4.2 Compton Effect

When a beam of monochromatic radiation such as x-rays, γ-rays etc., of light frequency is allowed to fall on a small particle then the beam is scattered into two

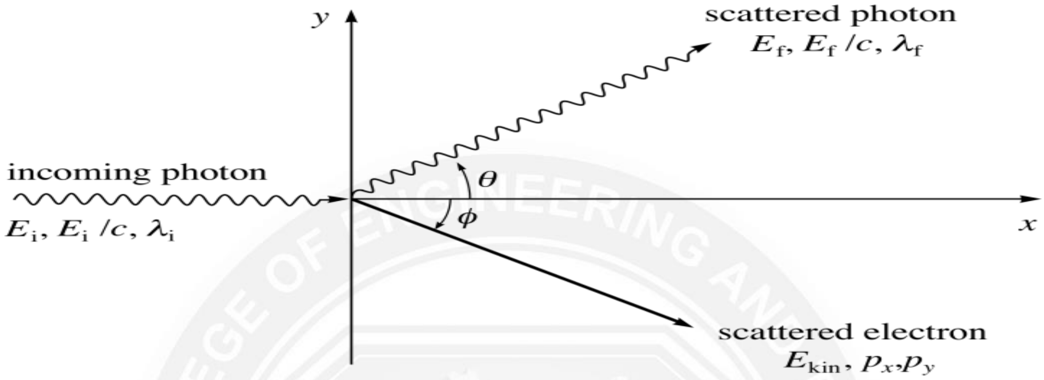


Fig :4.2.1Compton Effect

components. One component has the same wavelength as that of the incident radiation and the other component has a slightly longer wavelength.

This effect of scattering is called Compton Effect. The shift in wavelength is called Compton Shift.

Theory of Compton Effect

Principle

In Compton scattering the collision between a photon and an electron is considered. Then by applying the laws of conservation of energy and momentum, the expression for Compton wavelength is derived.

Assumptions

The collision occurs between the photon and an electron in the scattering material.

The electron is free and is at rest before collision with in incident photon.

With these assumptions let us consider a photon of energy ‘h*ν*’ colliding with an electron at rest. During the collision process, a part of energy is given to the electron, which in turn increases the kinetic energy of the electron and hence it recoils at an angle of ϕ as shown in figure. The scattered photon moves with an energy hν' (less than hν), at an angle θ with respect to the original direction.

