

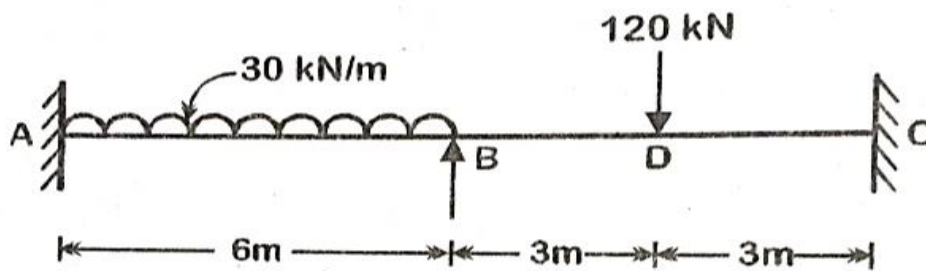
### 3.2 ANALYSIS OF CONTINUOUS BEAMS IN MOMENT DISTRIBUTION METHOD.

#### 3.2.1 NUMERICAL EXAMPLES ON( CONTINUOUS BEAMS ):

##### PROBLEM NO:01

For the continuous beam shown in figure, Calculate the support moments distribution method. Draw the SF and BM diagrams.

Solutions:



- **Fixed End Moments:**

$$MF_{AB} = -Wl^2/12 = -30 \times 6^2/12 = -90 \text{ kNm}$$

$$MF_{BA} = Wl^2/12 = 30 \times 6^2/12 = 90 \text{ kNm}$$

$$MF_{BC} = -Wl/8 = -120 \times 6/8 = -90 \text{ kNm}$$

$$MF_{CB} = Wl/8 = 120 \times 6/8 = 90 \text{ kNm}$$

- **Distribution Factor Table:**

Joint	Member	k	$\Sigma k$	Distribution factor ( $k/\Sigma k$ )
B	BA	$4EI/l = 4EI/6$	$4EI/3$	0.5
	BC	$4EI/l = 4EI/6$		0.5

