ROHINI

COLLEGE OF ENGINEERING & TECHNOLOGY

GE 8292- ENGINEERING MECHANICS

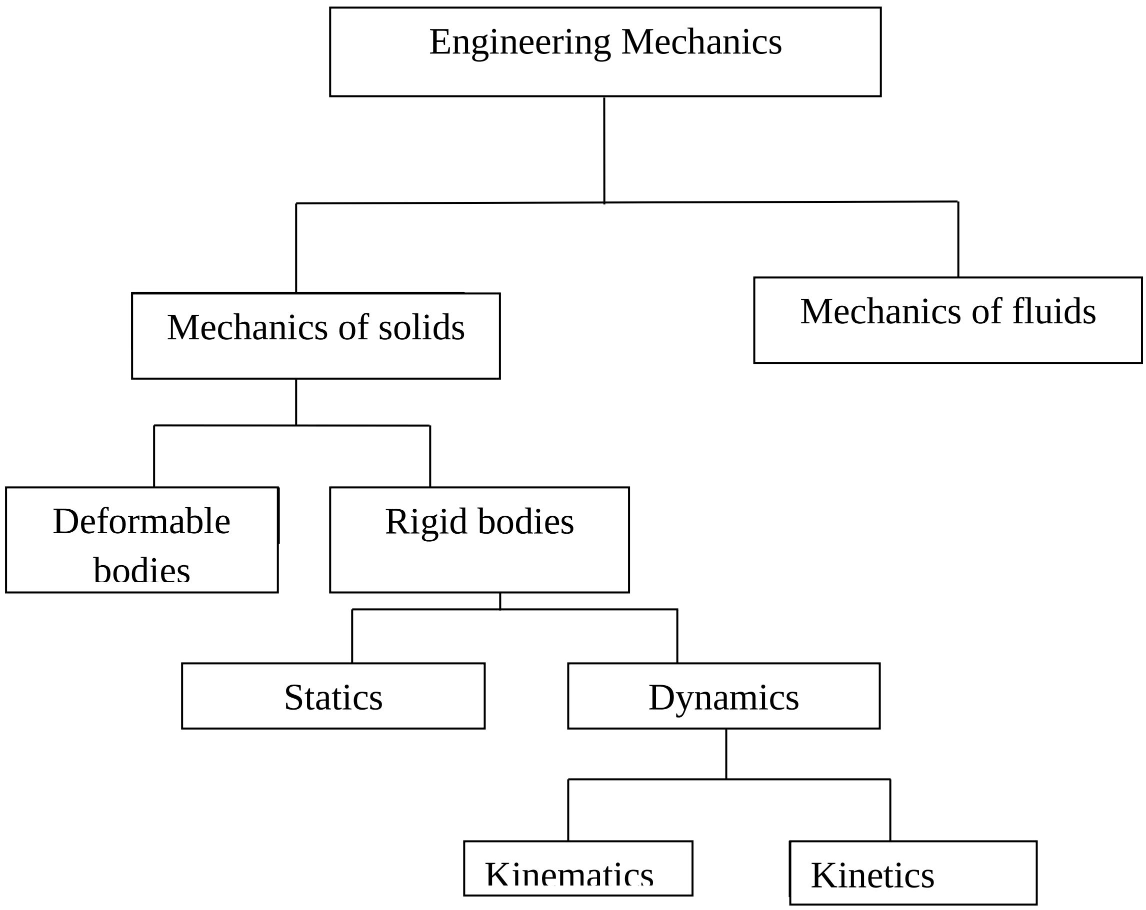
OBJECTIVES:

To develop capacity to predict the effect of force and motion in the course of carrying out the design function of Engineering.

UNIT-I BASIC & STATICS OF PARTICLES

Engineering Mechanics:

Engineering Mechanics may be defining as a branch of science which deals with the behavior of a body with the state of rest or motion, subjected to action of force.



Rigid Bodies:

When a body does not undergo any deformation under the application of forces then it is known as Rigid body.

