MODULE – 1

ELECTRICAL PROPERTIES OF MATERIALS

1.6.Effective mass of Electron

The mass acquired by an electron when it is accelerated in a periodic potential is called effective mass of an electron.It is denoted by m^*.

Explanation:

When an electron is accelerated the mass of the electron is not constant,but it varies.This varying mass is called effective mass (m^*).

Derivation of effective mass of electron:

When electric field is applied to a crystal the electron gains velocity described by wave vector k.

Group velocity $$v_g = \frac{dw}{dk} \quad \text{-----------------------(1)}$$

Where ω – angular frequency of the electron.

k- wave vector

we know that E = hϒ

$$E = \frac{h\,\omega}{2\pi}$$

$$= \hbar\,\omega$$

$$\omega = \frac{E}{\hbar} \quad \text{--------------------------(2)}$$

Substituting (2) in(1)

$$v_g = \frac{d}{dk}\left(\frac{E}{\hbar}\right)$$

$$v_g = \frac{1}{\hbar}\frac{dE}{dk}$$

The acceleration 'a' is

$$a = \frac{d}{dt}(v_g)$$

$$= \frac{d}{dt}\left[\frac{1}{\hbar}\left(\frac{dE}{dk}\right)\right]$$

$$= \frac{1}{\hbar}\frac{d^2E}{dk^2}\frac{dk}{dt} \text{------------------------}(4)$$

Momentum 'p' of an electron

$$p = \frac{h}{\lambda}$$

$$= \frac{h}{2\pi}\frac{2\pi}{\lambda}$$

$$= \hbar k \text{------------------------}(5)$$

Differentiating (5) w.r.t 't'

$$\frac{dp}{dt} = \hbar\frac{dk}{dt}$$

Or

$$F = \hbar\frac{dk}{dt}$$

$$\frac{dk}{dt} = \frac{F}{\hbar} \text{ -----------------(6)}$$

Substituting (6) in (4)

$$a = \frac{1}{\hbar}\frac{d^2E}{dk^2}\frac{F}{\hbar}$$

$$= \frac{1}{\hbar^2}\frac{d^2E}{dk^2}F$$

$$F = \left[\frac{\hbar^2}{\frac{d^2E}{dk^2}}\right] a \text{ -------------------(7)}$$

When an electrical field is applied ,acceleration is

$$a = \frac{eE}{m^*} = \frac{F}{m^*}$$

$$F = m^* a \text{ ----------------------(8)}$$

Comparing (7) & (8)

$$m^* a = \left[\frac{\hbar^2}{\frac{d^2E}{dk^2}}\right] a$$

$$m^* = \frac{\hbar^2}{\frac{d^2E}{dk^2}}$$

From (9) effective mass is not constant but depends on $\frac{d^2E}{dk^2}$

Special Cases:

Case i :

If $\frac{d^2E}{dk^2}$ is positive then m^* is also positive.

Case ii :

If $\frac{d^2E}{dk^2}$ is negative then m^* is also negative.

Case iii :

If $\frac{d^2E}{dk^2}$ is more then electrons behave as light particles.

Case iv :

If $\frac{d^2E}{dk^2}$ is very small,then the electrons behave as heavy particles.

1.6.1.Concept of hole (or) Effective or Negative mass of electron:

The effective mass m^* is negative near the zone edges of filled valence bands.The electrons in these regions are accelerated in a direction opposite to the direction of the applied field. This is called the negative mass behavior of the electrons.

The electrons with negative effective mass is considered as the same positive mass of that of an electron, but with positive charge. This new entity is given the name "hole".

The positive hole conduction and effective negative electron mass conduction are in equilibrium. The calculation made on the hole appear to be more convenient and hence the hole concept is retained.

Several phenomena like Hall effect, Thomson Effect etc find explanation on the basis of the hole concept.