Narrow Band Noise

Definition:

A random process X(t) is bandpass or narrowband random process if its power spectral density SX(f) is nonzero only in a small neighborhood of some high frequency fc Deterministic signals: defined by its Fourier transform Random processes: defined by its power spectral density.

1. Since X(t) is band pass, it has zero mean: E[(X(t)] = 0.
2. fc needs not be the center of the signal bandwidth, or in the signal bandwidth at all.

Narrowband Noise Representation:

In most communication systems, we are often dealing with band-pass filtering of signals. Wideband noise will be shaped into bandlimited noise. If the bandwidth of the bandlimited noise is relatively small compared to the carrier frequency, we refer to this as narrowband noise. We can derive the power spectral density Gn(f) and the auto-correlation function Rnn(τ) of the narrowband noise and use them to analyse the performance of linear systems.

In practice, we often deal with mixing (multiplication), which is a non-linear operation, and the system analysis becomes difficult. In such a case, it is useful to express the narrowband noise as $n(t) = x(t) \cos 2\pi f_c t - y(t) \sin 2\pi f_c t$.

where fc is the carrier frequency within the band occupied by the noise. x(t) and y(t) are known as the quadrature components of the noise n(t). The Hibert transform of n(t) is $\hat{n}(t) = H[n(t)] = x(t) \sin 2\pi f_c t + y(t) \cos 2\pi f_c t$.

- Generation of quadrature components of n(t).

x(t) and y(t) have the following properties:

1. E[x(t) y(t)] = 0. x(t) and y(t) are uncorrelated with each other.
2. x(t) and y(t) have the same means and variances as n(t).
3. If n(t) is Gaussian, then x(t) and y(t) are also Gaussian.

4. x(t) and y(t) have identical power spectral densities, related to the power spectral density of n(t) by Gx(f) = Gy(f) = Gn(f- fc) + Gn(f+ fc) (28.5)

for fc - 0.5B < | f | < fc + 0.5B and B is the bandwidth of n(t).

Inphase and Quadrature Components:

In-Phase & Quadrature Sinusoidal Components

$$\sin(A+B) = \sin(A)\cos(B) + \cos(A)\sin(B)$$

From the trig identity, we have

$$\begin{aligned} x(t) &\stackrel{\Delta}{=} A\sin(\omega t + \phi) = A\sin(\phi + \omega t) \\ &= [A\sin(\phi)]\cos(\omega t) + [A\cos(\phi)]\sin(\omega t) \\ &\stackrel{\Delta}{=} A_1\cos(\omega t) + A_2\sin(\omega t). \end{aligned}$$

From this we may conclude that every sinusoid can be expressed as the sum of a sine function phase zero) and a cosine function (phase 2). If the sine part is called the ``in-phase'' component, the cosine part can be called the ``phase-quadrature'' component. In general, ``phase quadrature'' means ``90 degrees out of phase,'' i.e., a relative phase shift of ± 2. It is also the case that every sum of an in-phase and quadrature component can be expressed as a single sinusoid at some amplitude and phase. The proof is obtained by working the previous derivation backwards. Figure 4.5.1 illustrates in-phase and quadrature components overlaid. Note that they only differ by a relative 0 degree phase shift.

PHASOR REPRESENTATION OF SIGNAL AND NOISE:

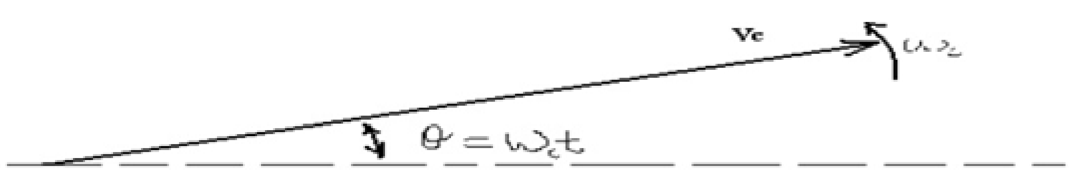

Figure 4.5.1 Phasor Diagram - 1

Diagram Source Brain Kart

The phasor represents a signal with peak value Vc, rotating with angular frequencies ωc rads per sec and with an angle ωct some reference axis at time t=0.