Deformable body:

When a body undergoes a temporary or permanent change in its dimensions due to application of force it is known as deformable body.

Statics:

It is the branch of science, which deals with the study of a body at rest.

Dynamics:

It is the branch of science which deals with the study of a body in motion.

Kinematics:

It is the study of a body in motion without considering the forces, that cause the motion.

Kinetics:

It is the study of a body in motion, with considering the forces, that causes the motion.

Application of Mechanics:

Engineering mechanics has application in many areas of engineering projects. To cite of few examples, engineering mechanics is applied in the design of spacecrafts and rockets. Analysis of structural stability and machine strength, vibrations, robotics, electrical machines, flow and automatic controls.

Mass and weight

Mass:

The Quantity of matter contained in a body is called mass. The force with which a body is attracted towards the centre of the earth is called weight.

Weight = Mass of body × acceleration due to gravity

W= mg

G=9.81 m/s^2.

Difference between Mass and Weight

	Mass	Weight
1.	It is a Quantity of matter contained in a body	It is the force with which the body is attracted towards the centre of the earth
2.	It is constant at all places	It is not constant at all places
3.	It resist motion in body	It provides motion in body
4.	It is a scalar quantity	It is a vector quantity
5.	It is never zero	It is zero at the centre of earth
6.	It is measured in kg both in MKS and SI units.	It is measured in Kgf in MKS units and Newton (N in SI units)

UNITS OF MEASUREMENTS

Measurement:

A physical Quantity can be measured by comparing the sample with a known standard amount.

Unit:

The known amount used in the measurement of physical quantity is called a unit.

1.Fundamental Units:

The units which are used for the measurement of basic or fundamental quantities (Mass, Length, Time) are known as fundamental units.

Eg. i) Mass ii) Length iii) Time.

2.Derived Units:

All units which are used for the measurement of physical quantities other than fundamental ones are called derived units.

Eg. Area, Volume, Speed, Velocity, etc….

System of Units:

1. Foot pound second system [FPS system]
2. Centimeter gram second system [CGS system]
3. Metre kilogram second system [MKS system]
4. System of International [SI system]

Six Fundamental Units in SI system:

1. The metre as the fundamental unit of length.
2. The kilogram as the fundamental unit of mass.
3. The second as the fundamental unit of time.
4. The ampere as the fundamental unit of electric current.
5. Kelvin as the fundamental unit of thermodynamics temperature.
6. The candela as the fundamental unit of luminous intensity.

LAW OF MECHANICS

Newton's First law:

It states that "A body continues in its states of rest or of uniform motion in astraight line, unless acted upon by some external force". This is known as First law of inertia.

Newton's second Law of motion:

It states "The rate of change of momentum is directly propositional to the impressed force and tasks places in same direction in which the force acts".

$$F = \text{Mass} \times \text{Acceleration} = ma$$

Newton's third Law of motion:

It states "To every action there is always an equal and opposite reaction".

- A particle remains in its position (rest or motion) if the resultant force acting on the particle is zero.
- Acceleration of a particle will be proportional to the resultant force and in the same direction if the resultant force is not zero.
- Action and reaction b/w interacting bodies are in the same line of action equal in magnitude but act in the opposite direction.

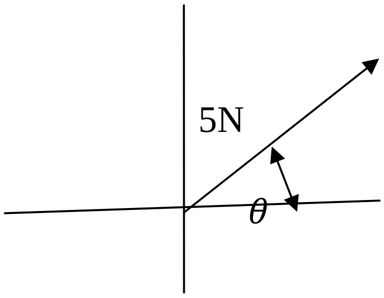
FORCE

Force is an agent which changes or tends to change the states (or) uniform motion of a body upon which it acts. Force is a vector quantity.

