### 5.3 METHOD OF SECTIONS

When the forces in a few members of a truss are to be determined, then the method of section is mostly used. This method is very quick as it does not involve the solution of other joints of the truss.

In this method, a section line is passed through the members, in which the forces are to be determined. The section line should be drawn in such a way that it does not cut more than three members in which the forces are unknown. The part of the truss on any one side of the section line is treated as free body in equilibrium under the action of external forces on that part and forces in the members cut by the section line. The unknown forces in the members are then determined by using equations of equilibrium as
$\Sigma \mathrm{Fx}=0, \Sigma \mathrm{Fy}=0$ and $\Sigma \mathrm{M}=0$,
Example 5.3.1 A truss of span 6 m is loaded as shown in Fig.5.6. Find the reactions and forces in the members of the truss by method of section.


Fig 5.6.a

Solution:
Reaction at $\mathrm{R}_{\mathrm{C}}=1.5 \mathrm{kN}$
Reaction at $\mathrm{R}_{\mathrm{B}}=2 \mathrm{kN}$

Determine the reactions at A and $\mathrm{D}\left(\mathrm{R}_{\mathrm{A}}\right.$ and $\left.\mathrm{R}_{\mathrm{D}}\right)$


Fig 5.6.b
From $\triangle$ CED,

$$
\mathrm{Eq}=\mathrm{qD}=1.5 \mathrm{~m}
$$

From $\triangle \mathrm{ABE}$,

$$
\begin{aligned}
\sin 60^{\circ} & =\frac{A B}{A E} \\
\mathrm{AB} & =\mathrm{AE} \times \sin 60^{\circ}=3 \times 0.866
\end{aligned}
$$

$\mathrm{AB}=\mathbf{2 . 5 9} \mathrm{m}$
From $\triangle \mathrm{ABp}$,

$$
\begin{aligned}
\cos 30^{\circ} & =\frac{A p}{A B} \\
\mathrm{Ap} & =\mathrm{AB} \times \cos 30^{\circ}=2.59 \times 0.866 \\
\mathbf{A p} & =\mathbf{2 . 2 5} \mathbf{~ m}
\end{aligned}
$$

Taking moment about A :


Fig 5.6.c
We know that,
Clockwise moment $=$ Anticlockwise moment

$$
\begin{aligned}
\mathrm{R}_{\mathrm{B}} \times \mathrm{Ap}+\mathrm{R}_{\mathrm{C}} \times \mathrm{Aq} & =\mathrm{R}_{\mathrm{D}} \times \mathrm{AD} \\
\mathrm{R}_{\mathrm{B}} \times 2.25+\mathrm{R}_{\mathrm{C}} \times 4.5 & =\mathrm{R}_{\mathrm{D}} \times 6 \\
2 \times 2.25+1.5 \times 4.5 & =\mathrm{R}_{\mathrm{D}} \times 6 \\
\mathbf{R}_{\mathrm{D}} & =\mathbf{1 . 8 7 5} \mathbf{K N}
\end{aligned}
$$

We know that,
Upward reaction = Downward reaction

$$
\begin{gathered}
\mathrm{R}_{\mathrm{A}}+\mathrm{R}_{\mathrm{D}}=\mathrm{R}_{\mathrm{B}}+\mathrm{R}_{\mathrm{C}} \\
\mathrm{RA}+1.875=2+1.5 \\
\mathrm{R}_{\mathrm{A}}=1.625 \mathrm{KN}
\end{gathered}
$$

Draw the section line $(1,1)$ cutting the members AB and AE


Fig 5.6.d
Now consider the equilibrium of the left part of the truss ( since, it is similar than right part).

Taking moments of all forces acting from the left of the section ( $\mathrm{R}_{\mathrm{A},}, \mathrm{F}_{\mathrm{AE}}$ and $\mathrm{F}_{\mathrm{AB}}$ ) about point E .

Since force $\mathrm{F}_{\mathrm{AE}}$ passing through the point E , moment about point E is zero. So, we have to consider $\mathrm{R}_{\mathrm{A}}$ and $\mathrm{F}_{\mathrm{AB}}$ forces only.

