delta y_0 + \frac{u(u-1)}{2!} \Delta^2 y_0 + \frac{u(u-1)(u-2)}{3!} \Delta^3 y_0 + \dots$$

$$0.375x^2 - 2.75x + 6$$

$$f(5) = 9.375 - 13.75 + 6$$

$$f(5) = 29.125$$

- 14 Using Newton's forward interpolation formula, find the cubic polynomial which takes the following values:

$$x : 0 \quad 1 \quad 2 \quad 3$$

$$f(x) : 1 \quad 2 \quad 1 \quad 10$$

Solution:

x	y	Δy	$\Delta^2 y$	$\Delta^3 y$
0	1			
		1		
1	2		-2	
		-1		12
2	1		10	
		9		
3	10			

$$y(x) = y_0 + \frac{u}{1!} \Delta y_0 + \frac{u(u-1)}{2!} \Delta^2 y_0 + \frac{u(u-1)(u-2)}{3!} \Delta^3 y_0$$

here $x_0 = 0; y_0 = 1; h = 1; u = \frac{x - x_0}{h} = x$

hence $y(x) = 2x^3 - 7x^2 + 6x + 1 = f(x)$

$f(4) = 41$

15 **From the following table find the value at $\tan 45^\circ 15'$**

x°	45	46	47	48	49	50
$\tan x^\circ$	1	1.03553	1.07237	1.11061	1.15037	1.19175

Solution:

$$y(x) = y_0 + \frac{u}{1!} \Delta y_0 + \frac{u(u-1)}{2!} \Delta^2 y_0 + \frac{u(u-1)(u-2)}{3!} \Delta^3 y_0 + \frac{u(u-1)(u-2)(u-3)}{4!} \Delta^4 y_0 + \frac{u(u-1)(u-2)(u-3)(u-4)}{5!} \Delta^5 y_0 + \dots$$

θ°	y	Δy	$\Delta^2 y$	$\Delta^3 y$	$\Delta^4 y$	$\Delta^5 y$
45	1.0000	0.03553				
46	1.03553	0.03684	0.00131	0.0009		
47	1.07237	0.03824	0.00140	0.00012	0.00003	
48	1.11061	0.03976	0.00152	0.00010	-0.00002	
49	1.15037	0.04138	0.00162			-0.00005
50	1.19175					

here $u = \frac{x - x_0}{h} = \frac{45^\circ 15' - 45^\circ}{1} = 15' = \frac{1}{4}$

hence $y(45^\circ 15') = 1.008764609$

16 **From the given data, find the number of students whose weight is between 60 and 70.**

Weight in lbs :	0 – 40	40-60	60-80	80-100	100-120
No of students :	250	120	100	70	50

Solution:

x	y	Cumulative frequency	Δy	$\Delta^2 y$	$\Delta^3 y$	$\Delta^4 y$
Below 40	250	250	120	-20		
Below 60	120	370	100	-30	-10	
Below 80	100	470	70	-20	10	20
Below 100	70	540	50			
Below 120	50	590				

$$y(x) = y_0 + \frac{u}{1!} \Delta y_0 + \frac{u(u-1)}{2!} \Delta^2 y_0 + \frac{u(u-1)(u-2)}{3!} \Delta^3 y_0 + \frac{u(u-1)(u-2)(u-3)}{4!} \Delta^4 y_0$$

$$\text{here } u = \frac{x - x_0}{h} = \frac{70 - 40}{20} = 1.5$$

$$\text{hence } y(70) = 423.5937$$

now we take $x = 60$

$$u = \frac{60 - 40}{20} = 1$$

$$y(60) = 370$$

Therefore, the no. of students whose weight below 60=370

The no. of students whose weight below 70=423.5937

No. Of students whose weight is between 60 & 70=423.5937-370=53.5937=54 approximately.

