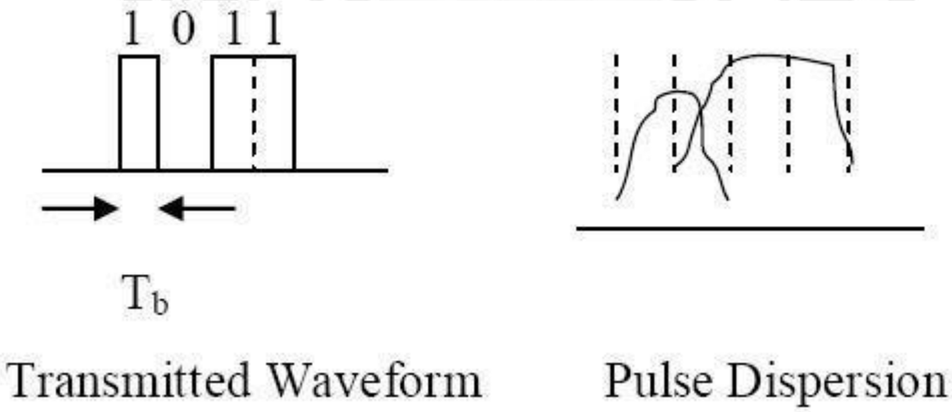
Inter symbol Interference

Generally, digital data is represented by electrical pulse, communication channel is always band limited. Such a channel disperses or spreads a pulse carrying digitized samples passing through it. When the channel bandwidth is greater than bandwidth of pulse, spreading of pulse is very less. But when channel bandwidth is close to signal bandwidth, i.e. if we transmit digital data which demands more bandwidth which exceeds channel bandwidth, spreading will occur and cause signal pulses to overlap. This overlapping is called **Inter Symbol Interference**. In short it is called ISI. Similar to interference caused by other sources, ISI causes degradations of signal if left uncontrolled. This problem of ISI exists strongly in Telephone channels like coaxial cables and optical fibers.

The main objective is to study the effect of ISI, when digital data is transmitted through band limited channel and solution to overcome the degradation of waveform by properly shaping pulse



(Source:Brainkart)

The effect of sequence of pulses transmitted through channel is shown in fig. The Spreading of pulse is greater than symbol duration, as a result adjacent pulses interfere. i.e. pulses get completely smeared, tail of smeared pulse enter into adjacent symbol intervals making it difficult to decide actual transmitted pulse. First let us have look at different formats of transmitting digital data.

In base band transmission best way is to map digits or symbols into pulse waveform. This waveform is generally termed as **Line codes**. To proceed with a mathematical study of intersymbol interference, consider a *baseband binary PAM system*, a generic form of which is depicted in Figure. The term "baseband" refers to an information-bearing signal whose spectrum extends from (or near)

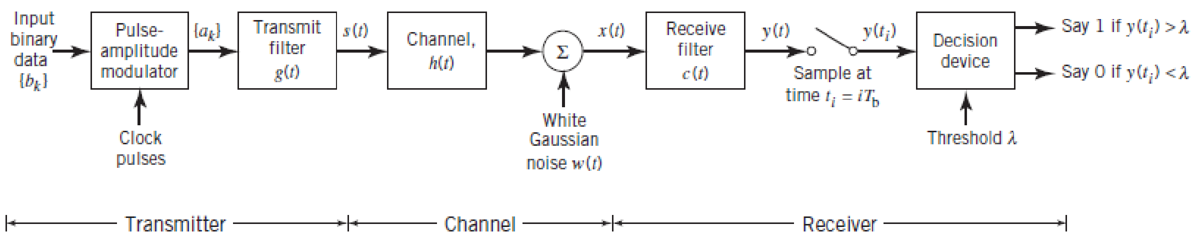


Fig: Baseband Binary data Transmission Systems

(Source: S. Haykin, ―Digital Communications‖, John Wiley, 2005-Page- 448)

