

UNIT I

AMPLITUDE MODULATION

Amplitude modulation or AM as it is often called, is a form of modulation used for radio transmissions for broadcasting and two-way radio communication applications. Although one of the earliest used forms of modulation it is still used today, mainly for long, medium and short wave broadcasting and for some aeronautical point to point communications.

One of the key reasons for the use of amplitude modulation was its ease of use. The system simply required the carrier amplitude to be modulated, but more usefully the detector required in the receiver could be a simple diode based circuit. This meant that AM radios did not need complicated demodulators and costs were reduced - a key requirement for widespread use of radio technology, especially in the early days of radio when ICs were not available.

Amplitude modulation history

The first amplitude modulated signal was transmitted in 1901 by a Canadian engineer named Reginald Fessenden. He took a continuous spark transmission and placed a carbon microphone in the antenna lead. The sound waves impacting on the microphone varied its resistance and in turn this varied the intensity of the transmission. Although very crude, signals were audible over a distance of a few hundred meters, although there was a rasping sound caused by the spark.

With the introduction of continuous sine wave signals, transmissions improved significantly, and AM soon became the standard for voice transmissions. Nowadays, amplitude modulation, AM is used for audio broadcasting on the long medium and short wave bands, and for two way radio communication at VHF for aircraft. However as there now are more efficient and convenient methods of

modulating a signal, its use is declining, although it will still be very many years before it is no longer used.

Amplitude modulation applications

Amplitude modulation is used in a variety of applications. Even though it is not as widely used as it was in previous years in its basic format it can nevertheless still be found.

Broadcast transmissions: AM is still widely used for broadcasting on the long, medium and short wave bands. It is simple to demodulate and this means that radio receivers capable of demodulating amplitude modulation are cheap and simple to manufacture. Nevertheless, many people are moving to high quality forms of transmission like frequency modulation, FM or digital transmissions.

Air band radio: VHF transmissions for many airborne applications still use Amplitude Modulation. It is used for ground to air radio communications as well as two-way radio links for ground staff as well.

Single sideband: Amplitude modulation in the form of single sideband is still used for HF radio links. Using a lower bandwidth and providing more effective use of the transmitted power this form of modulation is still used for many point to point HF links.

Quadrature amplitude modulation: AM is widely used for the transmission of data in everything from short range wireless links such as Wi-Fi to cellular telecommunications and much more. Effectively it is formed by having two carriers 90° out of phase. These form some of the main uses of amplitude modulation.

Need for Amplitude modulation

In order that a radio signal can carry audio or other information for broadcasting or for two-way radio communication, it must be modulated or changed in some way. Although there are a number of ways in which a radio signal may be modulated, one of the easiest is to change its amplitude in line with variations of the sound. In this way the amplitude of the radio frequency signal varies in line with the instantaneous value of the intensity of the modulation. This means that the radio frequency signal has a representation of the sound wave superimposed in it.

In view of the way the basic signal "carries" the sound or modulation, the radio frequency signal is often termed the "carrier". A continuous-wave goes on continuously without any intervals and it is the baseband message signal, which contains the information. This wave has to be modulated. According to the standard definition, "The amplitude of the carrier signal varies in accordance with the instantaneous amplitude of the modulating signal." Which means, the amplitude of the carrier signal containing no information varies as per the amplitude of the signal containing information, at each instant. This can be well explained by the following figures 1.1.1, 1.1.2, 1.1.3 and figure 1.1.4.

