5.MULTI STAGE GRAPH

A multistage graph **G = (V, E)** is a directed graph where vertices are partitioned into **k** (where $k > 1$) number of disjoint subsets $S = \{s_1, s_2, \ldots, s_k\}$ such that edge *(u, v)* is in E, then $u \in s_i$ and $v \in s_{1+1}$ for some subsets in the partition and $|s_1| = |s_k| = 1$.

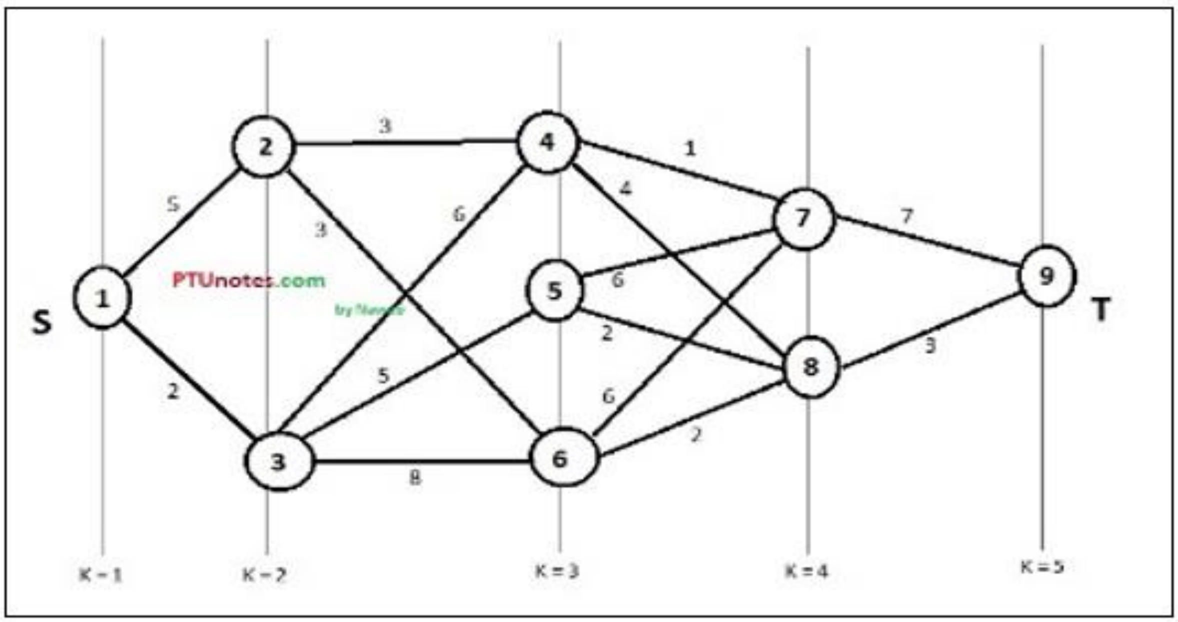
The vertex $s \in s_1$ is called the **source** and the vertex $t \in s_k$ is called **sink**.

G is usually assumed to be a weighted graph. In this graph, cost of an edge *(i, j)* is represented by *c(i, j)*. Hence, the cost of path from source ***s*** to sink ***t*** is the sum of costs of each edges in this path.

The multistage graph problem is finding the path with minimum cost from source ***s*** to sink ***t***.

Example

Consider the following example to understand the concept of multistage graph.



According to the formula, we have to calculate the cost **(i, j)** using the following steps

Step-1: Cost (K-2, j)

In this step, three nodes (node 4, 5. 6) are selected as **j**. Hence, we have three options to choose the minimum cost at this step.

Cost (3, 4) = min {c(4, 7) + Cost(7, 9),c(4, 8) + Cost(8, 9)} = 7

Cost (3, 5) = min {c(5, 7) + Cost(7, 9),c(5, 8) + Cost(8, 9)} = 5

Cost (3, 6) = min {c(6, 7) + Cost(7, 9),c(6, 8) + Cost(8, 9)} = 5

Step-2: Cost (K-3, j)

Two nodes are selected as j because at stage k - 3 = 2 there are two nodes, 2 and 3. So, the value i = 2 and j = 2 and 3.

Cost (2, 2) = min {c (2, 4) + Cost (4, 8) + Cost (8, 9), c (2, 6) +

Cost (6, 8) + Cost (8, 9)} = 8

Cost (2, 3) = {c (3, 4) + Cost (4, 8) + Cost (8, 9), c (3, 5) + Cost (5, 8) + Cost (8, 9), c (3, 6) + Cost (6, 8) + Cost (8, 9)} = 10

Step-3: Cost (K-4, j)

Cost (1, 1) = {c (1, 2) + Cost (2, 6) + Cost (6, 8) + Cost (8, 9), c (1, 3) + Cost (3, 5) + Cost (5, 8) + Cost (8, 9))} = 12

c (1, 3) + Cost (3, 6) + Cost (6, 8 + Cost (8, 9))} = 13

Hence, the path having the minimum cost is **1→ 3→ 5→ 8→ 9**.