The *pulse-amplitude modulator* changes the input binary data stream {*bk*} into a new sequence of short pulses, short enough to approximate impulses. More specifically, the pulse amplitude *ak* is represented in the polar form:

$$a_k = \begin{cases} +1 \text{ if } b_k \text{ is symbol } 1 \\ -1 \text{ if } b_k \text{ is symbol } 0 \end{cases}$$

The sequence of short pulses so produced is applied to a *transmit filter* whose impulse response is denoted by *g*(*t*). The transmitted signal is thus defined by the sequence

$$s(t) = \sum_k a_k g(t - kT_b)$$

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A binary data stream represented by the sequence {*ak*}, where *ak* = +1 for symbol 1 and *ak* = –1 for symbol 0, *modulates* the basis pulse *g*(*t*) and superposes *linearly* to form the transmitted signal *s*(*t*). The signal *s*(*t*) is naturally modified as a result of transmission through the *channel* whose impulse response is denoted by *h*(*t*). The noisy received signal *x*(*t*) is passed through a *receive filter* of impulse response *c*(*t*). The resulting filter output *y*(*t*) is sampled *synchronously* with the transmitter, with the sampling instants being determined by a *clock* or *timing signal* that is usually extracted from the receive-filter output. Finally, the sequence of samples thus obtained is used to reconstruct the original data sequence by means of a *decision device*. Specifically, the amplitude of each sample is compared with a *zero threshold*, assuming that the symbols 1 and 0 are equiprobable. If the zero threshold is exceeded, a decision is made in favor of symbol 1; otherwise a decision is made in favor of symbol 0. If the sample amplitude equals the zero threshold exactly, the receiver simply makes a random guess.

Except for a trivial scaling factor, we may now express the receive filter output as

$$y(t) = \sum_k a_k p(t - kT_b)$$

where the pulse $p(t)$ is to be defined. To be precise, an arbitrary time delay t_0 should be included in the argument of the pulse $p(t - kT_b)$ in (8.6) to represent the effect of transmission delay through the system. To simplify the exposition, we have put this delay equal to zero in (8.6) without loss of generality; moreover, the channel noise is ignored. The scaled pulse $p(t)$ is obtained by a double convolution involving the impulse response $g(t)$ of the transmit filter, the impulse response $h(t)$ of the channel, and the impulse response $c(t)$ of the receive filter, as shown by

$$p(t) = g(t) \star h(t) \star c(t)$$

where, as usual, the star denotes convolution. We assume that the pulse $p(t)$ is *normalized* by setting which justifies the use of a scaling factor to account for amplitude changes incurred in the course of signal transmission through the system. Since convolution in the time domain is transformed into multiplication in the frequency domain, we may use the Fourier transform to change (8.7) into the equivalent form

$$P(f) = G(f)H(f)C(f)$$

where $P(f)$, $G(f)$, $H(f)$, and $C(f)$ are the Fourier transforms of $p(t)$, $g(t)$, $h(t)$, and $c(t)$, respectively.

The receive filter output $y(t)$ is sampled at time $t_i = iT_b$, where i takes on integer values,

$$y(t_i) = \sum_{k=-\infty}^{\infty} a_k p[(i-k)T_b]$$

$$= a_i + \sum_{\substack{k=-\infty \\ k \neq i}}^{\infty} a_k p[(i-k)T_b]$$

The first term *ai* represents the contribution of the *i*th transmitted bit. The second term represents the residual effect of all other transmitted bits on the decoding of the *i*th bit. This residual effect due to the occurrence of pulses before and after the sampling instant *ti* is called *intersymbol interference* (ISI).