- **Reactions:**

**Moment About B,**

$$M_B = R_A \times 6 - 30 \times 6 \times 6/2 + M_A$$

$$-90 = 6R_A - 540 - 90$$

$$6R_A = 540; R_A = 90\text{KN}$$

**Moment About B,**

$$M_B = R_C \times 6 - 120 \times 3 + M_C$$

$$-90 = 6R_C - 360 - 90$$

$$6R_C = 360; R_C = 60\text{KN}$$

$$R_B = \text{Total load} - (R_A + R_C) = 30 \times 6 + 120 - (90 + 60)$$

$$= 300 - 150 = 150\text{KN}$$

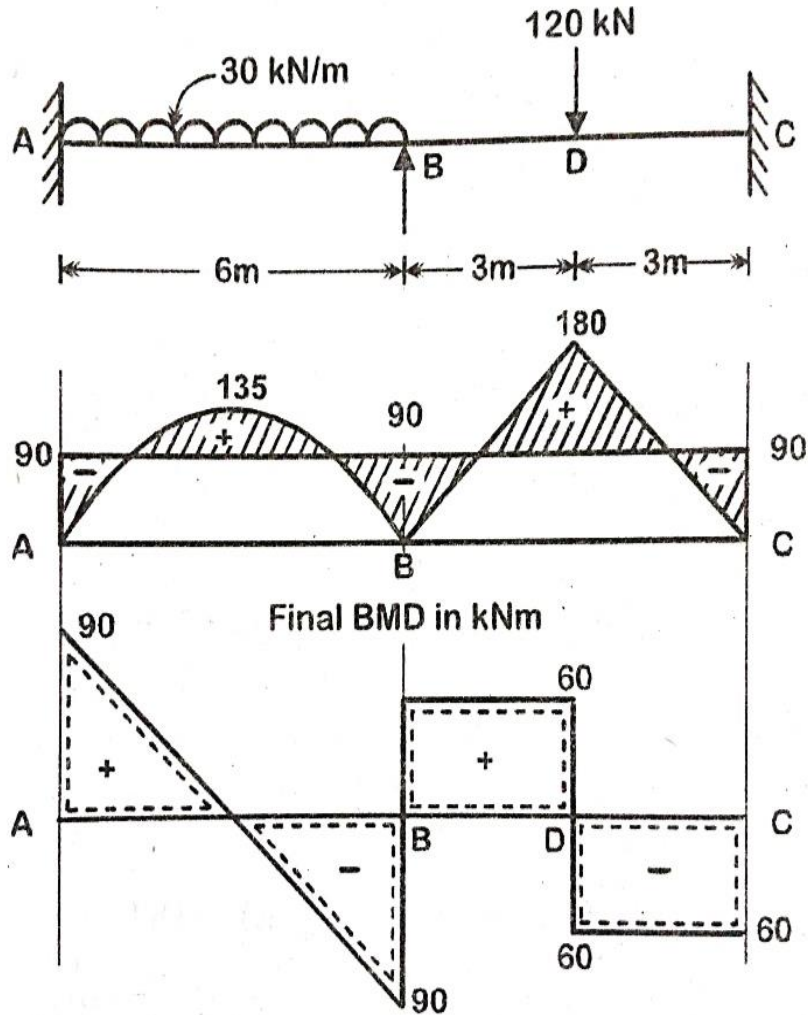
• **Moment Distribution Table:**

Joint	A	B		C
Member	AB	BA	BC	CB
D.F	-	0.5	0.5	-
F.E.M	-90	90	-90	90
Distribute	-	0	0	-
Final Moments	-90	90	-90	90
Conventional Moments	-90	-90	90	-90

• **Free BMD:**

$$M_{AB} = Wl^2/8 = 30 \times 6^2/8 = 135\text{kNm}$$

$$M_{BC} = Wl/4 = 120 \times 6/4 = 180\text{kNm}$$



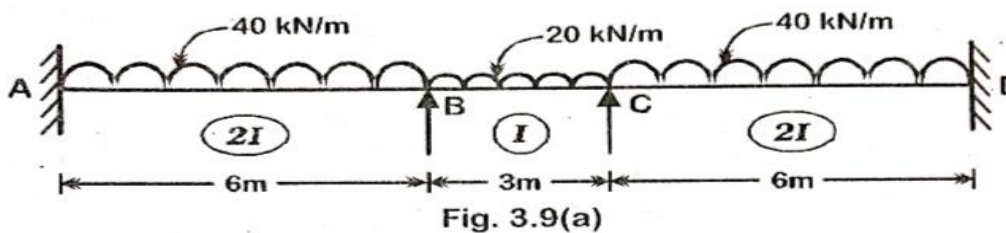
• **Final Bending Moments:**

$MA = -90\text{kNm}; \quad MB = -90\text{kNm}; \quad MC = -90\text{kNm}$

**PROBLEM NO:02**

For the continuous beam as shown in fig; Find the support moment carry out two cycles of distribution.

Solutions:



- **Fixed End Moments:**

$$MF_{AB} = -Wl^2/12 = -40 \times 6^2/12 = -120 \text{ kNm};$$

$$MF_{BA} = Wl^2/12 = 40 \times 6^2/12 = 120 \text{ kNm};$$

$$MF_{BC} = -Wl^2/12 = -20 \times 3^2/12 = -15 \text{ kNm};$$

$$MF_{CB} = Wl^2/12 = 20 \times 3^2/12 = 15 \text{ kNm};$$

$$MF_{CD} = -Wl^2/12 = -40 \times 6^2/12 = -120 \text{ kNm};$$

$$MF_{DC} = Wl^2/12 = 40 \times 6^2/12 = 120 \text{ kNm};$$

- **Distribution Factor Table:**

Joint	Member	k	$\Sigma k$	Distribution factor ( $k/\Sigma k$ )
B	BA	$4E(2I)/l = 4EI/3$	$7EI/3$	0.57
	BC	$3EI/l = 3EI/3$		0.43
C	CB	$3EI/l = 3EI/3$	$7EI/3$	0.43
	CD	$4E(2I)/l = 4EI/3$		0.57