If we now consider a carrier with a noise voltage with ―peak‖ value superimposed we may represents this as:

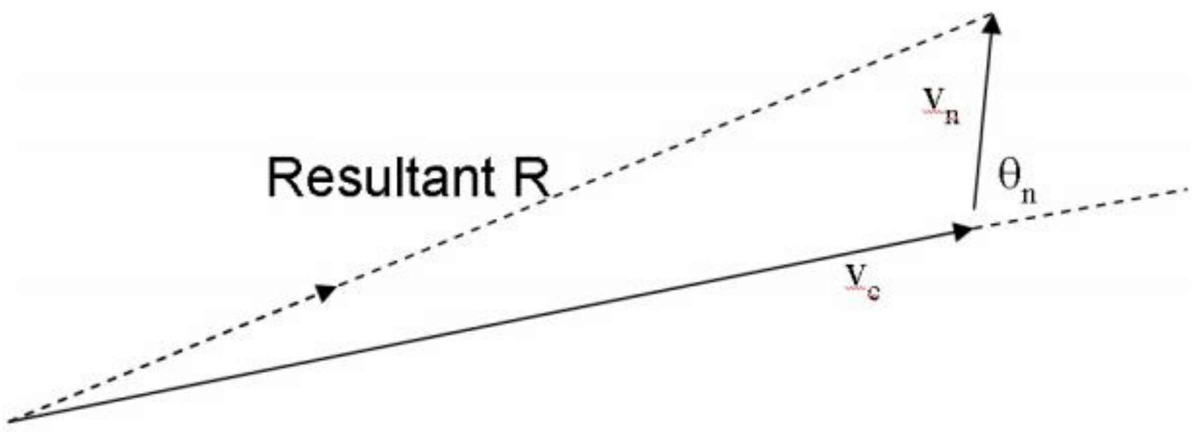

Figure 4.5.2 Phasor Diagram-2

Diagram Source Brain Kart

In this case Vn is the peak value of the noise and is the phase of the noise relative to the carrier. Both Vn and θ n are random variables, the above phasor diagram represents a snapshot at some instant in time in figure 4.5.2 and 4.5.3. The resultant or received signal R, is the sum of carrier plus noise. If we consider several snapshots overlaid as shown below we can see the effects of noise accompanying the signal and how this affects the received signal R.

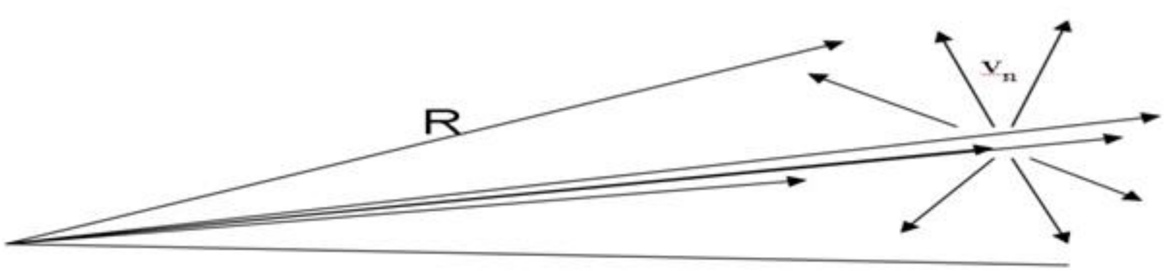

Fig 4.5.3 Phasor Diagram-3 , Diagram Source Brain Kart

Thus the received signal has amplitude and frequency changes (which in practice occur randomly) due to noise.We may draw, for a single instant, the phasor with noise resolved into 2 components, which are in the figure 4.5.4:

a) x(t) in phase with the carriers

x(t) Vn Cos θ n

b) y(t) in quadrature with the carrier

y(t) = Vn Sin θ n

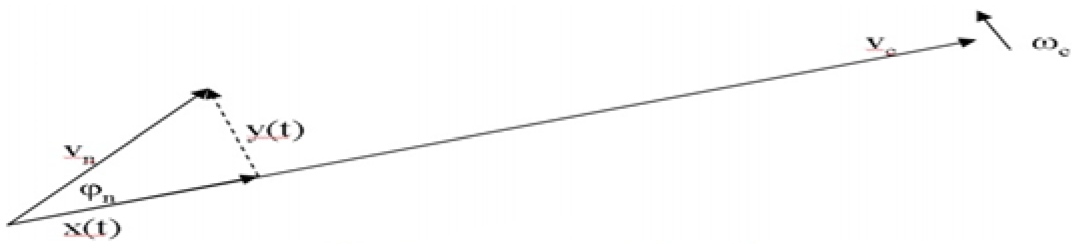

Fig 4.5.4 Phasor Diagram-4 , Diagram Source Brain Kart

The reason why this is done is that x(t) represents amplitude changes in Vc (amplitude changes affect the performance of AM systems) and y(t) represents phase (i.e. frequency) changes (phase / frequency changes affect the performance of FM/PM systems)

We note that the resultant from x(t) and y(t) i.e.