Characteristics of a Force:

1. Magnitude
2. Line of action
3. Direction

Graphical Representation of force:



Newton's second law of motion

Momentum = Mass × velocity

M= mass of the body

u = Initial velocity of body

v = final velocity of body

a = Constant acceleration

t = time required to change velocity from u to v

∴ change of momentum $= mv - mu$

Rate of change of momentum $= \frac{mv-mu}{t} = \frac{m(v-u)}{t} = ma$

By Newton's second Law

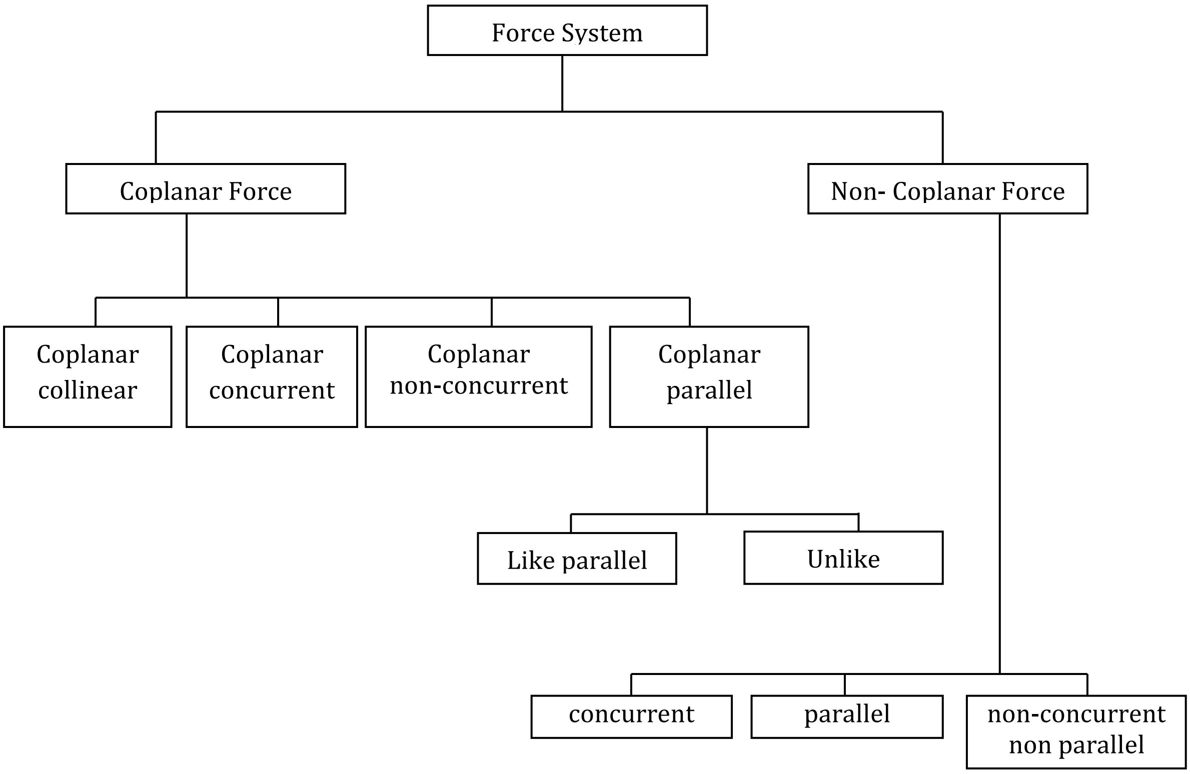
Force = Rate of change of momentum

$F = ma$

Unit of Force:

In SI system unit of force is (N) Newton. One Newton may be defined as the force while acting upon a mass of one kg. Produces an acceleration of 1 m/s^2 in the direction in which it acts.

$1\ N = 1\ kg \times 1 m/s^2 = 1 kgm/s^2$



<u>Parallel</u>

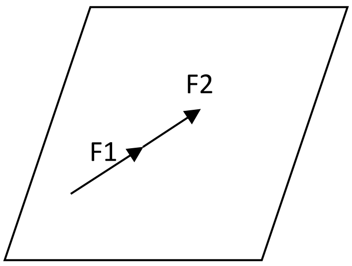
The forces do not line on same plane but their line of action is parallel to each other.

