UNIT IV

INITIAL VALUE PROBLEMS FOR ORDINARY DIFFERENTIAL EQUATIONS

PROBLEMS BASED ON EULER METHOD

PROBLEMS BASED ON EULER METHOD AND EULER MODIFIED METHOD

1. *Euler Method*

$$y_{n+1} = y_n + hf(x_n, y_n)$$

1. *Using Euler Method solve* $y' = x + y + xy$, $y(0) = 1$, *compute* y *at* $x = 0.05$

***solution*:**

Given $y' = f(x, y) = x + y + xy$ &

$$x_0 = 0 \text{ and } y_0 = 1$$
$$x_1 = 0.05 \text{ and } y_1 = ?$$

$$h = x_1 - x_0 = 0.05 - 0 = 0.05$$

$$y_1 = y_0 + hf(x_0, y_0)$$

$$y_1 = 1 + 0.05\, f(0, 1)$$
$$= 1 + 0.05\, [0 + 1 + (0)(1)]$$
$$= 1 + 0.05(1)$$
$$y_1 = 1.05$$

2. Using Euler Method find $y(0.2)$ & $y(0.4)$ from $\frac{dy}{dx} = x + y$, $y(0) = 1$

solution:

Given $\frac{dy}{dx} = f(x, y) = x + y$ and $x_0 = 0$ and $y_0 = 1$

$$x_1 = 0.2 \text{ and } y_1 = ?$$

$$x_2 = 0.4 \text{ and } y_2 = ?$$

$$h = x_1 - x_0 = 0.2 - 0 = 0.2$$

$$y_1 = y_0 + hf(x_0, y_0)$$

$$y_1 = 1 + 0.2\, f(0, 1)$$

$$= 1 + 0.2\, [0 + 1]$$

$$= 1 + 0.2$$

$$y_1 = 1.2$$

Again : $y_2 = y_1 + hf(x_1, y_1)$

$$y_2 = 1.2 + 0.2\, f(0.2, 1.2)$$

$$= 1.2 + 0.2\, [0.2 + 1.2]$$

$$= 1.2 + 0.2[1.4]$$

$$= 1.2 + 0.28$$

$$y_2 = 1.48$$

3. Using Euler Method find $y(4.1)$ & $y(4.2)$ from

$$5x\frac{dy}{dx} + y^2 - 2 = 0, y(4) = 1$$

Solution:

$$Given\ 5x\frac{dy}{dx} + y^2 - 2 = 0$$

$$5x\frac{dy}{dx} = 2 - y^2$$

$$\frac{dy}{dx} = 2 - y^2/5x$$

Given $y(4) = 1$ To find $y(4.1)$ & $y(4.2)$

$$x_0 = 4\ and\ y_0 = 1$$

$$x_1 = 4.1\ and\ y_1 = ?$$

$x_2 = 4.2\ and\ y_2 = ?$ h=0.1

By Euler Algorithm

$$y_1 = y_0 + hf(x_0, y_0)$$

$$= 1 + (0.1)\frac{2 - {y_0}^2}{5x_0}$$

$$= 1 + (0.1)\frac{2 - 1}{20}$$

$$= 1.005$$

$$y_1 = y(4.1) = 1.005$$

$$Again\ : y_2 = y_1 + hf(x_1, y_1)$$

$$= 1.005 + (0.1)\ \frac{2-1.005^2}{5(4.1)}$$

$$= 1.005 + (0.1)\ \frac{2-1.010025}{20.5}$$

$$y_2 = 1.0098$$

Modified Euler Method

$$y_{n+1} = y_0 + h\left[f\left(x_n + \frac{h}{2}, y_n + \frac{h}{2} f(x_n, y_n)\right)\right]$$

1. Solve $y' = 1 - y$, $y(0) = 0$ by Modified Euler Method find $y(0.1)$

solution:

Given $y' = f(x, y) = 1 - y$

$x_0 = 0$ and $y_0 = 0$

$x_1 = 0.1$ and $y_1 = ?$

$h = x_1 - x_0 = 0.1 - 0 = 0.1$

$$y_1 = y_0 + h\left[f\left(x_0 + \frac{h}{2}, y_0 + \frac{h}{2} f(x_0, y_0)\right)\right]$$

$$y_1 = 0 + 0.1\left[f\left(0 + \frac{0.1}{2}, 0 + \frac{0.1}{2} f(0,0)\right)\right]$$

$$= 0.1\left[f\left(0.05, \frac{0.1}{2}(1-0)\right)\right]$$

$$= 0.1[f(0.05, 0.05(1))]$$

$$= 0.1[f(0.05, 0.05)]$$

$$= 0.1[1 - 0.05]$$

$$= 0.1[0.95]$$

$$y_1 = 0.095$$

$$2.\ Using\ Modified\ Euler\ Method\ \ compute\ y(0) = 1$$

$$from\ \ y' = y - \frac{2x}{y}\ with\ y(0.1)$$

solution:

$$Given\ y' = f(x, y) = y - \frac{2x}{y}\ and$$

$$x_0 = 0\ and\ y_0 = 1$$

$$x_1 = 0.1\ and\ y_1 = ?$$

$$h = x_1 - x_0 = 0.1 - 0 = 0.1$$

$$y_1 = y_0 + h\left[f\left(x_0 + \frac{h}{2}, y_0 + \frac{h}{2} f(x_0, y_0)\right)\right]$$

$$y_1 = 1 + 0.1\left[f\left(0 + \frac{0.1}{2}, 1 + \frac{0.1}{2} f(0, 1)\right)\right]$$

$$= 1 + 0.1\left[f\left(0.05, 1 + \frac{0.1}{2}\left(1 - \frac{(2)(0)}{1}\right)\right)\right]$$

$$= 1 + 0.1[f(0.05, 1 + 0.05(1))]$$

$$= 1 + 0.1[f(0.05, 1.05)]$$

$$= 1 + 0.1\left[1.05 - \frac{2(0.05)}{1.05}\right]$$

$$= 1 + 0.1[1.05 - 0.0952]$$

$$= 1 + 0.1[0.9548]$$

$$= 1 + 0.09548$$

$$y_1 = 1.09548$$

3. $Solve\ y' = log_{10}(x+y)$, $y(0) = 2$ by $Modified\ Euler\ Method$ **and find the values of** $y(0.2), y(0.4)$,**and** $y(0.6)$, **taking h=0.2**