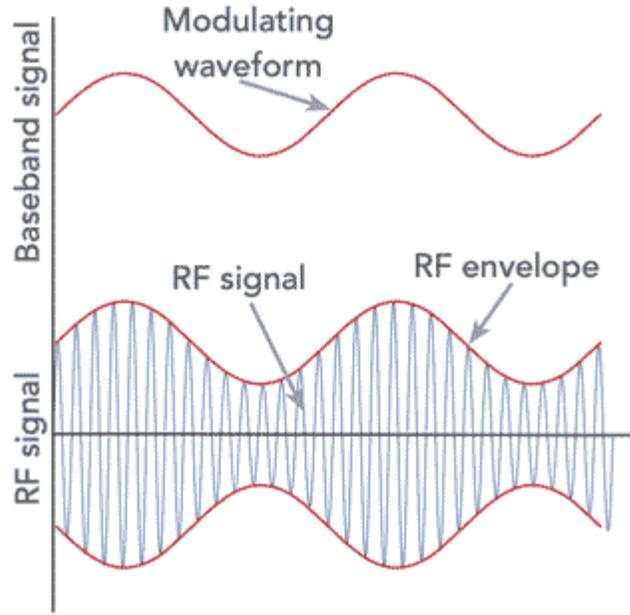


Figure 1.1.1 Modulating wave form and Modulated RF Signal

Diagram Source : Electronic Tutorials

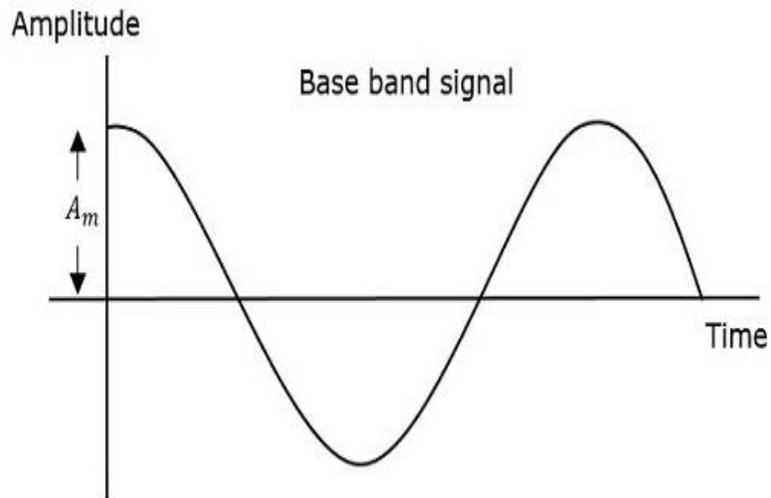


Figure: 1.1.2 Base Band Signal

Diagram Source : Electronic Tutorials

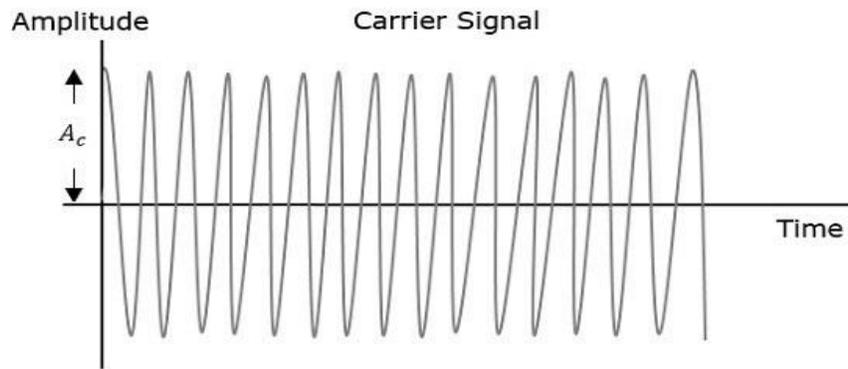


Fig1.1.3 Carrier Signal

Diagram Source : Electronic Tutorial

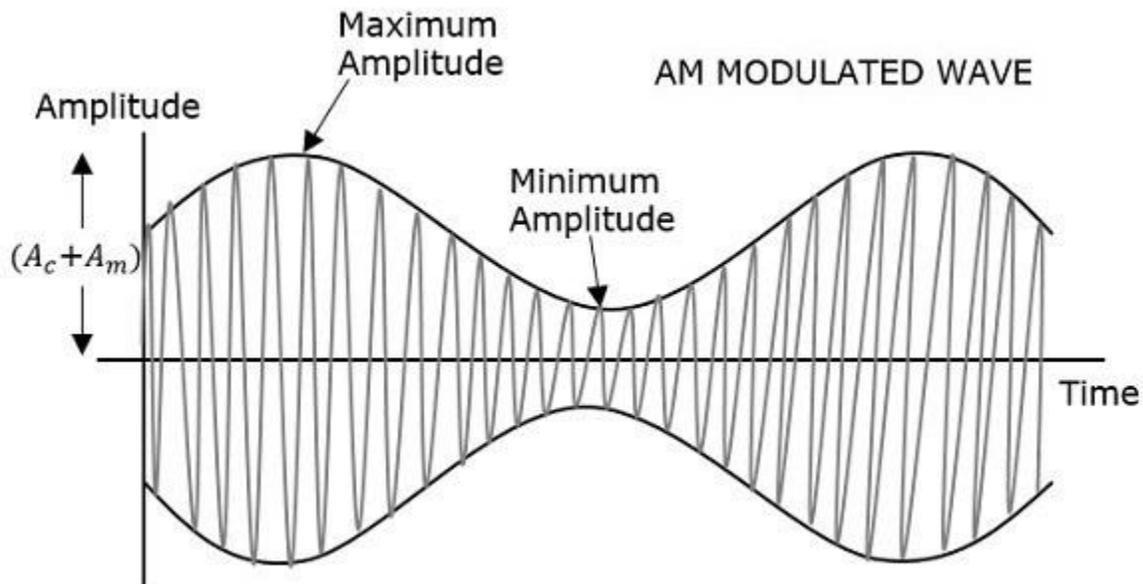


Figure 1.1.4 AM Modulated Signal

Diagram Source : Electronic Tutorial

The figure 1.1.4 shows the modulating wave, which is the message signal. The next one is the carrier wave, which is a high frequency signal and contains no information. While, the last one is the resultant modulated wave.

It can be observed that the positive and negative peaks of the carrier wave, are interconnected with an imaginary line. This line helps recreating the exact shape of the modulating signal. This imaginary line on the carrier wave is called as Envelope. It is the same as that of the message signal.

Mathematical Expressions

Following are the mathematical expressions for these waves. Time-domain Representation of the Waves Let the modulating signal be,

$$m(t)=A_m \text{Cos}(2\pi f_m t) \quad (1)$$

and the carrier signal be,

$$c(t)=A_c \text{Cos}(2\pi f_c t) \quad (2)$$

Where,

A_m and A_c are the amplitude of the modulating signal and the carrier signal respectively.

f_m and f_c are the frequency of the modulating signal and the carrier signal respectively.

Then, the equation of Amplitude Modulated wave will be

$$s(t)=[A_c+A_m \text{cos}(2\pi f_m t)]\text{Cos}(2\pi f_c t) \quad (3)$$

Modulation Index

A carrier wave, after being modulated, if the modulated level is calculated, then such an attempt is called as Modulation Index or Modulation Depth. It states the level of modulation that a carrier wave undergoes.

Rearrange the Equation as below.

$$s(t)=A_c[1+(A_m/A_c)\text{Cos}(2\pi f_m t)]\text{Cos}(2\pi f_c t) \quad (4)$$

$$\Rightarrow s(t) = A_c [1 + \mu \cos(2\pi f_m t)] \cos(2\pi f_c t) \quad (5)$$