We know that,
Sum of Clockwise moment = Sum of Anticlockwise moment
Force $\left(R_{A}\right) \times$ perpendicular distance between the force $\left(R_{A}\right)$ and a point $E+$ Force $\left(R_{A B}\right)$ $\times$ perpendicular distance between the force $\left(\mathrm{R}_{A B}\right)$ and a point $\mathrm{E}=0$
(since, there is no anticlockwise moment)

$$
\mathrm{R}_{\mathrm{A}} \times 3+\mathrm{F}_{\mathrm{AB}} \times \mathrm{BE}=0
$$

$$
\begin{equation*}
1.625 \times 3+\mathrm{F}_{\mathrm{AB}} \times \mathrm{BE}=0 \tag{1}
\end{equation*}
$$

From $\triangle$ AEB,

$$
\begin{aligned}
& \sin 30^{\circ}=\frac{B E}{A E} \\
& \mathrm{BE}=\mathrm{AE} \times \sin 30^{\circ} \\
& =3 \times 0.5 \\
& \mathrm{BE}=1.5 \mathrm{~m} \\
& 1.625 \times 3+\mathrm{F}_{\mathrm{AB}} \times 1.5=0 \\
& \mathrm{~F}_{\mathrm{AB}}=-3.25 \mathrm{kN} \text { (Compression) }
\end{aligned}
$$

(since, we are assuming all the forces are tensile forces. If we get negative value, the force in that member is compressive.)

Now taking moment of all forces acting to the left of $\operatorname{section}\left(\mathrm{R}_{\mathrm{A}}, \mathrm{F}_{\mathrm{AB}}\right.$ and $\left.\mathrm{F}_{\mathrm{AE}}\right)$ about point C.

Since force $\mathrm{F}_{\mathrm{AB}}$ passing through the point C , the moment about point C is zero. So, we have to consider $\mathrm{R}_{\mathrm{A}}$ and $\mathrm{F}_{\mathrm{AE}}$ force only.
We know that, sum of Clockwise moment = sum of Anticlockwise moment
Or Force ( $\mathrm{R}_{\mathrm{A}} \mathrm{X}$ perpendicular distance between the force $\mathrm{R}_{\mathrm{A}}$ and a point $\mathrm{C}+$ Force $\left(\mathrm{F}_{\mathrm{AE}}\right) \mathrm{X}$ perpendicular distance between the force $\mathrm{F}_{\mathrm{AE}}$ and a point $\mathrm{C}=0$
Or

$$
\mathrm{R}_{\mathrm{A}} \times \mathrm{A}_{\mathrm{q}}=\mathrm{F}_{\mathrm{AE}} \times \mathrm{C}_{\mathrm{q}}
$$

Or

$$
\begin{equation*}
1.625 \times 4.5=\text { FAE X Cq } \tag{2}
\end{equation*}
$$

From $\Delta \mathrm{CqD}$

$$
\operatorname{Cos} 60^{\circ}=\frac{q D}{C D}=\frac{1.5}{C D}
$$

$\therefore \quad \mathrm{CD}=3 \mathrm{~m}$

$$
\text { in } 60^{\circ}=\frac{C q}{C D}
$$

Or

$$
\mathrm{Cq}=\mathrm{CD} \mathrm{X} \sin 60=3 \mathrm{X} 0.866
$$

$=2.59 \mathrm{~m}$
From equation (2)

$$
\begin{aligned}
1.625 \times 4.5 & =\mathrm{F}_{\mathrm{AE}} \mathrm{X} 2.59 \\
\mathrm{~F}_{\mathrm{AE}} & =2.8 \mathrm{kN}(\text { Tension })
\end{aligned}
$$

Draw a section line $(2,2)$ cutting the members $\mathrm{BC}, \mathrm{BE}$ and AE .


Fig 5.6.e
(only two unknown forces are permitted while considering a section. Here, BC,BE unknown forces AE - known Force)
Now taking moment of all forces acting to the left of section ( $\mathrm{R}_{\mathrm{A}}, \mathrm{F}_{\mathrm{AE}}, \mathrm{F}_{\mathrm{BC}}, \mathrm{R}_{\mathrm{B}}$ and $\left.\mathrm{F}_{\mathrm{BE}}\right)$ about point A.
Consider $\mathrm{R}_{\mathrm{B}}$ and $\mathrm{F}_{\mathrm{BE}}$ forces only.
(Since $\mathrm{R}_{\mathrm{A}}, \mathrm{F}_{\mathrm{AE}}$ and $\mathrm{F}_{\mathrm{BC}}$ passing through a point A )
We know that,
Sum of Clockwise moment = Sum of Anticlockwise moment
Or Force ( $\mathrm{F}_{\mathrm{BE}}$ ) X perpendicular distance between the force $\left(\mathrm{F}_{\mathrm{BE}}\right)$ and a point $\mathrm{A}+$ Force
$R_{B} X$ perpendicular distance between the force $R_{B}$ and a point $A=0$
Or
$\mathrm{F}_{\mathrm{BE}} \mathrm{XAB}+\mathrm{R}_{\mathrm{B}} \mathrm{X} \mathrm{A}_{\mathrm{p}}=0$
Or

$$
\begin{aligned}
\mathrm{F}_{\mathrm{BE}} \times 2.59+2 \times 2.25 & =0 \\
\mathrm{~F}_{\mathrm{BE}} & =-1.73 \mathrm{kN} \text { Compression }
\end{aligned}
$$