Solution :

$$Given\ = log_{10}(x+y)$$

$$x_0 = 0\ and\ y_0 = 2$$

$$x_1 = 0.2\ and\ y_1 = ?$$

$$x_2 = 0.4\ and\ y_2 = ?$$

$$x_3 = 0.6\ and\ y_3 = ?$$

$$h = 0.2$$

$$y_1 = y_0 + h\left[f\left(x_0 + \frac{h}{2}, y_0 + \frac{h}{2}f(x_0, y_0)\right)\right]$$

$$y_1 = 2 + 0.2\left[f\left(\frac{0.2}{2}, 2 + \frac{0.2}{2}log2\right)\right]$$

$$y_1 = 2 + 0.2[f(0.1, 2.0301)]$$

$$y_1 = 2 + 0.2[log(0.1 + 2.0301)]$$

$$y(0.2) = 2.0657$$

$$y_2 = y_1 + h\left[f\left(x_1 + \frac{h}{2}, y_1 + \frac{h}{2}f(x_1, y_1)\right)\right]$$

$$y_2 = 2.0657 + 0.2\left[f\left(0.2 + \frac{0.2}{2}, 2.0657 + \frac{0.2}{2}f(0.2, 2.0657)\right)\right]$$

$$y_2 = 2.0657 + 0.2[f(0.3, 2.0657 + 0.1log(0.2 + 2.0657))]$$

$$= 2.0657 + 0.2[f(0.3, 2.1012))]$$

$$= 2.0657 + 0.2[log(0.3 + 2.1012))]$$
$$= 2.1418$$

$$y_3 = y_2 + h\left[f\left(x_2 + \frac{h}{2}, y_2 + \frac{h}{2}f(x_2, y_2)\right)\right]$$

$$y_2 = 2.1418 + 0.2\left[f\left(0.4 + \frac{0.2}{2}, 2.1418 + \frac{0.2}{2}f(0.4, 2.1418)\right)\right]$$

$$y_2 = 2.1418 + 0.2[f(0.5, 2.1418 + 0.1log(0.4 + 2.1418))]$$

$$y_2 = 2.1418 + 0.2[f(0.5, 2.1823))]$$

$$y_2 = 2.1418 + 0.2[log(0.5 + 2.1823))]$$

y(0.6) =2.2275

4. *Solve* $y' = y - x^2 + 1$, $y(0) = 0.5$ *by Modified Euler Method find* $y(0.2)$

Solution:

$$y' = y - x^2 + 1$$

$$x_0 = 0 \text{ and } y_0 = 0.5$$
$$x_1 = 0.2 \text{ and } y_1 = ?$$

$$y_1 = y_0 + h\left[f\left(x_0 + \frac{h}{2}, y_0 + \frac{h}{2}f(x_0, y_0)\right)\right]$$

$$y_1 = 0.5 + 0.2\left[f\left(0 + \frac{0.2}{2}, 0.5 + \frac{0.2}{2} f(0, 0.5)\right)\right]$$

$$y_1 = 0.5 + 0.2[f(0.1, 0.5 + (0.1)(0.5 - 0 + 1)]$$

$$y_1 = 0.5 + 0.2[f(0.1, 0.5 + (0.1)(1.5)]$$

$$y_1 = 0.5 + 0.2[f(0.1, 0.5 + (0.15))]$$

$$y_1 = 0.5 + 0.2[f(0.1, 0.65)]$$

$$y_1 = 0.5 + 0.2[0.65 - (0.1)^2 + 1]$$

5. *Cosider the initial value problem* $\frac{dy}{dx} = y - x^2 + 1$

with $y(0) = 0.5$

Using the **modified Euler method ,find y(0.2)**

Solution :

$(x, y) = y - x^2 + 1 \,, x_0 = 0$

$y_0 = 0.5$, **h=0.2** $x_1 = 0.2$

Modified Euler Method

$$y_{n+1} = y_0 + h\left[f\left(x_n + \frac{h}{2}, y_n + \frac{h}{2} f(x_n, y_n)\right)\right]$$

$$y_1 = y_0 + h\left[f\left(x_0 + \frac{h}{2}, y_0 + \frac{h}{2} f(x_0, y_0)\right)\right]$$

$$y_1 = 0.5 + 0.2\left[f\left(0 + \frac{0.2}{2}, 0.5 + \frac{0.2}{2} f(0, 0.5)\right)\right]$$

$$y_1 = 0.5 + 0.2[f(0.1, 0.5 + (0.1)(0.5 + 1))]$$

$$y_1 = 0.5 + 0.2[f(0.1, 0.65)]$$

$$y_1 = 0.5 + 0.2[0.65 - (0.1)^2 + 1\,]$$

$$y_1 = 0.5 + 0.328$$

$$y_1 = 0.828$$