Where, μ is Modulation index and it is equal to the ratio of A_m and A_c .
Mathematically, we can write it as

$$\mu = A_m / A_c \quad (6)$$

Hence, we can calculate the value of modulation index by using the above formula, when the amplitudes of the message and carrier signals are known. Now, let us derive one more formula for Modulation index. We can use this formula for calculating modulation index value, when the maximum and minimum amplitudes of the modulated wave are known.

Let A_{max} and A_{min} be the maximum and minimum amplitudes of the modulated wave.

We will get the maximum amplitude of the modulated wave, when $\cos(2\pi f_m t)$

$$\Rightarrow A_{max} = A_c + A_m \quad (7)$$

We will get the minimum amplitude of the modulated wave, when $\cos(2\pi f_m t)$ is -1.

$$\Rightarrow A_{min} = A_c - A_m$$

$$\Rightarrow A_{min} = A_c - A_m$$

Adding

$$A_{max} + A_{min} = A_c + A_m + A_c - A_m = 2A_c$$

$$A_c = (A_{max} + A_{min}) / 2 \Rightarrow A_c = (A_{max} + A_{min}) / 2 \quad (8)$$

Subtracting

$$A_{max} - A_{min} = A_c + A_m - (A_c - A_m)$$

$$=2A_m A_{\max} - A_{\min} = A_c + A_m - (A_c - A_m) = 2A_m$$

$$A_m = (A_{\max} - A_{\min}) / 2 \Rightarrow A_m = (A_{\max} - A_{\min}) / 2$$

$$\mu = A_{\max} - A_{\min} / A_{\max} + A_{\min}$$

$$\mu = A_{\max} - A_{\min} / A_{\max} + A_{\min} \quad (9)$$

Therefore, the above Equations are the two formulas for Modulation index. The modulation index or modulation depth is often denoted in percentage called as Percentage of Modulation. We will get the percentage of modulation, just by multiplying the modulation index value with 100.

For a perfect modulation, the value of modulation index should be 1, which implies the percentage of modulation should be 100%.

For instance, if this value is less than 1, i.e., the modulation index is 0.5, then the modulated output would look like the following figure. It is called as Under-modulation. Such a wave is called as an under-modulated wave. Fig 1.1.5 shows Under Modulated Wave.

