CONVOLUTION

- Mathematical way of combining two signals to form a third signal.
- It is the most important technique in DSP because convolving the two sequences in the time domain is equivalent to multiplying the sequences in the frequency domain.
- It is used for determining a system output given an input signal x(n) & system impulse response h(n).

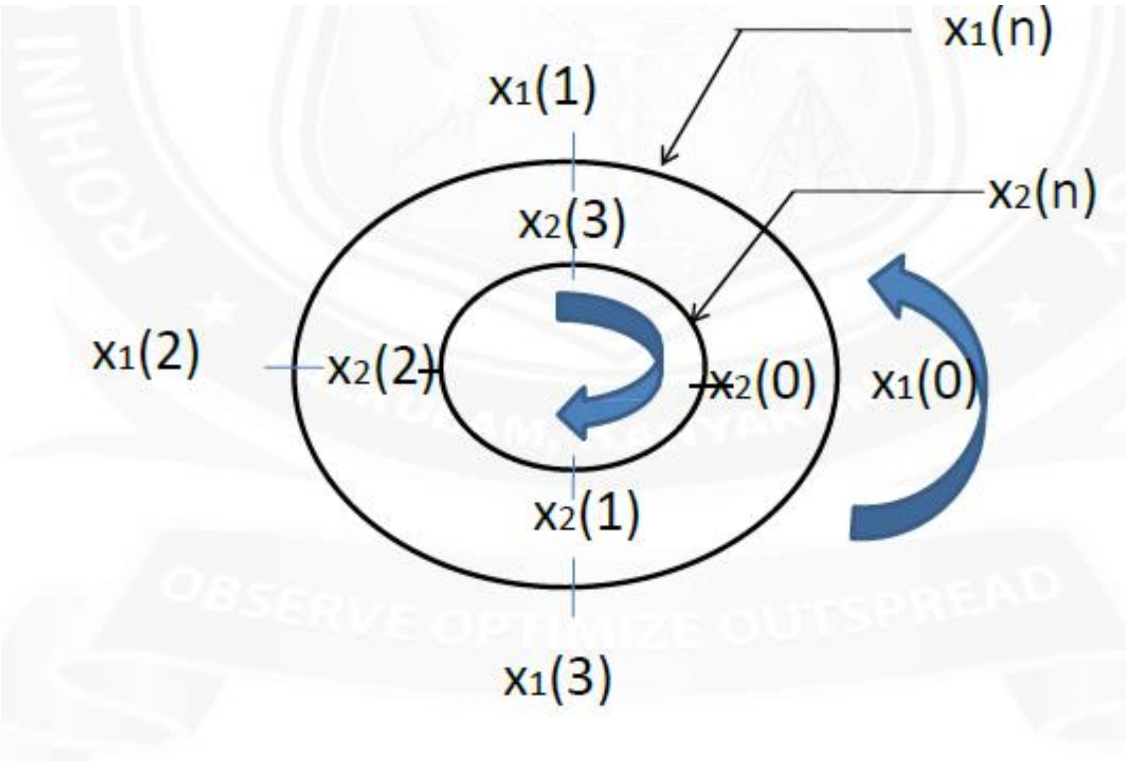
TYPES OF CONVOLUTION

- Linear convolution
  - Graphical method
  - Cross table method
- Circular convolution
  - Concentric circle method
  - Matrix multiplication method
- Section convolution
  - Overlap save method
  - Overlap add method