In the absence of ISI—and, of course, channel noise—we observe from (8.10) that the summation term is zero, thereby reducing the equation to which shows that, under these ideal conditions, the *i*th transmitted bit is decoded correctly.

$$y(t_i) = a_i$$

Nyquist criterion for distortion less transmission

Consider then the sequence of samples {*p*(*nT*b)}, where *n* = 0,

In particular, we may write

$$P_\delta(f) = R_b \sum_{n=-\infty}^{\infty} P(f - nR_b)$$

where *R*b = 1□*T*b is the *bit rate* in bits per second; *P*□□(*f*) on the left-hand side of (8.12) is the Fourier transform of an infinite periodic sequence of delta functions of period *T*b whose individual areas are weighted by the respective sample values of *p*(*t*). That is, *p* (*f*) is given by

$$P_\delta(f) = \int_{-\infty}^{\infty} \sum_{m=-\infty}^{\infty} [p(mT_b)\delta(t - mT_b)]\exp(-j2\pi ft)\, dt$$

Let the integer *m* = *i* – *k*. Then, *i* = *k* corresponds to *m* = 0 and, likewise, *i* □ *k* corresponds to *m* □ 0 . Accordingly, imposing the conditions of (8.11) on the sample values of *p*(*t*) in the integral in (8.13), we get

$$P_\delta(f) = p(0)\int_{-\infty}^{\infty} \delta(t)\exp(-j2\pi ft)\, dt$$

$$= p(0)$$

where we have made use of the sifting property of the delta function. Since from (8.8) we have $p(0) = 1$, it follows from (8.12) and (8.14) that the frequency-domain condition for zero ISI is satisfied, provided that

$$\sum_{n=-\infty}^{\infty} P(f - nR_b) = T_b$$

where $T_b = 1\square R_b$. We may now make the following statement on the *Nyquist criterion*[1] *for distortionless baseband transmission* in the frequency domain:

The frequency function $P(f)$ eliminates intersymbol interference for samples taken at intervals T_b Correspondingly, the baseband pulse $p(t)$ for distortionless transmission described in (8.18) is called the *ideal Nyquist pulse*, ideal in the sense that the bandwidth requirement is one half the bit rate.

There are two practical difficulties that make it an undesirable objective for signal design:

1. It requires that the magnitude characteristic of $P(f)$ be flat from $-W$ to $+W$, and zero elsewhere. This is physically unrealizable because of the abrupt transitions at the band edges $\pm W$, in that the Paley–Wiener criterion discussed in Chapter 2 is violated.
2. The pulse function $p(t)$ decreases as $1/|t|$ for large $|t|$, resulting in a slow rate of decay. This is also caused by the discontinuity of $P(f)$ at $\pm W$. Accordingly, there is practically no margin of error in sampling times in the receiver.

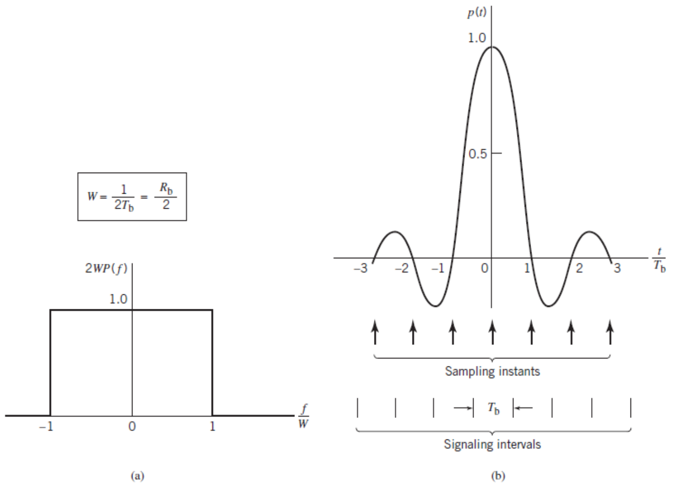

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Fig: a) Ideal Magnitude Response b)Ideal basic pulse shape
(Source: S. Haykin, ―Digital Communications‖, John Wiley, 2005-Page- 452)