We can regard x(t) as a phasor which is in phase with Vc Cos c t , i.e a phasor rotating at ωc . i.e. x(t)Cosω*c* t

and by similar reasoning, y(t) in quadrature i.e. y(t)Sinω*c* t

Hence we may write

Vn (*t)* + x(t) Cosω*c* t + y(t) Sinω*c* t

This equation is algebraic representation of noise and since

$$V_n = \sqrt{y(t)^2 + x(t)^2}$$
$$= \sqrt{V_n^2\, Cos^2\phi_n + V_n^2\, Sin^2\phi_n}$$
$$= V_n \qquad (Since\ Cos^2\theta + Sin^2\theta = 1)$$

We can regard x(t) as a phasor which is in phase with $V_c\, Cos\, \omega_c t$, i.e a phasor rotating at ω_c.

i.e. x(t)Cos ω_c t

and by similar reasoning, y(t) in quadrature

i.e. y(t)Sinω_c t

Hence we may write

$$V_n(t) = x(t)\, Cos\omega_c t + y(t)\, Sin\omega_c t$$

Or – alternative approach

$$V_n(t) = V_n\, Cos(\omega_c t - \phi_n)$$

$$V_n(t) = V_n\, Cos\phi_n\, Cos\omega_c t + V_n\, Sin\phi_n\, Sin\omega_c t$$

$$V_n(t) = x(t)\, Cos\omega_c t + y(t)\, Sin\omega_c t$$

This equation is algebraic representation of noise and since

$$x(t)=V_n\,Cos\,\phi_n = \sqrt{2\,p_o B_n}\,Cos\,\phi_n$$

the peak value of x(t) is $\sqrt{2\,p_o B_n}$ (i.e. when $Cos\,\phi_n = 1$)

Similarly the peak value of y(t) is also $\sqrt{2\,p_o B_n}$ (i.e. when $Sin\phi_n = 0$)

The mean square value in general is $\left(\frac{V_{peak}}{\sqrt{2}}\right)^2 = (V_{rms})^2$

and thus the mean square of x(t), i.e $\overline{x(t)^2} = \left(\frac{\sqrt{2p_o B_n}}{\sqrt{2}}\right)^2 = p_o B_n$

also the mean square value of y(t), i.e $\overline{y(t)^2} = \left(\frac{\sqrt{2p_o B_n}}{\sqrt{2}}\right)^2 = p_o B_n$

The total noise in the bandwidth, Bn is

$$N = \left(\frac{V_v}{\sqrt{2}}\right)^2 = p_o B_n = \frac{\overline{x(t)^2}}{2} + \frac{\overline{y(t)^2}}{2}$$

i.e. NOT $\overline{x(t)^2} + \overline{y(t)^2}$ as might be expected.

The reason for this is due to the Cos ϕn and Sinϕn relationship in the representation e.g. when say ―x(t)‖ contributes po Bn , the ―y(t)‖ contribution is zero, i.e. sum is always equal to po Bn .

The algebraic representation of noise discussed above is quite adequate for the analysis of many systems, particularly the performance of ASK, FSK and PSK modulated systems.

When considering AM and FM systems, assuming a large (S/N) ratio, i.e Vc>> Vn, the following may be used.