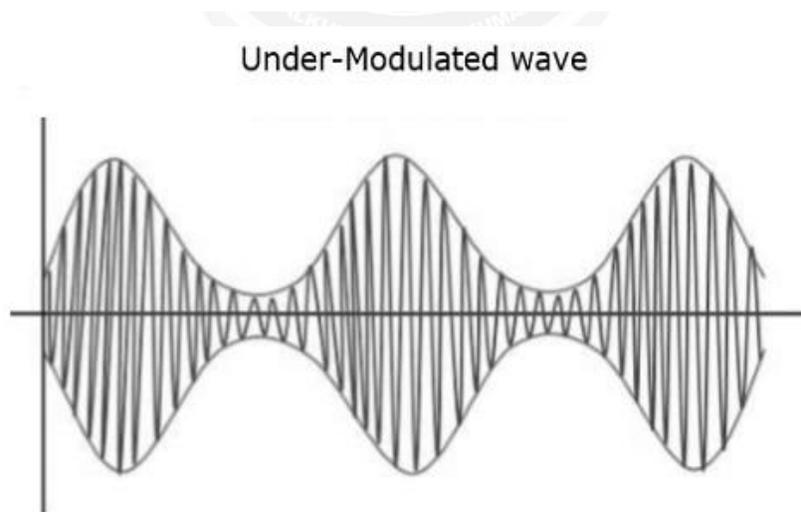


Figure 1.1.5 Under Modulated Wave

Diagram Source : Brain Kart

If the value of the modulation index is greater than 1, i.e., 1.5 or so, then the wave will be an over-modulated wave. It would look like the following figure 1.1.6.

Over-Modulated wave

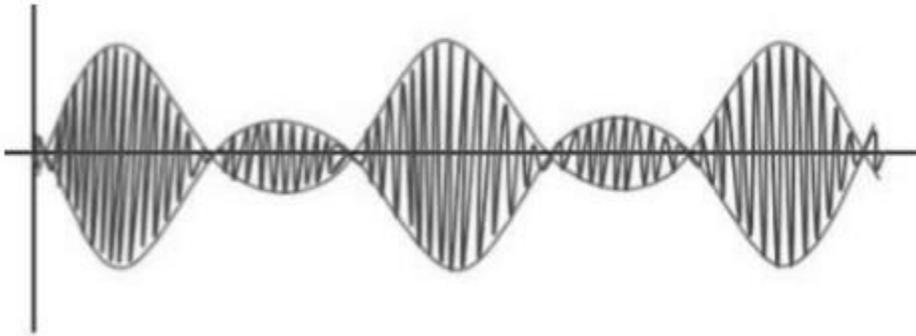


Figure 1.1.6 Over Modulated Signal

Diagram Source : Electronic Tutorials

As the value of the modulation index increases, the carrier experiences a 180° phase reversal, which causes additional sidebands and hence, the wave gets distorted. Such an over-modulated wave causes interference, which cannot be eliminated.

Let us consider that a carrier signal $A \cos \omega_c t$ is amplitude-modulated by a single-tone modulating signal

$$x(t) = V_m A \cos \omega_m t. \quad (9)$$

Then the unmodulated or carrier power

$$PC = \text{mean square (rms value)}$$

Power Content In Multiple-tone Amplitude Modulation

Mathematical Expression

Let us consider that a carrier signal $A \cos \omega_c t$ is modulated by a baseband or modulating signal $x(t)$ which is expressed as :

$$x(t) = V_1 \cos \omega_1 t + V_2 \cos \omega_2 t + V_3 \cos \omega_3 t \quad (10)$$

We know that the general expression for AM wave is

$$s(t) = A \cos \omega_c t + x(t) \cos \omega_c t \quad (11)$$

Putting the value of $x(t)$, we get

$$s(t) = A \cos \omega_c t + [V_1 \cos \omega_1 t + V_2 \cos \omega_2 t + V_3 \cos \omega_3 t] \cos \omega_c t$$

The expression for AM wave can further be expanded as under:

$$s(t) = A \cos \omega_c t + m_1 A \cos \omega_c t \cos \omega_1 t + m_2 A \cos \omega_c t \cos \omega_2 t + m_3 A \cos \omega_c t \cos \omega_3 t$$

Now we know that the total power in AM is given as ,

$$P_t = \text{carrier power} + \text{sideband power}$$

$$P_t = P_C + P_S$$

The carrier power P_C is given as considering additional resistance like antenna resistance R .