<u>Non concurrent,Non parallel</u>

The forces neither lie on same plane not their line of action meet at common point.

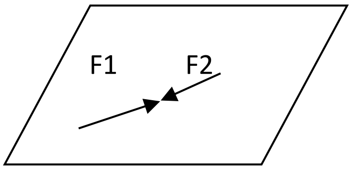
<u>Like collinear coplanar forces</u>

Forces acting in the same direction, lies on a common on line of action and acts in a single plane



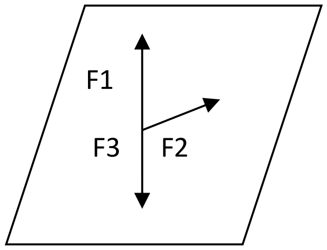
Unlike collinear coplanar forces:

Forces acting in the different lies on a common line of a action and act is a single plane.



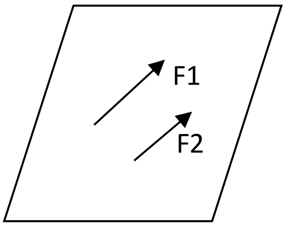
Coplanar concurrent forces:

Force intersects at a common point and lies in a single plane.



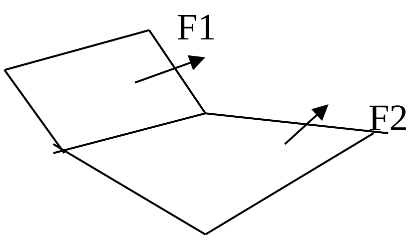
Coplanar Non concurrent flow:

Forces which do not intersects at a common point but acts in one plane. They may be parallel or non parallel



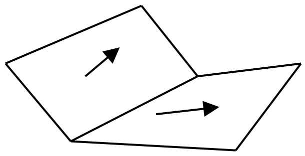
Non coplanar concurrent forces

Forces intersect at a common point but either line of action do not lie on same plane.



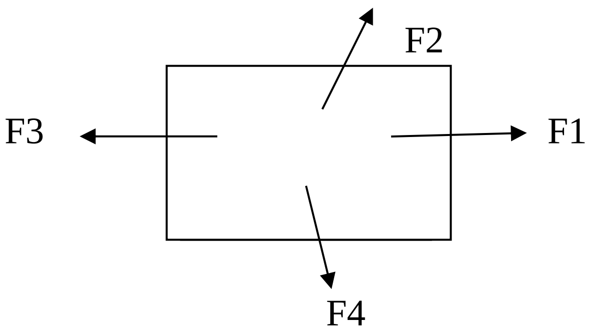
<u>Non Coplanar Non concurrent force:</u>

Forces do not intersect at one point and also their lines of action do not lie on same plane.



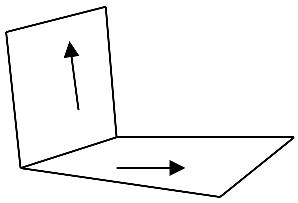
<u>Coplanar forces:</u>

The line of action of the forces lies on same plane.



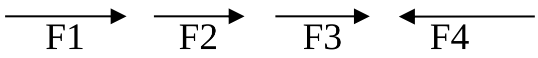
<u>Non coplanar forces:</u>

The lines of action of the forces not lie on same plane.



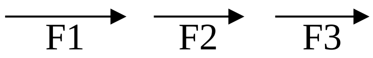
<u>Collinear:</u>

The Line of action of the forces lie on same plane.



<u>Like collinear:</u>

The line of action of forces lies on a same line and in same direction.

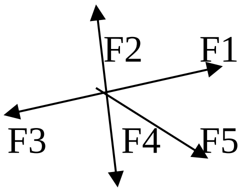


Unlike collinear

The lines of action of forces lie on same line but are in opposite direction.