CIRCULAR CONVOLUTION

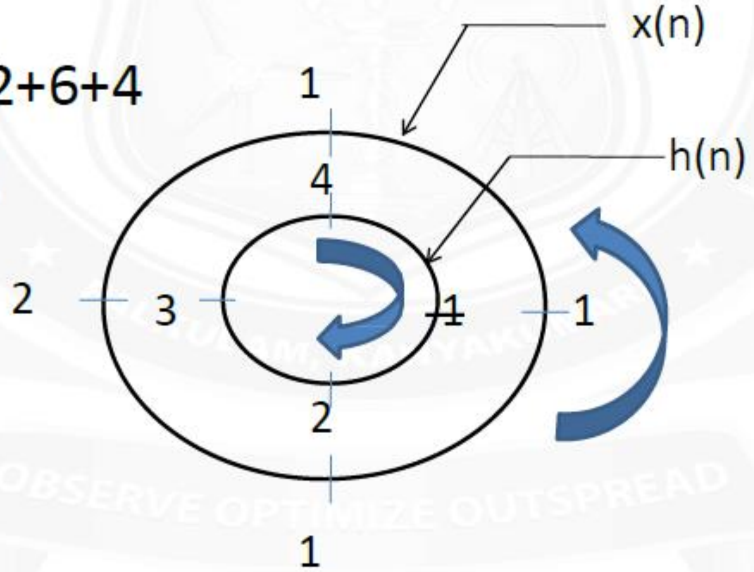
- It is a Periodic convolution
- Length of the two sequence must be same
- $x_1(n)$ & $x_2(n)$--->inputs
- $x_3(n)$---->output
- $x_3(n)=x_1(n) \circledN x_2(n)$
- Zero padding: if the two sequence length are not equal, we should add zeros to equate the length of the sequences.

CONCENTRIC CIRCLE METHOD

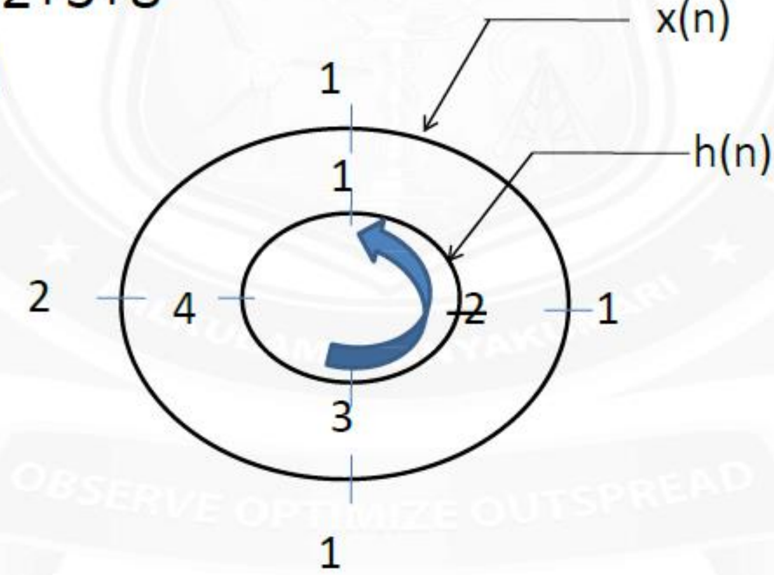


Perform the circular convolution of the following sequences, x(n)={1,1,2,1} and h(n)={1,2,3,4}

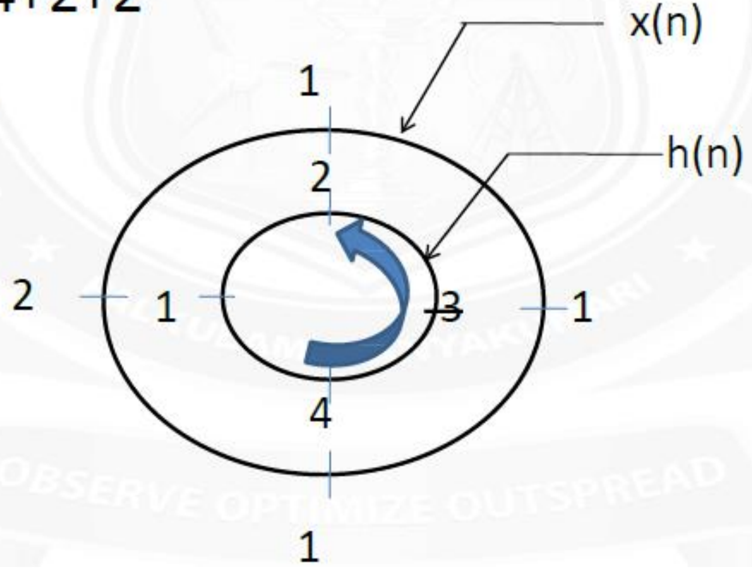
- Soln: y(n)=x(n) Ⓝ h(n)
- Step:1
- y(0)=1+2+6+4
- y(0)=13