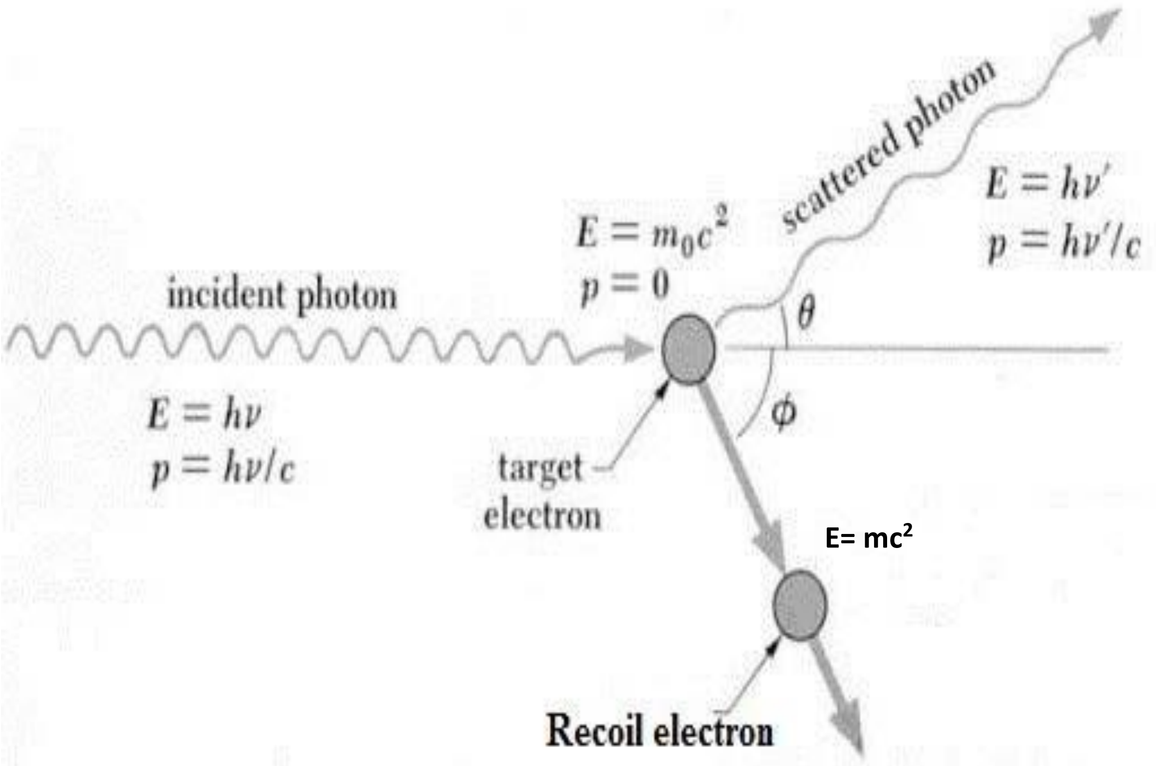


Fig : 4.2.2Compton Scattering

Let us find the energy and momentum components before and after collision process.

Energy before collision

Energy of the incident photon = hν

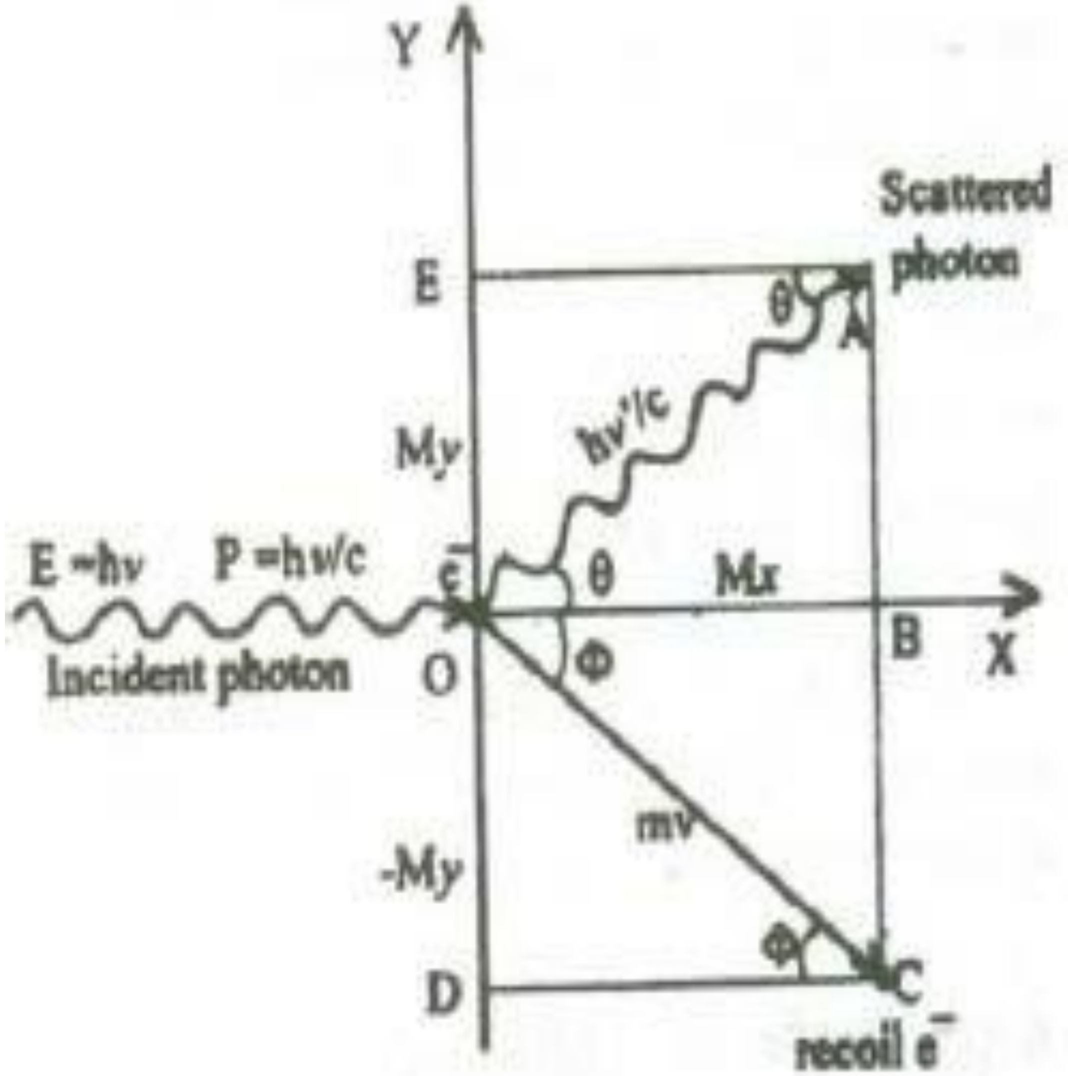
Energy of the electron at rest = m_0c^2

Where, m_0 is the rest mass energy of the electron.

