Vector Analysis

The quantities that we deal in electromagnetic theory may be either scalar or vectors. There are other class of physical quantities called Tensors: where magnitude and direction vary with co ordinate axes]. Scalars are quantities characterized by magnitude only and algebraic sign. A quantity that has direction as well as magnitude is called a vector. Both scalar and vector quantities are function of time and position . A field is a function that specifies a particular quantity everywhere in a region. Depending upon the nature of the quantity under consideration, the field may be a vector or a scalar field.
Example of scalar field is the electric potential in a region while electric or magneticfields at any point is the example of vector field.

A vector $\vec{A}$ can be written as, $\vec{A} = \hat{a} A$, where, $A = |\vec{A}|$ is the magnitude and $\hat{a} = \frac{\vec{A}}{|A|}$ is the unit vector which has unit magnitude and same direction as that $\vec{A}$.

Two vector $\vec{A}$ and $\vec{B}$ are added together to give another vector $\vec{C}$. We have

$$\vec{C} = \vec{A} + \vec{B} \quad ....(1.1)$$

Let us see the animations in the next pages for the addition of two vectors, which has tworules: **1: Parallelogram law** and **2: Head & tail rule** as shown in figure 1.1(a), 1.1(b) and 1.2

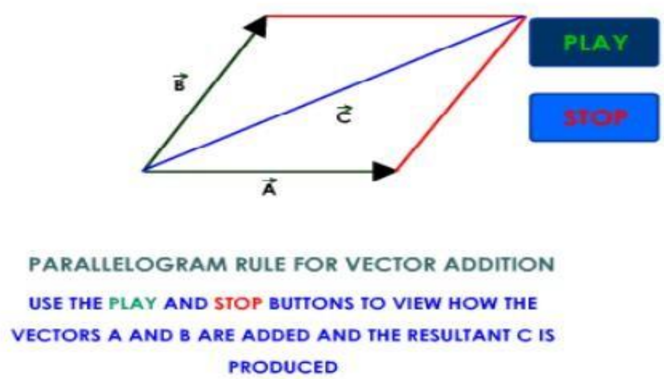

Fig 1.1(a):Vector Addition(Parallelogram Rule)

Fig 1.1(b): Vector Addition (Head & Tail Rule)

Vector Subtraction is similarly carried out: $\vec{D} = \vec{A} - \vec{B} = \vec{A} + (-\vec{B})$ ....(1.2)

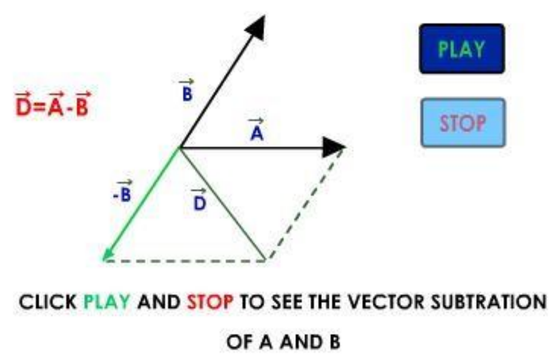

Fig 1.2: Vector subtraction

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Scaling of a vector is defined as $\vec{C} = \alpha\vec{B}$, where $\vec{C}$ is scaled version of vector $\vec{B}$ and α is a scalar.

Some important laws of vector algebra are:

$\vec{A} + \vec{B} = \vec{B} + \vec{A}$ Commutative Law .... (1.3)

$\vec{A} + (\vec{B} + \vec{C}) = (\vec{A} + \vec{B}) + \vec{C}$ Associative Law..(1.4)

$\alpha(\vec{A} + \vec{B}) = \alpha\vec{A} + \alpha\vec{B}$ Distributive Law..(1.5)

The position vector $\vec{r_Q}$ of a point *P* is the directed distance from the origin (*O*) to *P*, i.e., $\vec{r_Q} = \overrightarrow{OP}$ as shown in figure 1.3.

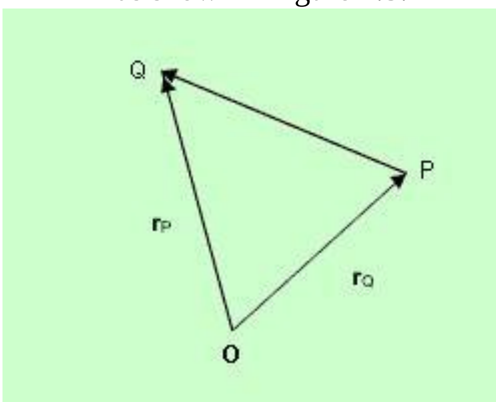

Fig 1.3: Distance Vector
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If $\vec{r_Q}$ = OP and $\vec{r_P}$ = OQ are the position vectors of the points P and Q then the distance vector

$$\overrightarrow{PQ} = \overrightarrow{OQ} - \overrightarrow{OP} = \vec{r_P} - \vec{r_Q}$$

Product of Vectors

The two types of vector multiplication are:

Scalar product $\vec{A} \cdot \vec{B}$, Vector product $\vec{A} \times \vec{B}$

The dot product between two vectors is defined as $\vec{A} \cdot \vec{B} = |A||B|cos\theta AB$ ....(1.6)