$$P_{\text{carrier}} = [(V_c/\sqrt{2})^2 / R] = V^2 C / 2R \quad (12)$$

Each side band has a value of $mV_c / 2$ and r.m.s value of $(mV_c/2)/\sqrt{2}$. Hence power in LSB and USB can be written as

$$P_{LSB} = P_{USB} = (mV_c/2)/\sqrt{2})^2/R = m^2V_c^2/8R = m^2/4 P_{carrier}$$

$$P_{total} = V_c^2/2R + [m^2V_c^2/8R] + [m^2V_c^2/8R]$$

$$= V_c^2/2R + [m^2V_c^2/4R]$$

$$= V_c^2/2R (1 + m^2/2)$$

$$P_{total} = P_{carrier} (1 + m^2/2) \quad (13)$$

In some applications, the carrier is simultaneously modulated by several sinusoidal modulating signals. In such a case, the total modulation index is given as

$$m_t = \sqrt{(m_1^2 + m_2^2 + m_3^2 + m_4^2 + \dots)}$$

If I_c and I_t are the r.m.s values of unmodulated current and total modulated current and R is the resistance through which these current flow, then

$$P_{total}/P_{carrier} = (I_t R / I_c R)^2 = (I_t / I_c)^2$$

$$P_{total}/P_{carrier} = (1 + m^2/2)$$

$$I_t / I_c = 1 + m^2/2 \quad (14)$$

Limitations of Amplitude Modulation:

1. Switching modulator Low Efficiency- Since the useful power that lies in the small bands is quite small, so the efficiency of AM system is low.
2. Limited Operating Range – The range of operation is small due to low efficiency. Thus, transmission of signals is difficult.
3. Noise in Reception – As the radio receiver finds it difficult to distinguish between the amplitude variations that represent noise and those with the signals, heavy noise is prone to occur in its reception.

4. Poor Audio Quality – To obtain high fidelity reception, all audio frequencies till 15 Kilo Hertz must be reproduced and this necessitates the bandwidth of 10 Kilo Hertz to minimize the interference from the adjacent broadcasting stations. Therefore in AM broadcasting stations audio quality is known to be poor.

The following two modulators generate AM wave.

- Square law modulator
- Switching Modulator

Square Law Modulator

Following is the block diagram of the square law modulator shown in figure 1.1.7

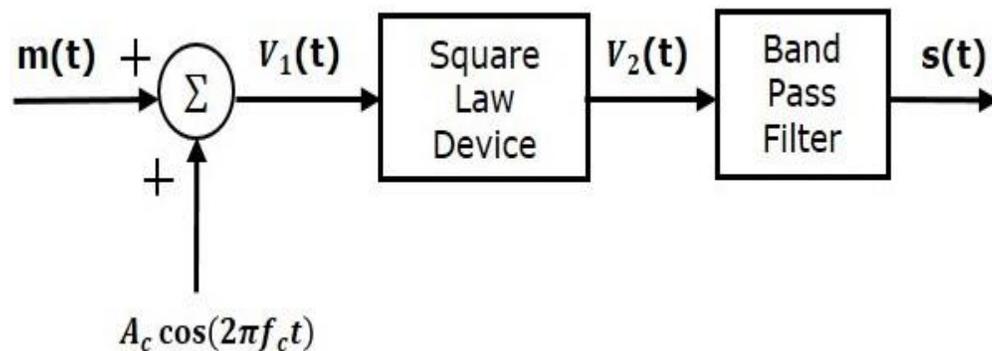


Figure 1.1.7 Block diagram of the square law modulator

Let the modulating and carrier signals be denoted as $m(t)$ and $A_c \cos(2\pi f_c t)$ respectively. These two signals are applied as inputs to the summer (adder) block. This summer block produces an output, which is the

addition of the modulating and the carrier signal. Mathematically, we can write it as

$$V_1(t) = m(t) + A_c \cos(2\pi f_c t) \quad (15)$$

This signal $V_1(t)$ is applied as an input to a nonlinear device like diode. The characteristics of the diode are closely related to square law.

$$V_2(t) = k_1 V_1(t) + k_2 V_1^2(t)$$

$$V_2(t) = k_1 V_1(t) + k_2 V_1^2(t)$$

Where, k_1 and k_2 are constants.