Total Energy before Collision = $hv + m_0c^2$ ....(1)

Energy after collision

Energy of the scattered photon = hv′



Energy of the recoil electron = mc^2

Where, m is the mass of the electron moving with velocity v'

Total energy after collision = $hv' + mc^2$ ---------**(2)**

According to the **law of conservation of energy**

Total energy before collision = Total energy after collision

Equation (1) = Equation (2)

$hv + m_0c^2 = hv' + mc^2$ ... .(3)

X – Component of Momentum before Collision

X –component momentum of the incident photon $= \frac{hv}{c}$

X – component momentum of the electron at rest $= 0$

Total X – component of momentum before collision is $= \frac{hv}{c}$------(4)

x- Component of momentum after collision

X- Component momentum of the scattered photon can be calculated from figure.

In Δ OAB

$$\cos\theta = \frac{Mx}{\frac{hv'}{c}}$$

X – Component momentum of the scattered photon is $= \frac{hv'}{c}\cos\theta$

X- Component momentum of the recoil electron can be calculated from figure.

In Δ OBC $\cos\phi = \frac{M_x}{mv}$

X – Component momentum of the recoil electron is $= mv\cos\phi$

Total X- Component of momentum after collision

$$= \frac{hv'}{c}\cos\theta + mv\cos\phi \text{.....................(5)}$$

According to the **law of conservation of momentum**

Total momentum before collision = Total momentum after collision

Equation (4) = Equation (5)

$$\frac{hv}{c} = \frac{hv'}{c}\cos\theta + mv\cos\phi \text{ ....(6)}$$

Y – Component of Momentum before collision

Y – Component momentum of the incident photon = 0

Y – Component momentum of the electron at rest = 0

Total Y – component of momentum before collision is = 0.........................(7)

Y – Component of momentum after collision

From figure, in Δ OAE,

$$\sin\theta = \frac{M_y}{\frac{hv'}{c}}$$

Y – Component momentum of the scattered photon is $= \frac{hv'}{c}\sin\theta$

From figure, in Δ OCD,

$$\sin\phi = \frac{-M_y}{mv}$$

Y- Component momentum of the recoil electron is $= -\,mv\sin\phi$

Total Y-Component of momentum after collision is

$$= \frac{hv'}{c}\sin\theta - mv\sin\phi \text{--------------} \quad (8)$$

According to the law conservation of momentum,

Equation (7) = Equation (8)

$$0 = \frac{hv'}{c}\sin\theta - mv\sin\phi \text{------------} \quad (9)$$

From equation (6), we can write

$$\frac{hv}{c} = \frac{hv'}{c}\cos\theta + mv\cos\phi$$

$mcv\cos\phi = h(v - v'\cos\theta)$................(10)

From equation (9) we can write

$$mcv \sin \phi = hv' \sin \theta \ldots\ldots\ldots\ldots (11)$$

Squaring and adding equation (10) and (11) we get

$$m^2c^2v^2 (\cos^2\phi + \sin^2 \phi) = h^2 [v^2 - 2vv' \cos \theta + (v')^2 \cos^2\theta] + h^2 (v')^2 \sin^2\theta$$

Since , $\cos^2\phi + \sin^2 \phi = 1$ and

$h^2 (v')^2[\cos^2\theta + \sin^2\theta] = h^2 (v')^2$ we get

$$m^2c^2v^2 = h^2 [v^2 - 2vv'\cos \theta + (v')^2] \ldots\ldots \quad (12)$$

From equation (3), we can write

$$mc^2 = m_0c^2 + h (v - v')$$

Squaring on both sides we get

$$m^2c^4=m_0^2c^4+2hm_0c^2+h(v - v')+h^2[v^2-2vv'+(v')^2] \ldots..(13)$$

Subtracting equation (12) from equation (13), we get

$$m^2c^2 (c^2 - v^2)=m_0^2c^4+2hm_0c^2+h(v - v')-2h^2vv'(1-\cos \theta) \ldots\ldots\ldots\ldots (14)$$