Vector product $\vec{A} \times \vec{B} = |A||B| \sin\theta_{AB} \cdot \vec{n}$

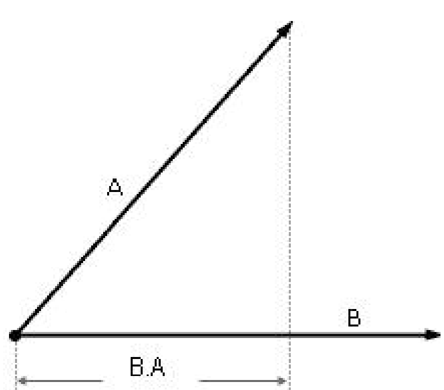

Fig 1.4: Vector dot product

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The dot product is commutative i.e., $\vec{A}\cdot\vec{B}=\vec{B}\cdot\vec{A}$ and distributive i.e., $\vec{A}\cdot\left(\vec{B}+\vec{C}\right)=\vec{A}\cdot\vec{B}+\vec{A}\cdot\vec{C}$. Associative law does not apply to scalar product as shown in figure 1.4

The vector or cross product of two vectors $\vec{A}$ and $\vec{B}$ is denoted by $\vec{A}\times\vec{B}$. $\vec{A}\times\vec{B}$ is a vector perpendicular to the plane containing $\vec{A}$ and $\vec{B}$, the magnitude is given by $|A||B|\sin\theta_{AB}$ and direction is given by right hand rule as explained in Figure 1.5.

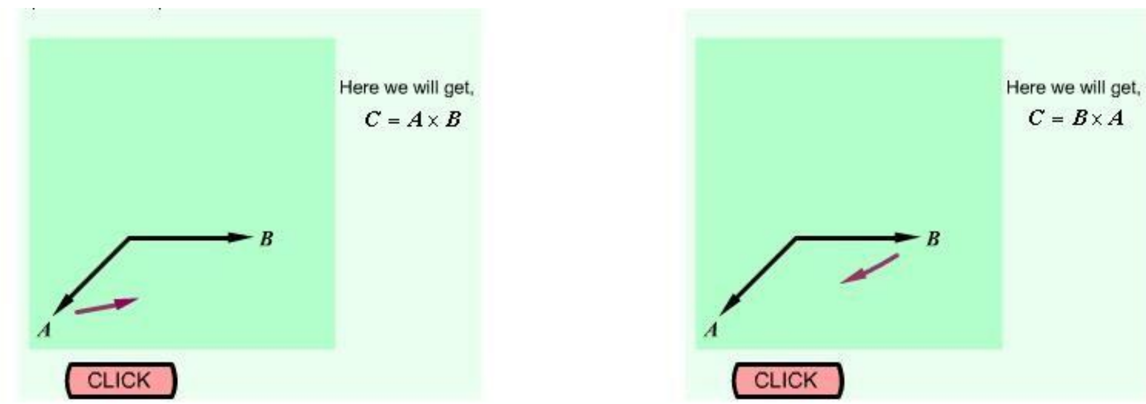

Fig 1.5 :Illustrating the left thumb rule for determining the vector cross product

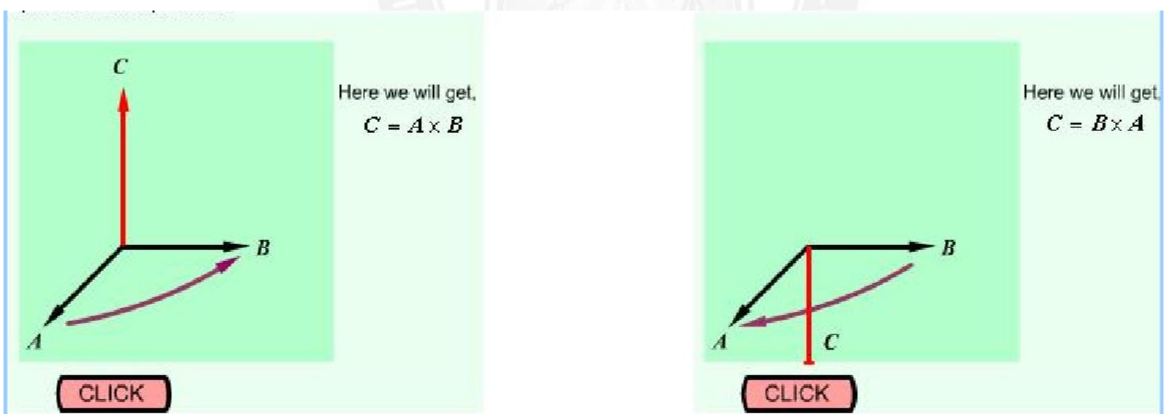

Fig 1.5 :Illustrating the left thumb rule for determining the vector cross product

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$\vec{A}\times\vec{B} = \hat{a}_n AB \sin\theta_{AB}$ ....(1.7)

$\hat{a}_n$ is the unit vector given by, $\hat{a}_n = \dfrac{\vec{A}\times\vec{B}}{\left|\vec{A}\times\vec{B}\right|}$.

The following relations hold for vector product.

$\vec{A}\times\vec{B} \quad -\vec{B}\times\vec{A}$ i.e., cross product is non commutative.......... (1.8)

$\vec{A}\times\left(\vec{B} + \vec{C}\right) = \vec{A}\times\vec{B} + \vec{A}\times\vec{C}$ i.e., cross product is distributive......................(1.9)

$\vec{A}\times\left(\vec{B} \times \vec{C}\right) \neq \left(\vec{A}\times\vec{B}\right)\times\vec{C}$ i.e., cross product is non associative .... (1.10)

Scalar and vector triple product :

Scalar triple product $\vec{A}\cdot\left(\vec{B} \times \vec{C}\right) = \vec{B}\cdot\left(\vec{C}\times\vec{A}\right) = \vec{C}\cdot\left(\vec{A}\times\vec{B}\right)$ ....(1.11)

Vector triple product $\vec{A}\times\left(\vec{B} \times \vec{C}\right) = \vec{B}\left(\vec{A}\cdot\vec{C}\right) - \vec{C}\left(\vec{A}\cdot\vec{B}\right)$