( Since $\mathrm{F}_{\mathrm{BE}}$ and $\mathrm{F}_{\mathrm{AE}}$ passing through a point E .)
We know that,
Sum of Clockwise moment = Sum of Anticlockwise moment
Force $\mathrm{R}_{\mathrm{A}} \mathrm{X}$ Perpendicular distance between the force $\mathrm{R}_{\mathrm{A}}$ and a point $\mathrm{E}+$ Force $\mathrm{F}_{\mathrm{BC}} \mathrm{X}$
Perpendicular distance between the force $F_{B C}$ and a point $E=\operatorname{Force}\left(R_{B}\right) X$
Perpendicular distance between $R_{B}$ and a point $E$
Or
$\mathrm{R}_{\mathrm{A}} \mathrm{XAE}+\mathrm{F}_{\mathrm{BC}} \mathrm{XBE}=\mathrm{R}_{\mathrm{B}} \mathrm{XpE}$
Or $\quad 1.625$ X $3+\mathrm{F}_{\mathrm{BC}} \mathrm{X} 1.5=\mathrm{R}_{\mathrm{B}} \mathrm{XpE}$
Or $\quad 1.625$ X $3+\mathrm{F}_{\mathrm{BC}} \mathrm{X} 1.5=2 \mathrm{X} 0.75$

$$
(\mathrm{AP}=2.25, \mathrm{AE}=3 \mathrm{~m} \text { and } \mathrm{pE}=3-2.25=0.75 \mathrm{~m})
$$

$$
\mathrm{F}_{\mathrm{BC}}=-2.25 \mathrm{kN} \text { Compression }
$$

Draw a section line $(3,3)$ cutting the members $\mathrm{BC}, \mathrm{CE}$ and ED. (Only two unknown forces are permitted while considering a section. Here, BC is known force, CE and ED are unknown forces).


Fig 5.6.f

Taking moment of all forces acting from the left of section $\left(\mathrm{R}_{\mathrm{A}}, \mathrm{R}_{\mathrm{B}}, \mathrm{F}_{\mathrm{BC}}, \mathrm{F}_{\mathrm{CE}}\right.$ and $\left.\mathrm{F}_{\mathrm{ED}}\right)$ about point D .
Consider $\mathrm{R}_{\mathrm{A}}, \mathrm{R}_{\mathrm{B}}, \mathrm{F}_{\mathrm{BC}}, \mathrm{F}_{\mathrm{CE}}$ forces only.
( Since $\mathrm{F}_{\mathrm{ED}}$ passing through a point D )
We know that, Sum of Clockwise moment = Sum of Anticlockwise moment
Force $R_{A} X$ perpendicular distance between the force $R_{A}$ and a point $D+$ Force $F_{B C} X$ perpendicular distance between the force $F_{B C}$ and a point $D+$ Force $F_{C E} X$ perpendicular distance between $F_{C E}$ and a point $D=$ Force $R_{B} X$ Perpendicular distance between $R_{B}$ and a point $D$
Or $\quad \mathrm{R}_{\mathrm{A}} \mathrm{X} \mathrm{AD}+\mathrm{F}_{\mathrm{BC}} \mathrm{XCD}+\mathrm{F}_{\mathrm{CE}} \mathrm{X} \times \mathrm{D}=\mathrm{R}_{\mathrm{B}} \mathrm{X} \mathrm{pD}$ or
$1.625 \times 6+-2.25 \times 3+F_{C E} X \times D=2 X 3.75$
( $\mathrm{ED}=\mathrm{CE}=\mathrm{CD}=3 \mathrm{~m}$ )
$\mathrm{ED}=3 \mathrm{~m}, \mathrm{Ap}=2.25 \mathrm{~m}, \mathrm{pD}=3+0.75=3.75 \mathrm{~m}$
From $\Delta \mathrm{CxD}$,

$$
\operatorname{Sin} 60^{\circ}=\frac{x D}{C E}
$$

$\mathrm{xD}=\mathrm{CD} X \operatorname{Sin} 60^{\circ}=3 \mathrm{X} \operatorname{Sin} 60^{\circ}$ or $\quad 1.625 \times 6-$
$2.25 \times 3+\mathrm{F}_{\mathrm{CE}} \mathrm{X} \times \mathrm{xD}=2 \mathrm{X} 3.75$ or 1.625 X $6-2.25 \mathrm{X} 3$
$+\mathrm{F}_{\mathrm{CE}} \mathrm{X} 3 \mathrm{X} \operatorname{Sin} 60^{\circ}=2 \mathrm{X} 3.75$

$$
\therefore \quad \mathrm{F}_{\mathrm{CE}}=0.96 \mathrm{kN} \text { (Tension) }
$$