Considering the general phasor representation shown in the figure 4.5.5 below:-

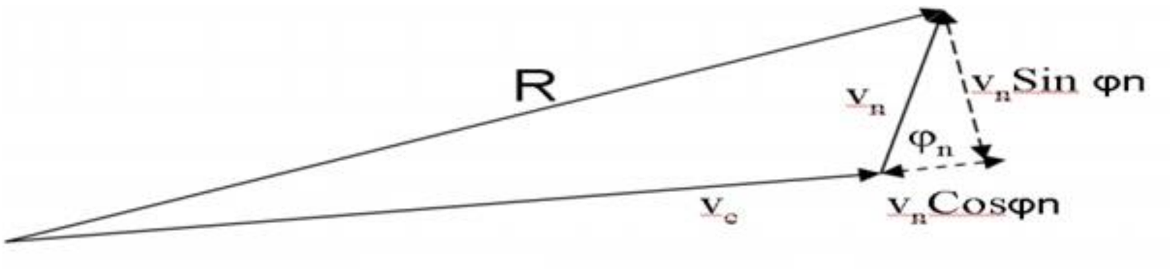

Figure 4.5.5 Phasor Diagram-5 ,

Diagram Source Brain Kart

Since AM is sensitive to amplitude changes, changes in the Resultant length are predominantly due to x(t) in the figure 4.5.6.

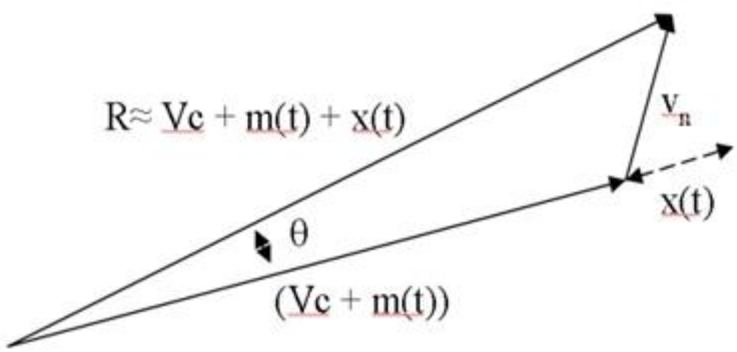

Figure 4.5.6 Phasor Diagram-6

Diagram Source Brain Kart

For FM systems the signal is of the form $Vc\,Cos\omega ct$ Noise will produce both amplitude changes (i.e. in Vc) and frequency variations – the amplitude variations are removed by a limiter in the FM receiver. Hence,

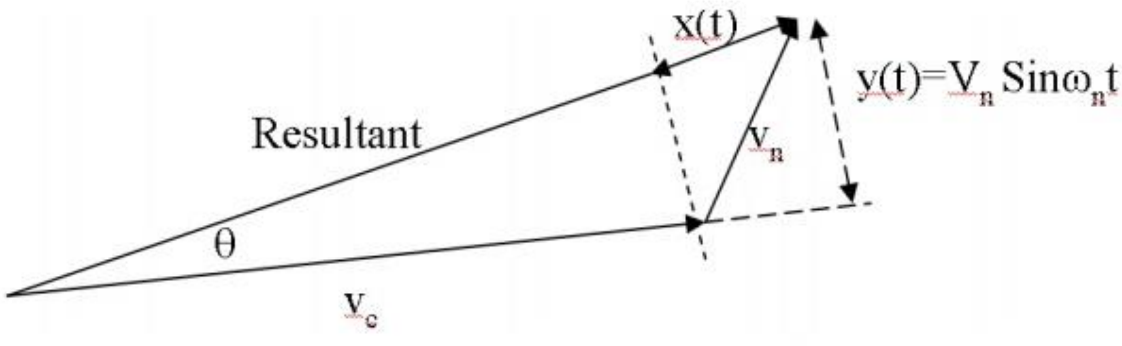

Figure 4.5.7 Phasor Diagram -7

Diagram Source Brain Kart

The angle theta represents frequency / phase variations in the received signal due to noise. From the diagram.

$$\theta = \tan^{-1}\left(\frac{V_n Sin\omega_n t}{V_c + V_n Cos\omega_n t}\right)$$

$$= \tan^{-1}\left(\frac{\frac{V_n}{V_c} Sin\omega_n t}{1 + \frac{V_n}{V_c} Cos\omega_n t}\right)$$

Since $V_c >> V_n$ (assumed) then $\frac{V_n}{V_c} Cos\omega_n t << 1$

So $\theta = \tan^{-1}\left(\frac{V_n}{V_c} Sin\omega_n t\right)$ {which is also obvious from diagram}

Since $\tan\theta = \theta$ for small θ and θ is small since $V_c >> V_n$

Then $\theta \approx \frac{V_n}{V_c} Sin\omega_n t$

The above discussion for AM and FM serve to show bow the ‘model‘ may be used to describe the effects of noise. Applications of this model to ASK, FSK and PSK demodulation, and AM and FM demodulation are discussed elsewhere.