From the theory of relativity, the relativistic formula for the variation of mass with the velocity of the electron is given by

$$m= \frac{m_0}{\sqrt{1-\frac{v^2}{c^2}}}$$

$$m^2= \frac{m_0^2}{1-\frac{v^2}{c^2}}$$

$$m^2= \frac{c^2m_0^2}{c^2-v^2}$$

$$(c^2 - v^2)m^2 = c^2 m_0^2 \ldots\ldots\ldots\ldots\ldots(15)$$

In order to make this equation similar to LHS of equation (14) multiply it by c^2 on both sides,

$$m^2 c^2 (c^2 - v^2) = m_0^2 c^4 \ldots\ldots\ldots\ldots (16)$$

Equating equation (16) and (14), we can write

$$m_0^2c^4 = m_0^2c^4 + 2h\, m_0c^2 + h\,(v - v') - 2h^2vv'\,(1 - \cos\theta)$$

$$2h\, m_0c^2 + h\,(v - v') = 2h^2vv'\,(1 - \cos\theta)$$

$$\frac{v - v'}{vv'} = \frac{h}{m_0 c^2}(1 - \cos\theta)$$

$$\frac{1}{v'} - \frac{1}{v} = \frac{h}{m_0 c^2}(1 - \cos\theta)$$

Multiplying both sides by 'c', we get

$$\frac{c}{v'} - \frac{c}{v} = \frac{h}{m_0 c^2}(1 - \cos\theta) \ldots\ldots\ldots\ldots\ldots\ldots(17)$$

Since $\lambda = \frac{c}{v}$ and $\lambda' = \frac{c}{v'}$ we can write equation (17) as

$$\lambda' - \lambda = \frac{h}{m_0 c^2}(1 - \cos\theta)$$

The change in wavelength

$$\Delta\lambda = \frac{h}{m_0 c^2}(1 - \cos\theta) \ldots\ldots\ldots (18)$$

Equation (18) represents the shift in wavelength or Compton Shift, which is independent of the incident radiation as well as the nature of the scattering substance. Thus the shift in wavelength or Compton Shift depends on the angle of scattering.

Special Cases:

Case:i

When $\theta = 0$; $\cos\theta = 1$

Equation (18) becomes $\Delta\lambda = 0$

This implies that at $\theta = 0$ the scattering is absent and the out coming radiation has the same wavelength or frequency as that of the incident radiation. Thus we get the output as a single peak.

Case (ii)

When $\theta = 90^0$; $\cos\theta = 0$

Equation (18) becomes $\Delta\lambda = \frac{h}{m_0\, c}$

Substituting the values of *h*, m_0 and *c* we get

$$\Delta\lambda = \frac{6.625x10^{-34}}{9.11x10^{-31}\; 3x10^{8}}$$

$$\Delta\lambda = 0.02424\ A^0$$

This wavelength is called *Compton wavelength*, which has a good agreement with the experimental results

Case (iii)

When $\theta = 180^0$; $\cos\theta = -1$

Equation (18) becomes $\Delta\lambda = \frac{h}{m_0\, c}[1-(-1)]$

$$\Delta\lambda = \frac{2h}{m_0\, c}$$

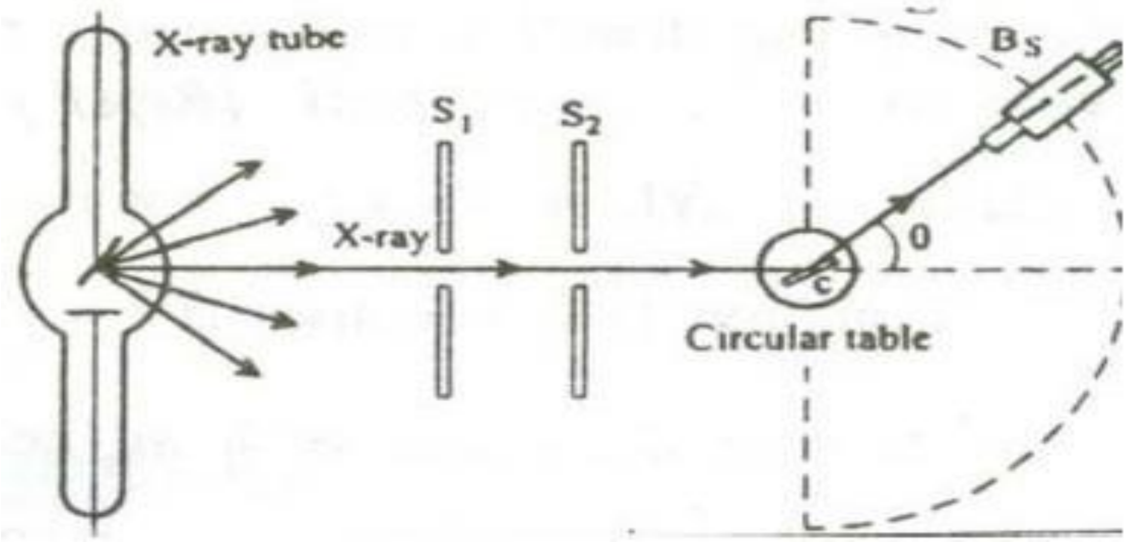
Substituting the values of *h*, m_0 and *c* we get