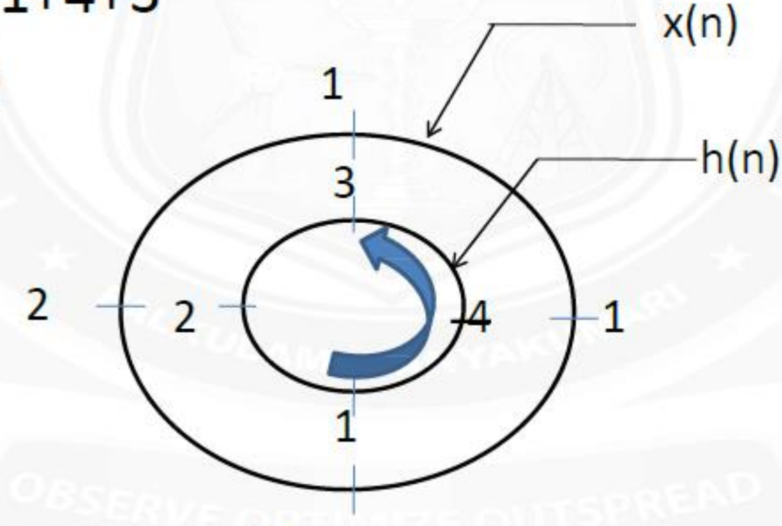
- Step:2
- y(1)=1+2+3+8
- y(1)=14



- Step:3
- y(2)=3+4+2+2
- y(2)=11



- Step:4
- y(3)=4+1+4+3
- y(3)=12



- y(n)={13,14,11,12}

Perform the circular convolution of the following sequences, x(n)={1,1,2,1} and h(n)={1,2,3,4}

- Matrix method:

h(n) x(n) y(n)

$$\begin{bmatrix} 1 & 4 & 3 & 2 \\ 2 & 1 & 4 & 3 \\ 3 & 2 & 1 & 4 \\ 4 & 3 & 2 & 1 \end{bmatrix} \begin{bmatrix} 1 \\ 1 \\ 2 \\ 1 \end{bmatrix} = \begin{bmatrix} 1+4+6+2 \\ 2+1+8+3 \\ 3+2+2+4 \\ 4+3+4+1 \end{bmatrix} = \begin{bmatrix} 13 \\ 14 \\ 11 \\ 12 \end{bmatrix}$$

- y(n)={13,14,11,12}

OVERLAP PROCESSING

For linear filtering we discussed the use of DFT. When the input data sequence is long, then it requires large time to get the output sequence. Hence other techniques are used to filter long data sequences. These techniques are overlap save method and overlap add method.

Instead of finding the output of complete input sequence. It is broken into small length sequences. The outputs due to these small length input sequences are computed fast. Since the filtering is linear, the outputs due to these small length sequences are fitted one after another (concatenated) to get the final output sequence.