17

Find the value of sec 310 from the following data

θ	31	32	33	34
$\tan\theta$	0.6008	0.6249	0.6494	0.6745

Solution:

The difference table is

θ	$\tan\theta$	Δy	$\Delta^2 y$	$\Delta^3 y$
31	0.6008			
		0.0241		
32	0.6249		0.0004	
		0.0245		0.0002
33	0.6494		0.0006	
		0.0251		
34	0.6745			

$$\frac{d}{d\theta}(\tan \theta) = \sec^2 \theta$$

By Newton's formula

$$\left(\frac{dy}{dx}\right)_{x=x_0} = \left(\frac{dy}{dx}\right)_{u=0} = \frac{1}{h} [\Delta y_0 - \frac{1}{2} \Delta^2 y_0 + \frac{1}{3} \Delta^3 y_0 + \dots]$$

$$\left(\frac{dy}{dx}\right)_{x=x_0} = \left(\frac{dy}{dx}\right)_{u=0} = \frac{1}{1^0} [0.0241 - \frac{1}{2}(0.0004) + \frac{1}{3}(0.0002) + \dots] = 1.3693$$

$$1^0 = \frac{\pi}{180}$$

$$\frac{d}{d\theta}(\tan 31^0) = \sec^2 31^0 = 1.3693$$

$$\sec 31^0 = 1.1702$$

- 18 Evaluate $I = \int_1^2 \frac{dx}{1+x^2}$ using Simpson's $\frac{1}{3}$ rule by taking four equal parts.

Solution:

x	1	1.25	1.5	1.75	2
f(x)	0.5	0.3902	0.3077	0.2462	0.2

Simpson's one - third rule is

$$\begin{aligned} \int_{x_0}^{x_n} f(x) dx &= \frac{h}{3} [(y_0 + y_n) + 2(y_2 + y_4) + 4(y_1 + y_3 + y_5)] \\ &= \frac{0.25}{3} [(0.5 + 0.2) + 2(0.3077) + 4(0.3902 + 0.2462)] \\ &= 0.2941 \end{aligned}$$

- 19 Evaluate $\int_0^1 \frac{x^2}{1+x^3} dx$ using Simpson's $\frac{1}{3}$ rule with $h = 0.25$

Solution:

x	0	0.25	0.5	0.75	1
f(x)	0	0.06154	0.22222	0.39560	0.50000

Simpson's one - third rule is

$$\begin{aligned} \int_{x_0}^{x_n} f(x) dx &= \frac{h}{3} [(y_0 + y_n) + 2(y_2) + 4(y_1 + y_3)] \\ &= \frac{0.25}{3} [0 + 0.5 + 2(0.2222) + 4(0.06154 + 0.39560)] = 0.231083 \end{aligned}$$

- 20 Evaluate $\int_0^6 \frac{e^x}{1+x} dx$ using Simpson's $\frac{1}{3}$ rule with $h = 1$

Solution:

x	0	1	2	3	4	5
f(x)	1	1.359	2.463	5.021	10.91	24.73

Simpson's one – third rule is

$$\int_{x_0}^{x_n} f(x) dx = \frac{h}{3} [(y_0 + y_n) + 2(y_2 + y_4) + 4(y_1 + y_3 + y_5)]$$

$$= \frac{1}{3} [1 + 57.632 + 2(2.463 + 10.91) + 4(1.359 + 5.021 + 24.735)]$$

$$= 69.946$$

21 Evaluate $I = \int_{-3}^3 x^4 dx$ correct to three decimals dividing the range of integration into 8 equal parts

using Trapezoidal, Simpson's $\frac{3}{8}$, Simpson's $\frac{1}{3}$ rule. And compare the result with actual

integration

Solution:

x	-3	-2	-1	0	1	2	3
y	81	16	1	0	1	16	81

Trapezoidal rule:

$$\int_{x_0}^{x_n} f(x) dx = \frac{h}{3} [(y_0 + y_n) + 2(y_1 + y_2 + y_3 + \dots + y_{n-1})]$$

$$= 115$$

Simpson's one – third rule:

$$\int_{x_0}^{x_n} f(x) dx = \frac{h}{3} [(y_0 + y_n) + 2(y_2 + y_4) + 4(y_1 + y_3 + y_5)]$$