Substitute $V_1(t)$ in $V_2(t)$,

$$V_2(t) = k_1 [m(t) + A_c \cos(2\pi f_c t)] + k_2 [m(t) + A_c \cos(2\pi f_c t)]^2$$

$$\Rightarrow V_2(t) = k_1 m(t) + k_1 A_c \cos(2\pi f_c t) + k_2 m^2(t) +$$

$$k_2 A_c^2 \cos^2(2\pi f_c t) + 2k_2 m(t) A_c \cos(2\pi f_c t)$$

$$\Rightarrow V_2(t) = k_1 m(t) + k_2 m^2(t) + k_2 A_c^2 \cos^2(2\pi f_c t) +$$

$$k_1 A_c \left[1 + \left(\frac{2k_2}{k_1} \right) m(t) \right] \cos(2\pi f_c t)$$

The last term of the above equation represents the desired AM wave and the first three terms of the above equation are unwanted. So, with the help of band pass filter, we can pass only AM wave and eliminate the first three terms.

Therefore, the output of square law modulator is

$$s(t) = k_1 A_c [1 + (2k_2 / k_1)m(t)] \cos(2\pi f_c t) \quad (16)$$

The standard equation of AM wave is

$$s(t) = A_c [1 + k_a m(t)] \cos(2\pi f_c t)$$

Where, K_a is the amplitude sensitivity

By comparing the output of the square law modulator with the standard equation of AM wave, we will get the scaling factor as $k_1 k_1$ and the amplitude sensitivity k_a as $2k_2 / 2k_1$

Switching Modulator

Following is the block diagram of switching modulator shown in Figure 1.1.8.

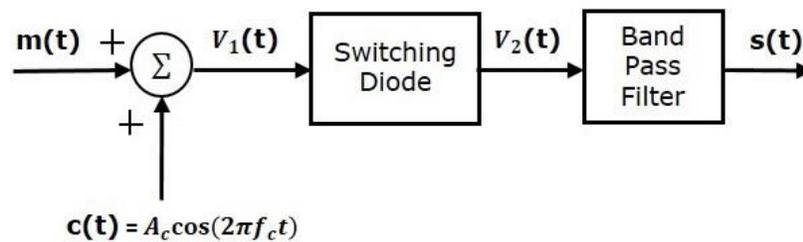


Figure 1.1.8 block diagram of the Switching modulator

Switching modulator is similar to the square law modulator. The only difference is that in the square law modulator, the diode is operated in a non-linear mode, whereas, in the switching modulator, the diode has to operate as an ideal switch.

Let the modulating and carrier signals be denoted as $m(t)$ and $c(t) = A_c \cos(2\pi f_c t)$ respectively. These two signals are applied as inputs to the summer (adder) block. Summer block produces an output, which is the addition of modulating and carrier signals. Mathematically, we can write it as

$$V_1(t) = m(t) + c(t)$$

$$V_1(t) = m(t) + A_c \cos(2\pi f_c t)$$

This signal $V_1(t)$ is applied as an input of diode. Assume, the magnitude of the modulating signal is very small when compared to the amplitude of carrier signal A_c . So, the diode's ON and OFF action is controlled by carrier signal $c(t)$. This means, the diode will be forward biased when $c(t) > 0$ and it will be reverse biased when $c(t) < 0$.

Therefore, the output of the diode is

$$V_2(t) = \begin{cases} V_1(t) & \text{if } c(t) > 0 \\ 0 & \text{if } c(t) < 0 \end{cases}$$

We can approximate this as

$$V_2(t) = V_1(t) x(t)$$

Where, $x(t)$ is a periodic pulse train with time period $T = \frac{1}{f_c}$

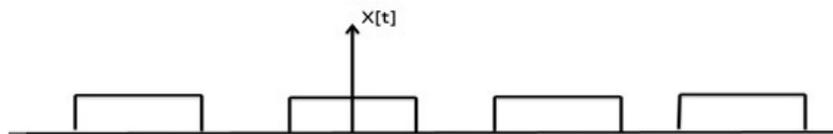


Figure 1.1.9 Periodic pulse train Diagram Source Brain Kart

Diagram Source : Brain Kart

The Fourier series representation of this periodic pulse train is in figure 1.1.9 is ,

$$x(t) = \frac{1}{2} + \frac{2}{\pi} \sum_{n=1}^{\infty} \frac{(-1)^n - 1}{2n - 1} \cos(2\pi(2n - 1)f_c t)$$

$$\Rightarrow x(t) = \frac{1}{2} + \frac{2}{\pi} \cos(2\pi f_c t) - \frac{2}{3\pi} \cos(6\pi f_c t) + \dots$$

Substitute, $V_1(t)$ and $x(t)$ ✓ 2.

$$V_2(t) = [m(t) + A_c \cos(2\pi f_c t)] \left[\frac{1}{2} + \frac{2}{\pi} \cos(2\pi f_c t) - \frac{2}{3\pi} \cos(6\pi f_c t) + \dots \right]$$

$$V_2(t) = \frac{m(t)}{2} + \frac{A_c}{2} \cos(2\pi f_c t) + \frac{2m(t)}{\pi} \cos(2\pi f_c t) + \frac{2A_c}{\pi} \cos^2(2\pi f_c t) - \frac{2m(t)}{3\pi} \cos(6\pi f_c t) - \frac{2A_c}{3\pi} \cos(2\pi f_c t) \cos(6\pi f_c t) + \dots$$

The 1st term of the above equation represents the desired AM wave and the remaining terms are unwanted terms. Thus, with the help of band pass filter, we can pass only AM wave and eliminate the remaining terms.