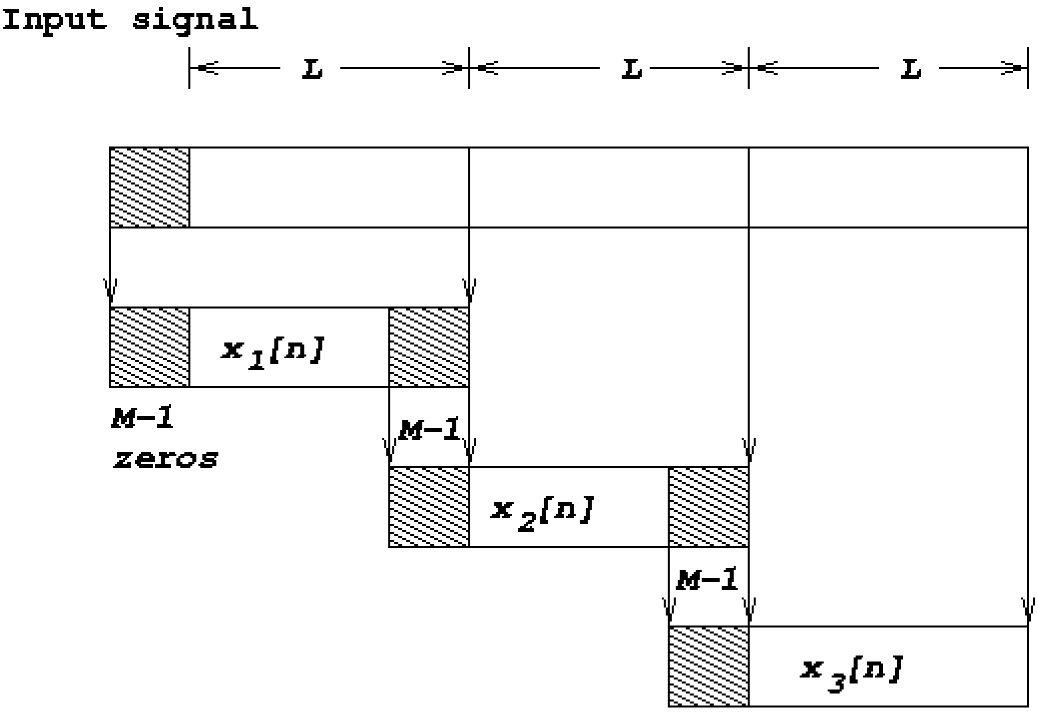
Let the unit sample response of FIR filter has length '*M*'. Let the input data sequence be segmented into blocks of '*L*' samples.

- Now consider filter response of length *P*, but assume input signal is of arbitrarily long length: need to run filter "on the fly" as blocks of input data become available
- Plan: break signal into consecutive blocks of length *L*, pad each with zeros to length *L*+*P*-1, and do FFT/multiply/IFFT