$$\Delta\lambda = 0.04848\ A^0$$

Thus for $\theta = 180^0$ the shift in wavelength is found to be maximum.

Experimental Verification of Compton Effect Principle:

When a beam of monochromatic radiation such as x-rays, γ-rays etc., of light frequency is allowed to fall on a small particle then the beam is scattered into two components. One component has the same wavelength as that of the incident radiation and the other component has a slightly longer wavelength. This effect of scattering is called Compton Effect. The shift in wavelength is



called Compton Shift.

Fig 4.2.3 Compton Effect

Construction

It consists of an X – ray tube for producing X – rays. A small block of carbon C (scattering element) is mounted on a circular table as shown in figure. A Bragg's spectrometer (B_s) is allowed to freely swing in an arc about the scattering element to catch the scattered photons. Slits S_1 and S_2 helps to focus the X – rays onto the scattering element.

Working

X - Rays of monochromatic wavelength ‘λ’ is produced from an X - rays tube and is made to pass through the slit S_1 and S_2. These X - rays are made to fall on the scattering element. The scattered X - rays are received with the help of the Bragg’s spectrometer and the scattered wavelength is measured. The experiment is repeated for various scattering angles and the scattered wavelengths are measured. The experimental results are plotted.

In this figure when the scattering angle $\theta = 0^0$, the scattered radiation peak will be the same as that of the incident radiation peak ‘A’. Now, when the scattering angle is increased, for one incident radiation peak A of wavelength (λ) we get two scattered peals A and B. Here the peak ‘A’ is found to be of same wavelength as that of the incident wavelength and the peak ‘B’ is of greater wavelength than the incident radiation.

The shift in wavelength or difference in wavelength (Δλ) of the two scattered beams is found to increase with respect to the increase in scattering angle. At $\theta = 90^0$, the Δλ is found to be 0.0236 = 0.02424 A^0, which has good agreement with the theoretical results. Hence this wavelength is called Compton wavelength and the shift in wavelength is called Compton shift.

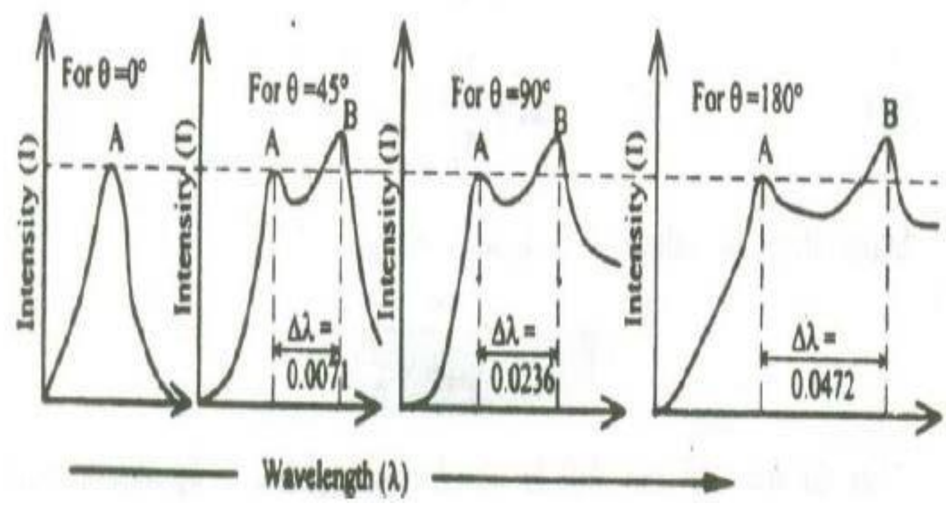


Fig:4.2.4 Compton shift