- **Free BMD:**

$$M_{AB} = Wl^2/8 = 40 \times 6^2/8 = 180 \text{ kNm};$$

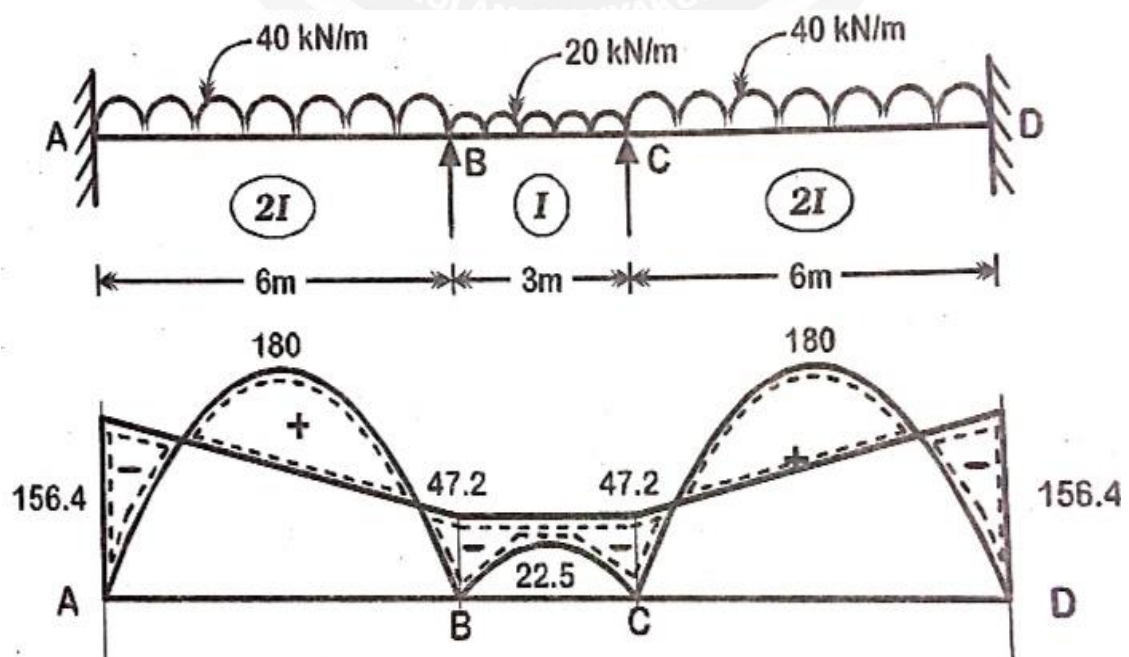
$$M_{BC} = Wl^2/8 = 20 \times 3^2/8 = 22.5 \text{ kNm};$$

$$M_{CD} = Wl^2/8 = 40 \times 6^2/8 = 180 \text{ kNm}.$$

- **Moment Distribution Table:**

Joint	A	B		C		D
Member	AB	BA	CB	BC	CD	DC
D.F	-	0.57	0.43	0.43	0.57	-
F.E.M	-120	120	-15	15	-120	120
Distribute		-59.9	-45.1	45.1	59.9	
Carry over	-29.95		22.6	-22.6		29.95
Distribute		-12.9	-9.7	9.7	12.9	
Carry over	-6.45					6.45
Final Moments	-156.4	47.2	-47.2	47.2	-47.2	156.4
Conventional Moments	-156.4	-47.2	-47.2	-47.2	-47.2	-156.4

- **Moment Diagram:**



- **Final Moments:**

$$M_A = -156.40\text{kNm}; \quad M_B = -47.20\text{kNm}; \quad M_C = -156.40\text{kNm}$$

**PROBLEM NO:03**

A continuous beam ABCD, simply supported at A,B,C and D is loaded as shown in fig. EI is constsnt.

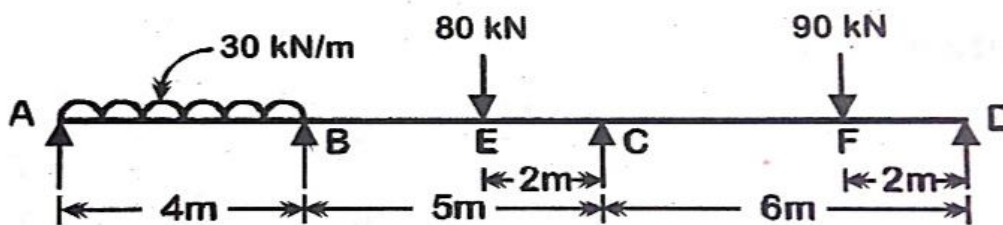


Fig. 3.11(a)

Solutions:

- **Fixed End Moments:**

$$M_{FAB} = -Wl^2/12 = -30 \times 4^2/12 = -40 \text{ kNm};$$

$$M_{FBA} = Wl^2/12 = 30 \times 4^2/12 = 40 \text{ kNm};$$

$$M_{FBC} = -Wab^2/l^2 = -80 \times 3 \times 2^2/5^2 = -38.4 \text{ kNm};$$

$$M_{FCB} = Wa^2b/l^2 = 20 \times 3^2 \times 2/5^2 = 57.6 \text{ kNm};$$

$$M_{FCD} = -Wab^2/l^2 = -90 \times 4 \times 2^2/6^2 = -40 \text{ kNm};$$

$$M_{FDC} = Wa^2b/l^2 = 90 \times 2^2 \times 4/6^2 = 80 \text{ kNm}.$$

- **Distribution Factor Table:**

Joint	Member	k	$\Sigma k$	Distribution factor ( $k/\Sigma k$ )
B	BA	$3EI/l = 3EI/4$	$27EI/20$	0.56
	BC	$3EI/l = 3EI/5$		0.44
C	CB	$3EI/l = 3EI/5$	$11EI/10$	0.5
	CD	$3EI/l = 3EI/6$		0.5

- **Free BMD:**

$$M_{AB} = Wl^2/8 = 30 \times 4^2/8 = 60\text{kNm};$$

$$M_{BC} = Wab/l = 80 \times 2 \times 3/5 = 96\text{kNm};$$

$$M_{CD} = Wab/l = 90 \times 4 \times 2/5 = 120\text{kNm}$$

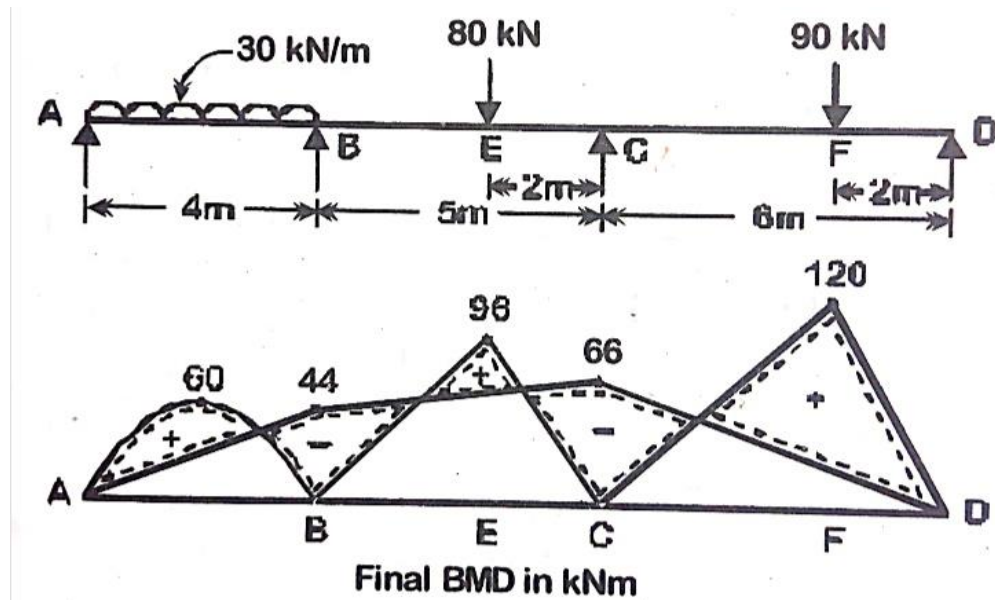
- **Moment Distribution Table:**

Joint	A	B		C		D
Member	AB	BA	CB	BC	CD	DC
D.F	-	0.56	0.44	0.5	0.5	-
F.E.M	-40	40	-38.4	57.6	-40	80
Release C & D	40					-80
carry over		20			-40	
Initial moments	0	60	-38.4	57.6	-80	0
Distribute		-12.096	-9.504	11.2	11.2	
Carry over		0	5.6	-4.552	0	
Distribute		-3.136	-2.464	2.376	2.376	
Carry over		0	1.188	-1.232	0	
Distribute		-0.66	-0.52	0.61	0.61	
Carry over			0.308	-0.26		
Final Moments	0	44	-44	66	-66	0
Conventional Moments	0	-44	-44	-66	-66	0