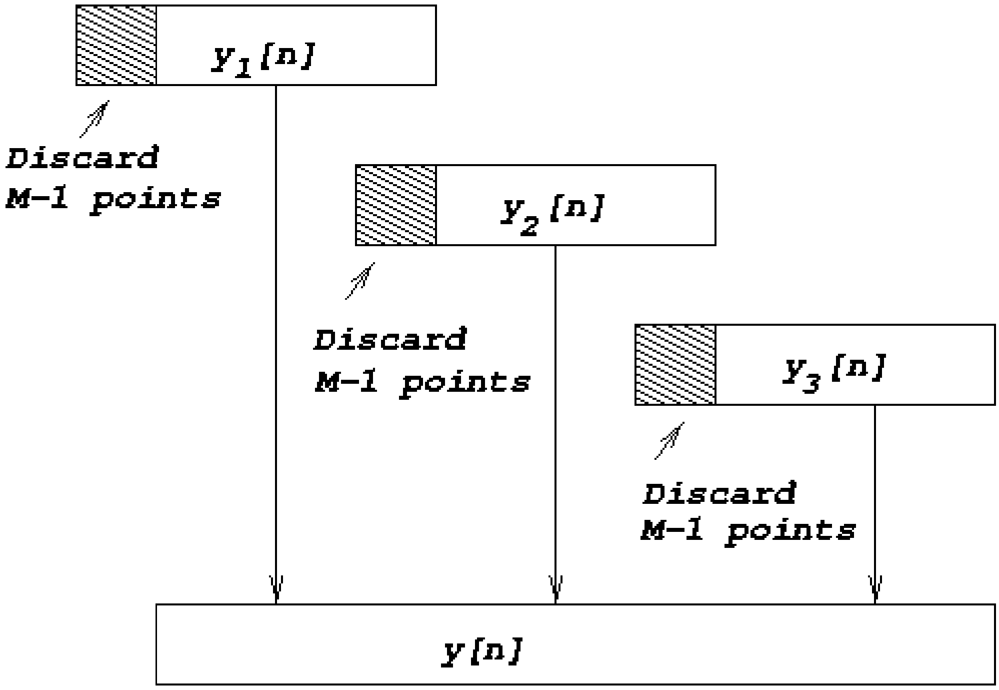
OVERLAP ALGORITHM

- *Note* that the last *P*-1 output samples will overlap the start of the next block, and the overlapping points must be added to get the proper response. This is known as the *overlap-add algorithm.*

OVERLAP SAVE METHOD



Output Signal



Find y(n) using overlap save method

x(n)={1,2,3,4,5,6,7,8,9} and h(n)={1,2,3}

- Soln: step-1
- x(n)={1,2,3,4,5,6,7,8,9} → L=3
- h(n)={1,2,3} → M=3
- x1(n)={0,0,1,2,3}   h(n)={1,2,3,0,0}

  M-1 zeros

- x2(n)={2,3,4,5,6}
- x3(n)={5,6,7,8,9}
- x4(n)={8,9,0,0,0}

- $y_1(n)=x_1(n) \circledN h(n)=\{12,9,1,4,10\}$
- $y_2(n)=x_2(n) \circledN h(n)=\{29,25,16,22,28\}$
- $y_3(n)=x_3(n) \circledN h(n)=\{47,43,34,40,46\}$ **Step-2**
- $y_4(n)=x_4(n) \circledN h(n)=\{8,25,42,27,0\}$
- Step-3:

12 9 1 4 10

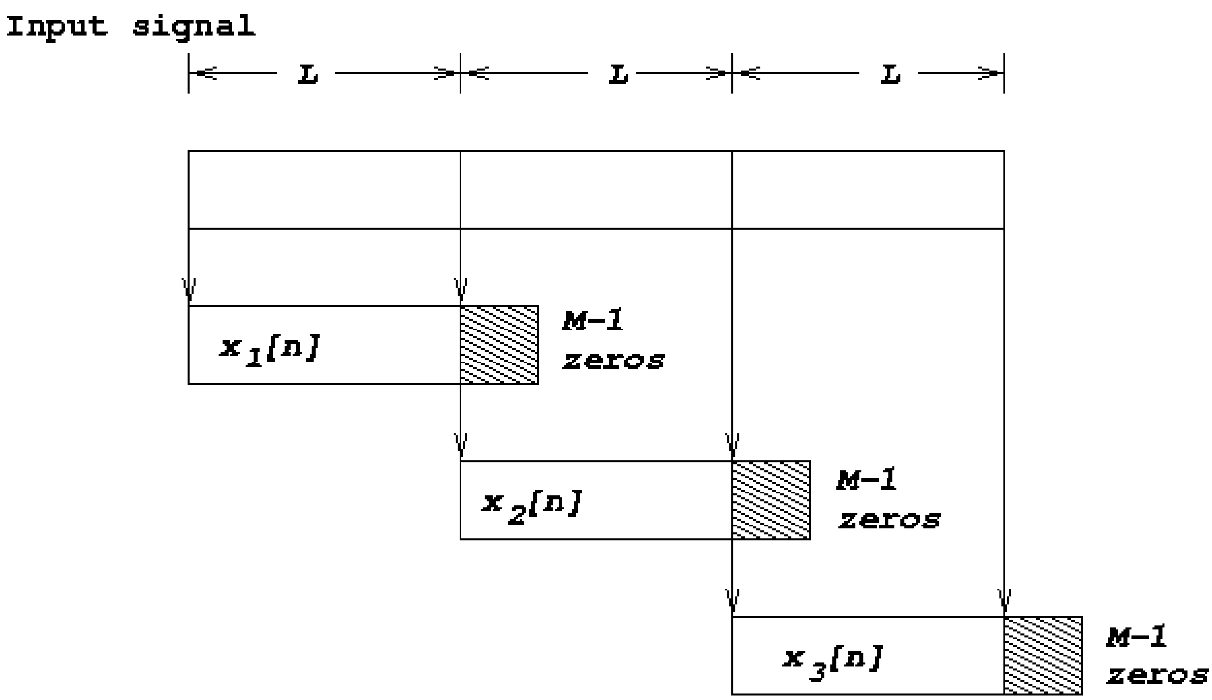
29 25 16 22 28

47 43 34 40 46

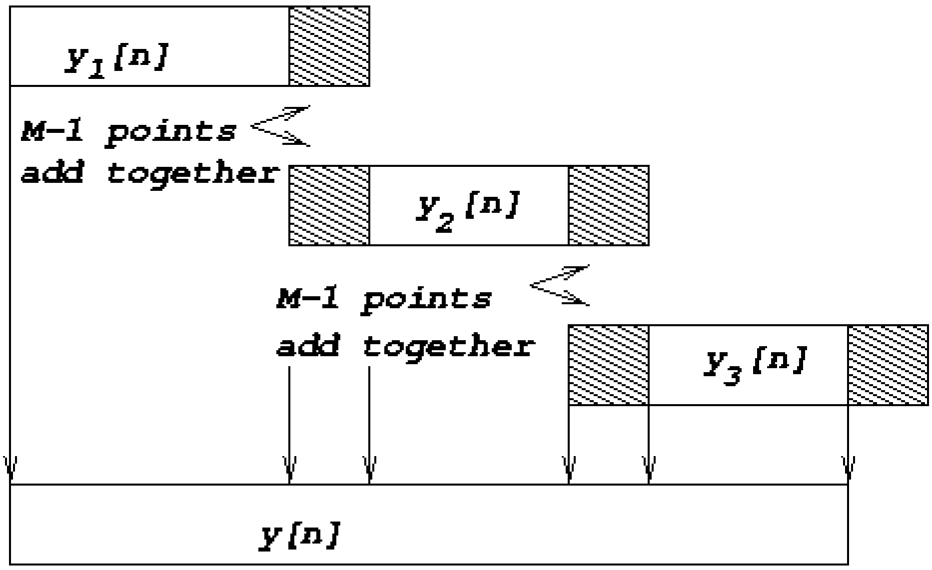
8 25 42 27 0

y(n)={1,4,10,16,22,28,34,40,46,42,27}

OVERLAP ADD METHOD



Output Signal



Find y(n) using overlap add method

x(n)={1,2,3,4,5,6,7,8,9} and h(n)={1,2,3}

- Soln: step-1
- x(n)={1,2,3,4,5,6,7,8,9} → L=3
- h(n)={1,2,3} → M=3
- x1(n)={1,2,3,0,0} h(n)={1,2,3,0,0}
- x2(n)={4,5,6,0,0}
- x3(n)={7,8,9,0,0}

- $y_1(n)=x_1(n) \textcircled{N} h(n)=\{1,4,10,12,9\}$
- $y_2(n)=x_2(n) \textcircled{N} h(n)=\{4,13,28,27,18\}$ **Step-2**
- $y_3(n)=x_3(n) \textcircled{N} h(n)=\{7,22,46,42,27\}$

- Step-3:

1	4	10	12	9						
			4	13	28	27	18			
						7	22	46	42	27

$y(n)=\{1,4,10,16,22,28,34,40,46,42,27\}$