ENGINEERING PHYSICS

UNIT II

WAVES AND FIBRE OPTICS

2.5.Forced Oscillation

2.5.1. Differential Equation And Its Solution Of Forced Oscillations

2.5.Forced Oscillation

Oscillation in which the body vibrates with a frequency other than natural frequency due to the external force applied in equal interval of time is called Forced Oscillation.

2.5.1.Differential Equation And Its Solution Of Forced Oscillations

Let as consider a mass "m" connected to a spring and an external force is applied to this. Then there are three types of forces acting on it.

Restoring Force, Frictional force External force as in figure 2.5.1

Restoring force:

A restoring force is the force which is opposite to the direction of displacement(y).

$$F_1 \, \alpha - y$$

$$F1 = - ky\text{----------------(i)}$$

Where, K is the force constant and y is the displacement. Here the negative sign indicates that the restoring force acts in the opposite direction to the displacement.

Friction force:

Friction force or damping force is due to presence of air resistance, which is opposite to the direction of velocity

$$F_2 = -r \frac{dy}{dt}\text{-----------------(2)}$$

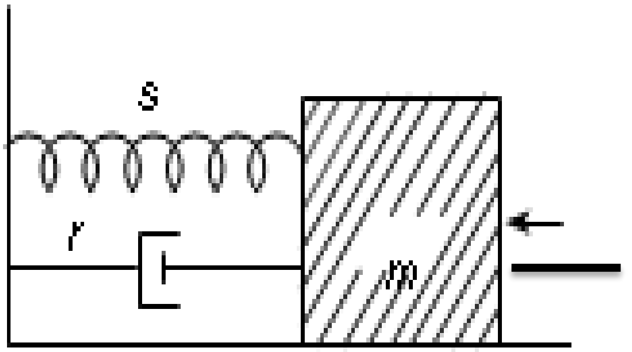

Fig 2.5.1 Forced Oscillations

(Source-"Physics of vibration and waves" by H.J.Pain page 57)

Applied force

$$F_3 = F_0 \sin\omega t \text{---------------(3)}$$

Total force

$$F = F_1 + F_2 + F_3 \text{----------------(4)}$$

Sub (i) & (2) & (3) in (4)

$$F = -ky - r\frac{dy}{dx} + F_0 \sin\omega t \text{-------------(5)}$$

But according to Newton's law

$$F = ma$$

$$\text{Here } F = m\frac{d^2y}{dt^2} \text{--------------(6)}$$

Where $\frac{d^2y}{dt^2}$ *is the acceleration*

From (4) & (5)

$$m\frac{d^2y}{dt^2} = -ky - r\frac{dy}{dt} + F_0 \sin\omega t$$

Divide by m

$$\frac{d^2y}{dt^2} = -\frac{k}{m}y - \frac{r}{m}\frac{dy}{dt} + \frac{F0}{m}\sin\omega t$$

$$\frac{d^2y}{dt^2} + \frac{r}{m}\frac{dy}{dt} + \frac{k}{m}y = \frac{F0}{m}\sin\omega t$$

Put r/m = 2b & k/m = ω_0^2 & F_0/m = f

$$\frac{d^2y}{dt^2} + 2b\frac{dy}{dt} + \omega_0^2 y = f\sin\omega t \text{---------------(7)}$$

The solution for this equation is

$$y = A\sin(\omega t-\theta) \text{------------(8)}$$

Where, A is the amplitude and **θ** is the angle at which the displacement lags behind the applied force.

Differentiating twice (8)

We get $$\frac{dy}{dt} = A\omega cos((\omega t-\theta) \text{------(9)}$$

$$\frac{d^2y}{dt^2} = -A\omega^2 sin(\omega t-\theta) \text{--------(10)}$$

Sub (8),(9) &(10) in (7)

$$-A\omega^2 sin(\omega t-\theta) + 2bA\omega cos(\omega t-\theta) + A\omega_0^2\sin(\omega t-\theta) = f\sin(\omega t-\theta+\theta)$$

$$= f\sin(\omega t-\theta)\cos\theta) + f cos(\omega t-\theta)\sin\theta \text{---(11)}$$

Comparing the equations

$$A(\omega_0^2 - \omega^2) = f cos\theta \text{------(12)}$$

$$2bA\omega = f sin\theta \text{------(13)}$$

Squaring & adding (12) & (13)

$$A^2(\omega_0^2 - \omega^2)^2 + 4b^2A^2\omega^2 = f^2cos^2\theta + f^2sin^2\theta \text{-------(14)}$$

$$A^2(\omega_0^2 - \omega^2)^2 + 4b^2\omega^2) = f^2$$

$$A^2 = \frac{f^2}{(\omega_0^2-\omega^2)^2+4b^2\omega^2)}$$

$$A = \frac{f}{\sqrt{(\omega_0^2-\omega^2)^2+4b^2\omega^2}}$$

Divide (13) by (12)

$$\frac{2bA\omega}{A({\omega_0}^2-\omega^2)}=\tan\theta$$

$$\theta = \tan^{-1}\left(\frac{2b\omega}{({\omega_0}^2-\omega^2)}\right)$$

The amplitude and phase of the forced oscillation depends on ${\omega_0}^2 - \omega^2$