UNIT-IV

Z – Transforms AND DIFFERENCE EQUATIONS

PART B

1. Using the Z transforms, Solve $u_{n+2} + 3u_{n+1} + 2u_n = 0$ *given* $u_0 = 1, u_1 = 2$.

Solution:

Given $u_{n+2} + 3u_{n+1} + 2u_n = 0$

$$Z[u_{n+2}] + Z[3u_{n+1}] + Z[2u_n] = 0$$

$$[z^2\,U(z) - z^2\,u(0) - zu(1)] + 3[zU(z) - zu(0)] + 2\,U(z) = 0$$

$(z^2 + 3z + 2)$ U (z)- z^2-2z-3z = 0 $[u_0 = 1, u_1 = 2]$

U (z) = $\frac{z^2+5z}{z^2+3z+2}$

U (z) = $\frac{z(z+5)}{z^2+3z+2}$

$\frac{U(z)}{z} = \frac{(z+5)}{(z+1)(z+2)} = \frac{A}{(z+1)} + \frac{B}{(z+2)}$ .... (1)Then

$$z + 5 = A(z+2) + B(z+1)$$

Put z = -1, we get4	Put z = -2, we get3
=A	= -B
A = 4	B = -3

(1) $\frac{U(z)}{z} = [\frac{4}{(z+1)}] - [\frac{3}{(z+2)}]$

⇨ U (z) = $4[\frac{z}{(z+1)}] - 3[\frac{z}{(z+2)}]$

$Z[u(n)] = 4[\frac{z}{(z+1)}]-3[\frac{z}{(z+2)}]$

$u(n) = 4Z^{-1}[\frac{z}{(z+1)}]-3\ Z^{-1}[\frac{z}{(z+2)}]$

$= 4(-1)^n - 3(-2)^n$

$= [4-3(2^n)]\ (-1)^n$

2. Solve the difference equation $y(n+3)-3y(n+1)+2y(n)=0$

$given\ that\ y(0)=4, y(1)=0\ and\ y(2)=8$

Solution:

Given $y(n+3)-3y(n+1)+2y(n)=0$

$Z[\ y(n+3)]-3Z[y(n+1)]+2Z[y(n)]=0$

$[z^3Y(z)-z^3y(0)-z^2\ y(1)-zy(2)]-3[zY(z)-2y(0)]+2Y(z)=0$

$[z^3Y(z)-4z^3-8z]-3[zY(z)-4z]+2Y(z)=0$ $\qquad [y(0)=4, y(1)=0, y(2)=8]$

$[z^3-3z+2]Y(z)-4z^3-8z+12z=0$

$[z^3-3z+2]Y(z)-4z^3-4z=0$

$[z^3-3z+2]Y(z)=4z^3-4z$

$Y(z) = \frac{4z^3-4z}{z^3-3z+2}$

$= \frac{4z(z^2-1)}{(z-1)^2(z+2)}$

$= \frac{(z+1)(z-1)}{(z-1)^2(z+2)}$

$= \frac{4Z(z+1)}{(z-1)(z+2)}$

$\frac{Y(z)}{z} = \frac{4(z+1)}{(z-1)(z+2)} = \frac{A}{z-1} + \frac{B}{z+2}$ ……….(1)

$$4(z+1) = A(z+2) + B(z-1)$$

Put z=1 , we get

8 = 3A

A=8/3

Put z =-2 , we get

-4 = -3B

B = 4/3

(1) $\frac{Y(z)}{z} = \frac{8/3}{z-1} + \frac{4/3}{z+2}$

$$\frac{Y(z)}{z} = \frac{8}{3}\left\{\frac{z}{z-1}\right\} + \frac{4}{3}\left\{\frac{z}{z+2}\right\}$$

Z[y(n)] = $\frac{8}{3}\left\{\frac{z}{z-1}\right\} + \frac{4}{3}\left\{\frac{z}{z+2}\right\}$

y(n) = $\frac{8}{3}Z^{-1}\left\{\frac{z}{z-1}\right\} + \frac{4}{3}Z^{-1}\left\{\frac{z}{z+2}\right\}$

= $\frac{8}{3}(1)^n + \frac{4}{3}(-2)^n$

3. Using Z transforms, Solve $u_{n+2} - u_{n+1} + 6u_n = 4^n$ given that $u_0 = 0, u_1 = 1$

Solution:

Given $u_{n+2} - u_{n+1} + 6u_n = 4^n$

$$Z[u_{n+2}] - Z[u_{n+1}] + 6Z[u_n] = Z[4^n]$$

$$[z^2 U(z) - z^2 u(0) - zu(1)] - 5[zU(z) - zu(0)] + 6U(z) = \frac{z}{z+4}$$

$(z^2 - 5z + 6)$ U (z)-z = $\frac{z}{z+4}$ $[u_0 = 0, u_1 = 1]$

$(z^2 - 5z + 6)$ U (z) = z+ $\frac{z}{z+4}$

$(z-3)(z-2)\ U(z) = \frac{z^2-4z+z}{z-4}$

$(z-3)(z-2)\ U(z) = \frac{z^2-3z}{z-4}$

$U(z) = \frac{z(z-3)}{(z-3)(z-2)(z-4)}$

$$U(z) = \frac{z}{(z-2)(z-4)}$$

$$\frac{U(z)}{z} = \frac{1}{(z-2)(z-4)}$$

$$\frac{U(z)}{z} = \frac{1}{(z-2)(z-4)} = \frac{A}{(z-2)} + \frac{B}{(z-4)} \quad \ldots\ldots\ldots\ldots\ldots (1)$$

$$1 = A(z-4) + B(z-2)$$

Put z = 2, we get | Put z = 4, we get

1 =-2A | 1= 2B

$A = -\frac{1}{2}$ | $B = \frac{1}{2}$

(2) $$\frac{U(z)}{z} = \left[\frac{-\frac{1}{2}}{(z-2)}\right] + \left[\frac{\frac{1}{2}}{(z-4)}\right]$$

$$\Rightarrow \quad U(z) = -\frac{1}{2}\left[\frac{z}{(z-2)}\right] + \left[\frac{z}{(z-4)}\right]$$

$$Z[u(n)] = -\frac{1}{2}\left[\frac{z}{(z-2)}\right] \frac{1}{2} + \left[\frac{z}{(z-4)}\right]$$

$$u(n) = -\frac{1}{2}Z^{-1}\left[\frac{z}{(z-2)}\right] + \frac{1}{2}Z^{-1}\left[\frac{z}{(z-4)}\right]$$

$$= -\frac{1}{2}(2)^n + \frac{1}{2}(4)^n$$

4.Using Z transforms, Solve $y_{n+2} + 6y_{n+1} + 9y_n = 2^n \text{ given that } y_0 = 0, \quad y_1 = 0$

Solution:

Given $y_{n+2} + 6y_{n+1} + 9y_n = 2^n$

$$Z[y_{n+2}] + 6Z[y_{n+1}] + 9Z[y_n] = Z[2^n]$$

$$[z^2 Y(z) - z^2 y(0) - zy(1)] + 6[zY(z) - zy(0)] + 9Y(z) = \frac{z}{z-2}$$

$$(z^2 + 6z + 9)\, Y(z) = \frac{z}{z-2} \qquad y_0 = 0 \;\; y_1 = 0$$

$$(z^2 + 6z + 9)\, Y(z) = \frac{z}{z-2}$$

$$(z+3)^2\, Y(z) = \frac{z}{z-2}$$

$$Y(z) = \frac{z}{(z-2)(z+3)^2}$$

$$\frac{Y(z)}{z} = \frac{1}{(z-2)(z+3)^2}$$

$$\frac{Y(z)}{z} = \frac{1}{(z-2)(z+3)^2} = \frac{A}{(z-2)} + \frac{B}{(z+3)} + \frac{C}{(z+3)^2} \quad \text{.............. (1)}$$

$$1 = A(z+3)^2 + B(z-2)(z+3) + c(z-2)$$

Put z = 2, we get

1 =25A

$A = \frac{1}{25}$

Put z = -3, we get

1= - 5CC

$= -\frac{1}{5}$

Equating z^2 co-eff.

on both sides, we get

0=A+B

B = -A, B = - $\frac{1}{25}$

(1) $\frac{Y(z)}{z} = \frac{1}{25}\frac{1}{z-2} - \frac{1}{25}\frac{1}{z+3} - \frac{1}{5}\frac{1}{(z+3)^2}$

$$Y(z) = \frac{1}{25}\frac{z}{z-2} - \frac{1}{25}\frac{z}{z+3} - \frac{1}{5}\frac{z}{(z+3)^2}$$

$$y(n) = \frac{1}{25}Z^{-1}[\frac{z}{z-2}] - \frac{1}{25}Z^{-1}[\frac{z}{z+3}] - \frac{1}{5}Z^{-1}[\frac{z}{(z+3)^2}]$$

ie,

$$y(n) = \frac{1}{25}(2)^n - \frac{1}{25}(-3)^n + \frac{1}{15}(-3)^n n$$

$$\left\lfloor 1 + a + a \right.$$

$+ \ldots + a$

$$= \quad a - 1 \quad |$$