$$= 98$$

Simpson's $\frac{3}{8}$ rule is

$$\int_{x_0}^{x_n} f(x) dx = \frac{h}{3} [(y_0 + y_n) + 2(y_3 + y_6 + \dots) + 3(y_1 + y_2 + y_4 + y_5 + \dots)]$$

$$= 99$$

Actual integration:

$$I = \int_{-3}^3 x^4 dx = \left[\frac{x^5}{5} \right]_{-3}^3 = \left[\frac{243}{5} - \frac{(-243)}{5} \right]$$

$$= 97.2$$

22 Evaluate $\int_1^2 \int_1^2 \frac{dx dy}{x+y}$ by using Trapezoidal and Simpson's rule with $h = k = 0.25$.

Solution:

y / x	1	1.25	1.5	1.75	2
1	0.5	0.4444	0.4	0.3636	0.3333
1.25	0.4444	0.4	0.3636	0.3333	0.3077

1.5	0.4	0.3636	0.3333	0.3077	0.2857
1.75	0.3636	0.3333	0.3077	0.2857	0.2667
2	0.3333	0.3077	0.2857	0.2667	0.25

By Trapezoidal rule,

$$I = \frac{hk}{4} \{ \text{sum of values of } f \text{ at the four corners} + 2(\text{sum of the values of } f \text{ at the remaining nodes on the boundary}) + 4(\text{sum of the values of } f \text{ at the interior nodes}) \}$$

$$I = 0.34065$$

- 23 Evaluate $I = \int_0^2 \int_0^2 \sin(9x + y) \, dx \, dy$ by using Simpson's $\frac{1}{3}$ rule by taking $h = 0.25$ $k = 0.5$

Solution:

y/x	0	0.25	0.5	0.75	1
0	0	0.7781	-0.9775	0.45	0.4121
0.5	0.4794	0.3817	-0.9589	0.8231	-0.07515
1	0.8415	-0.1082	-0.7055	0.9946	-0.5540

By Simpson's rule

$$I = \left\{ \frac{0.25 \times 0.5}{3 \times 3} \left[(0 + 0.4121 - 0.5540 + 0.8415) + 2(-0.9775 + 0.8415) + 4(0.7781 + 0.45 - 0.07515 + 0.9946 - 0.1082 + 0.4794) + \left\{ \begin{array}{l} 8(-0.9589) + \\ 16(0.3817 + 0.8231) \end{array} \right\} \right] \right\}$$

$$I = 0.2548$$

- 24 Evaluate numerically $\int_1^2 \int_1^2 \frac{xy}{x+y} \, dx \, dy$ by using Trapezoidal rule taking $h = k = 0.25$

Solution:

y\x	1	1.25	1.5	1.75	2
1	0.5	0.5556	0.6	0.6364	0.6667
1.25	0.5556	0.625	0.6818	0.7292	0.7692
1.5	0.6	0.6818	0.75	0.8077	0.8571
1.75	0.6364	0.7292	0.8077	0.875	0.9333
2	0.6667	0.7692	0.8571	0.9333	1

By using Trapezoidal rule

$$I = \frac{hk}{4} \{ \text{sum of values of } f \text{ at the four corners} + 2(\text{sum of the values of } f \text{ at the remaining nodes on the boundary}) + 4(\text{sum of the values of } f \text{ at the interior nodes}) \}$$

$$\begin{aligned}
&= \frac{(0.25)(0.25)}{4} [2.8334+2(8.7032)+4(6.6874)] \\
&= 0.0156[2.8334+17.4064+26.7496] \\
&= 0.73303
\end{aligned}$$

25 **Evaluate** $\int_0^1 \int_0^1 \frac{1}{x+y+1} dx dy$ **by using Trapezoidal rule taking h = 0.5 and k = 0.25**

Solution:

$$\begin{aligned}
I &= \frac{hk}{4} [(f_{00} + f_{04} + f_{20} + f_{24}) + 2(f_{01} + f_{02} + f_{03} + f_{10} + f_{21} + f_{22} + f_{23} + f_{14}) + 4(f_{11} + f_{12} + f_{13})] \\
&= 0.03125 [2.3333 + 8.622 + 6.0616] \\
&= 0.5318
\end{aligned}$$