Therefore, the output of switching modulator is

$$s(t) = A_c/2(1 + (4\pi A_c)m(t))\cos(2\pi f_c t) \quad (17)$$

We know the standard equation of AM wave is

$$s(t) = A_c[1 + k_a m(t)]\cos(2\pi f_c t)$$

Where, k_a is the amplitude sensitivity. By comparing the output of the switching modulator with the standard equation of AM wave, we will get the scaling factor as 0.5 and amplitude sensitivity k_a as $4/\pi A_c$.

The switching modulator using a diode has been shown in figures 1.1.10 (a) & (b).

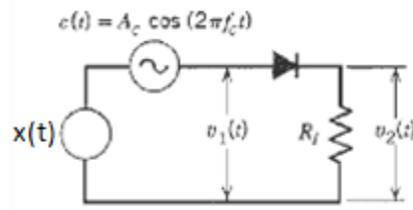


Figure 1.1.10 (a) Switching Modulator

Diagram Source : Brain Kart

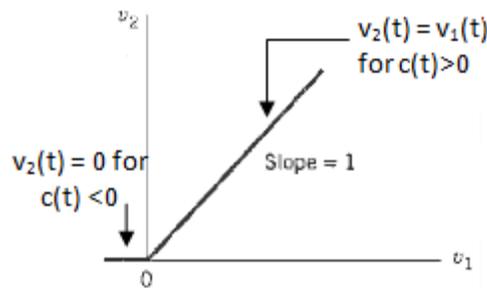


Figure1. 1.10 (b) Input and output voltage Characteristics

Diagram Source : Brain Kart

This diode is assumed to be operating as a switch . The modulating signal $x(t)$ and the sinusoidal carrier signal $c(t)$ are connected in series with each other. Therefore, the input voltage to the diode is given by

$$v_1(t) = c(t) + x(t) = E_c \cos(2\pi f_c t) + x(t) \quad (18)$$

The amplitude of carrier is much larger than that of $x(t)$ and $c(t)$ decides the status of the diode (ON or OFF) .

Working Operation and Analysis

Let us assume that the diode acts as an ideal switch .Hence, it acts as a closed switch when it is forward biased in the positive half cycle of the carrier and offers zero impedance .Whereas it acts as an open switch when it is reverse biased in the

negative half cycle of the carrier and offers an infinite impedance .Therefore, the output voltage $v_2(t) = v_1(t)$ in the positive half cycle of $c(t)$ and $v_2(t) = 0$ in the negative half cycle of $c(t)$.

Hence ,

$$\begin{aligned} v_2(t) &= v_1(t) & \text{for } c(t) > 0 \\ v_2(t) &= 0 & \text{for } c(t) < 0 \end{aligned} \quad (19)$$

In other words , the load voltage $v_2(t)$ varies periodically between the values $v_1(t)$ and zero at the rate equal to carrier frequency f_c .We can express $v_2(t)$ mathematically as under a pulse train. In figure 1.1.11 :

$$v_2(t) = v_1(t) \cdot g_p(t) = [x(t) + E_c \cos(2\pi f_c t)] g_p(t)$$

where, $g_p(t)$ is a periodic pulse train of duty cycle equal to one half cycle period i.e. $T_0 / 2$ (where $T_0 = 1/f_c$) .

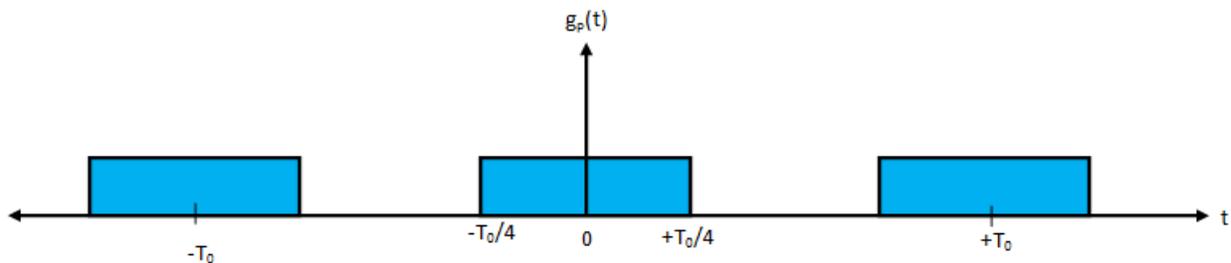


Figure 1.1.11 Periodic pulse train of duty cycle equal to one half cycle period

Diagram Source : Electronic Tutorial

Let us express $g_p(t)$ with the help of Fourier series as under :