- **Final Moments:**

$$M_A = 0\text{kNm}; \quad M_B = -44\text{kNm}; \quad M_C = -66\text{kNm}; \quad M_D = 0\text{kNm}.$$

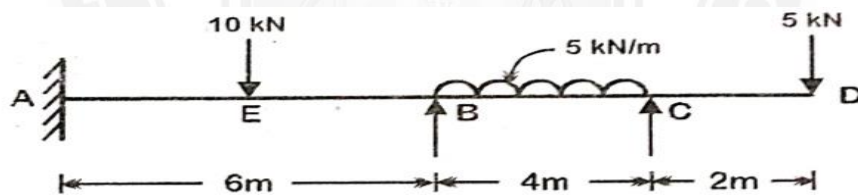
- Final Moments Diagram:



### PROBLEM NO:04

Find the support moments for the continuous beam using moment distribution method.

EI is constant.



### Solutions:

- Fixed End Moments:

$$M_{FAB} = -Wl/8 = -10 \times 6/8 = -7.5 \text{ kNm}$$

$$M_{FBA} = Wl/8 = 10 \times 6/8 = 7.5 \text{ kNm}$$

$$M_{FBC} = -Wl^2/12 = -5 \times 4^2/12 = -6.67 \text{ kNm}$$

$$M_{FCB} = Wl^2/12 = 5 \times 4^2/12 = 6.67 \text{ kNm}$$

$$M_{FCD} = -Wx1 = -5 \times 2 = -10 \text{ kNm}$$

- Free BMD:

$$M_{AB} = Wl/4 = 10 \times 6/4 = 15 \text{ kNm}$$

$$M_{BC} = Wl^2/8 = 5 \times 4^2/8 = 10 \text{ kNm}$$



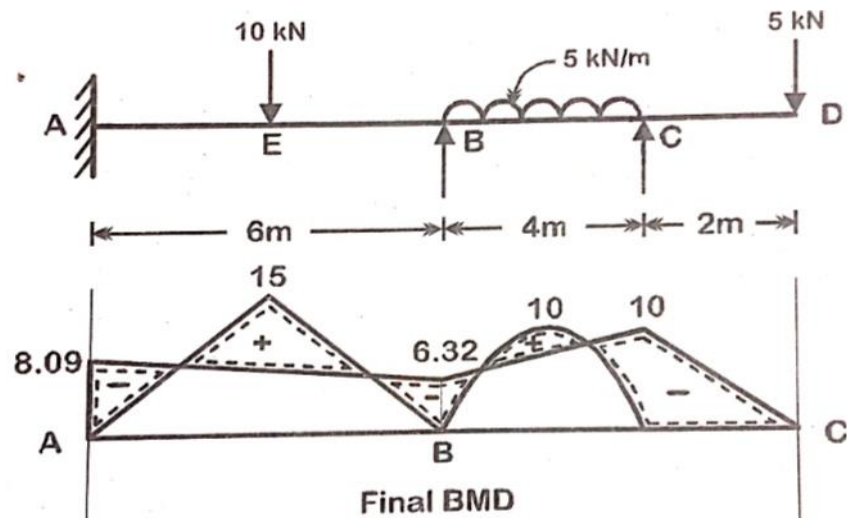
- **Distribution factor table:**

Joint	Member	k	$\Sigma k$	Distribution factor ( $k/\Sigma k$ )
B	BA	$4EI/1 = 4EI/6$	17EI/12	0.47
	BC	$3EI/1 = 3EI/4$		0.53
C	CB	$4EI/1 = 3EI/4$	3EI/4	1
	CD	0		0

- **Moment Distribution table:**

Joint	A	B		C	
Member	AB	BA	BC	CB	CD
D.F	-	0.47	0.53	1	0
F.E.M	-7.5	7.5	-6.67	6.67	-10
Release C & carry over	-	-	1.67	3.33	0
Initial moments	-7.5	7.5	-5	10	-10
Distribute	-	-1.18	-1.32	-	-
Carry over	-0.59	-	-	-	-
Final Moments	-8.09	6.32	-6.32	10	-10
Conventional Moments	-8.09	-6.32	-6.32	-10	-10

- **Final Moments Diagram:**



- **Final Moments:**

$$M_A = -8.09\text{kNm}; \quad M_B = -6.32\text{kNm}; \quad M_C = -10\text{kNm}$$