Now taking moment of all the forces acting from the left of the section $\left(\mathrm{R}_{\mathrm{A}}, \mathrm{R}_{\mathrm{B}}, \mathrm{F}_{\mathrm{BC}}\right.$, $F_{C E}$ and $F_{E D}$ ) about point $C$.
Consider $\mathrm{R}_{\mathrm{A}}, \mathrm{R}_{\mathrm{B}}, \mathrm{F}_{\mathrm{ED}}$ forces only.
(since $\mathrm{F}_{\mathrm{BC}}, \mathrm{F}_{\mathrm{CE}}$ passing through a point C )
Force $\mathrm{R}_{\mathrm{A}} \mathrm{X}$ Perpendicular distance between $\mathrm{R}_{\mathrm{A}}$ and a point $\mathrm{C}=$ Force $\mathrm{R}_{\mathrm{B}} \mathrm{X}$ Perpendicular distance between $R_{B}$ and a point $C+$ Force $F_{E D} X$ perpendicular distance between $F_{E D}$ and a point $C$
Or

$$
\mathrm{R}_{\mathrm{A}} \times \mathrm{A}_{\mathrm{q}}=\mathrm{R}_{\mathrm{B}} \times \mathrm{Pq}+\mathrm{F}_{\mathrm{ED}} \times \mathrm{Cq}
$$

Or
1.625 X 4.5=2X $2.25+\mathrm{F}_{\mathrm{ED}} \mathrm{X} 3 \mathrm{X} \sin 60^{\circ}$

$$
\mathrm{F}_{\mathrm{ED}}=1.08 \mathrm{kN} \text { (Tension) }
$$

We know that From fig (iii),

$$
\begin{gathered}
\mathrm{Aq}=4.5 \mathrm{~m} \\
\mathrm{Ap}=2.25 \\
(\mathrm{PE}=3-2.25=0.75 \mathrm{~m} \\
\mathrm{Pq}=\mathrm{PE}+\mathrm{Eq}=0.75+1.5=2.25 \mathrm{~m} \\
\left.\mathrm{Cq}=\mathrm{xD}=3 \mathrm{X} \sin 60^{\circ}\right)
\end{gathered}
$$

Consider a section line $(4,4)$ cutting the members CD and ED.


Fig 5.6.g
Consider the equilibrium of the right part of truss ( since It is smaller than left part). Now taking moment of all forces acting from the right of the section $\left(R_{D}, F_{C D}\right.$ and $\mathrm{F}_{\mathrm{CE}}$ ) about point E .
We know that, Sum of Clockwise moment = Sum of Anticlockwise moment
Or Force $R_{D} X$ perpendicular distance between the force $R_{D}$ and a point $E+$ Force $F_{C D}$ X
perpendicular distance beteen the force $\mathrm{F}_{\mathrm{CD}}$ and a point $\mathrm{E} .=0$
Or $\quad \mathrm{R}_{\mathrm{D}} \mathrm{XDE}+\mathrm{F}_{\mathrm{CD}} \mathrm{XxE}=0$
Or $1.875 \mathrm{X} 3+\mathrm{F}_{\mathrm{CD}} \mathrm{X} 3 \mathrm{X} \sin 60^{\circ}=0$
From $\triangle$ CEx, $\quad \operatorname{Sin} 60^{\circ}=\frac{x E}{C E}$
Or $x E=C E X \operatorname{Sin} 60^{\circ}=3 X \operatorname{Sin} 60^{\circ}$
$\therefore \mathrm{F}_{\mathrm{CD}}=-2.16 \mathrm{kN}$ Compression Result:

| Sl.No. | Member | Force <br> $(\mathrm{kN})$ | Nature of <br> force |
| :---: | :---: | :---: | :---: |
| 1 | AB | -3.25 | Compression |
| 2 | AE | 2.8 | Tension |
| 3 | BE | -1.73 | Compression |
| 4 | BC | -2.25 | Compression |
| 5 | CE | 0.96 | Tension |
| 6 | ED | 1.08 | Tension |
| 7 | CD | -2.16 | Compression |