$$\begin{aligned} g_p(t) &= \frac{1}{2} + \frac{2}{\pi} \sum_{n=1}^{\infty} \frac{(-1)^{n-1}}{2n-1} \cos [2\pi f_c t(2n-1)] \\ g_p(t) &= \frac{1}{2} + \frac{2}{\pi} \cos(2\pi f_c t) + \text{odd harmonic components} \end{aligned} \quad (20)$$

Substituting $g_p(t)$ into $v_2(t)$ equation , we get

$$v_2(t) = [x(t) + E_c \cos(2\pi f_c t)] \left\{ \frac{1}{2} + \frac{2}{\pi} \sum_{n=1}^{\infty} \frac{(-1)^{n-1}}{2n-1} \cos [2\pi f_c t(2n-1)] \right\}$$

Therefore,

$$v_2(t) = [x(t) + E_c \cos(2\pi f_c t)] \left\{ \frac{1}{2} + \frac{2}{\pi} \cos(2\pi f_c t) + \text{odd harmonics} \right\}$$

The odd harmonics in this expression are unwanted, and therefore, are assumed to be eliminated , Hence,

$$v_2(t) = \underbrace{\frac{1}{2} x(t)}_{\text{Modulating Signal}} + \underbrace{\frac{1}{2} E_c \cos(2\pi f_c t) + \frac{2}{\pi} x(t) \cos(2\pi f_c t)}_{\text{AM Wave}} + \underbrace{\frac{2E_c}{\pi} \cos^2(2\pi f_c t)}_{\text{Second harmonic of carrier}} \quad (21)$$

In this expression, the first and the fourth terms are unwanted terms whereas the second and third terms together represents the AM wave .Clubbing the second and third terms together , we obtain

$$v_2(t) = \frac{E_c}{2} \left[1 + \frac{4}{\pi E_c} x(t) \right] \cos(2\pi f_c t) + \text{unwanted terms} \quad (22)$$

This is the required expression for the AM wave with $m=[4/\pi E_c]$. The unwanted terms can be eliminated using a band-pass filter (BPF) .

AM Spectra and Band Width

The AM signal has three frequency components, Carrier, Upper Sideband and lower side Band

Bandwidth of AM

The bandwidth of a complex signal like AM is the difference between its highest and lowest frequency components and is expressed in Hertz (Hz). Bandwidth deals with only frequencies.

As shown in the following figure 1.1.12

$$\text{Bandwidth} = (f_c - f_m) - (f_c + f_m) = 2 f_m$$

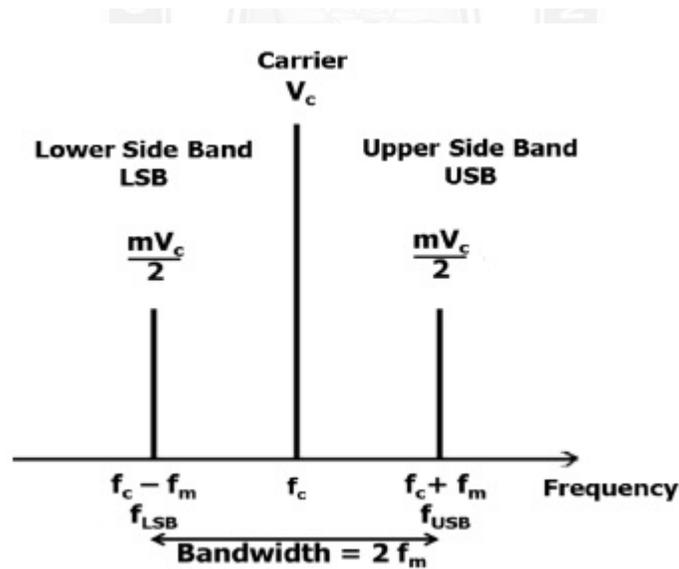


Figure 1.1.12 AM Spectra and Bandwidth

Diagram Source : *elprocus.com*