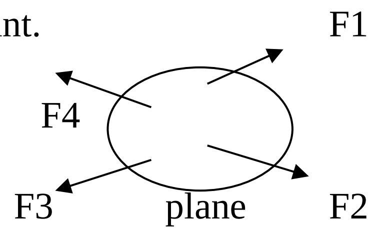
Concurrent forces:

The lines of action of all forces meet at a common point and lie in the same plane.



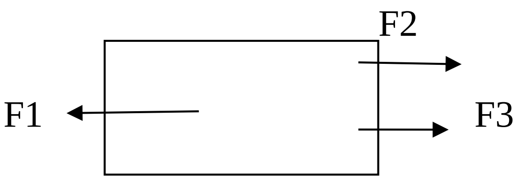
Non concurrent force system:

The forces will lie on same plane but their line of action will not intersect at a common point.



Parallel forces:

The forces lying on same plane whole line of action are parallel to each other.



Like parallel:

If the force acts in the same direction they are coplanar like parallel force system.

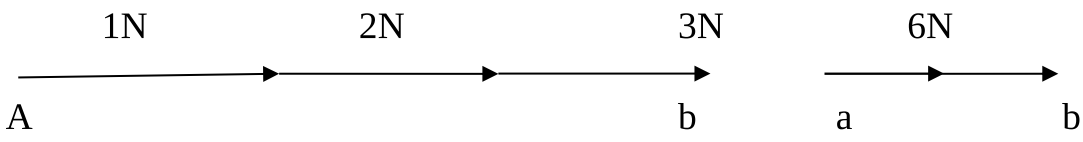
Unlike parallel: If the force acts in opposite direction, they are coplanar unlike parallel force system.

UNIT-1

Chapter-2 Statics of Particles in Two Dimensions- Resultant Force

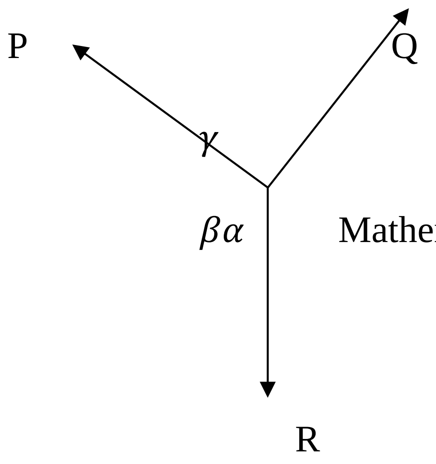
Resultant Force:

If a number of forces acting on a particle simultaneously are replaced by a single force which could produce the same effort as produced by the given forces, that single force is called resultant force.



Lami's Theorem:

It state that "if three coplanar forces acting at a point be in equilibrium, than each force is propositional to the sin of the angle between the other two forces.

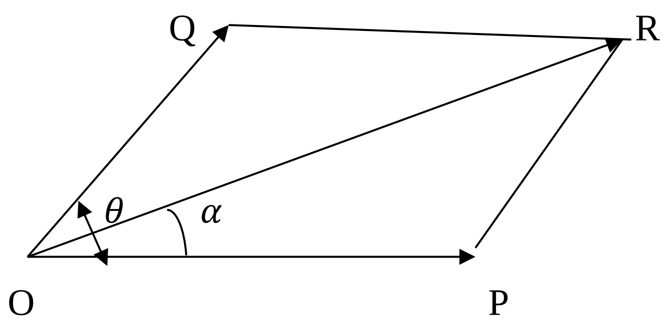


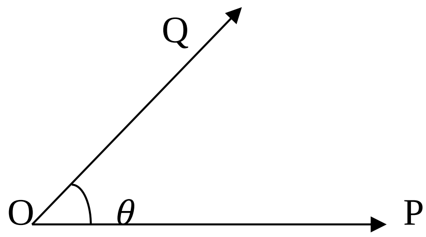
Mathematically

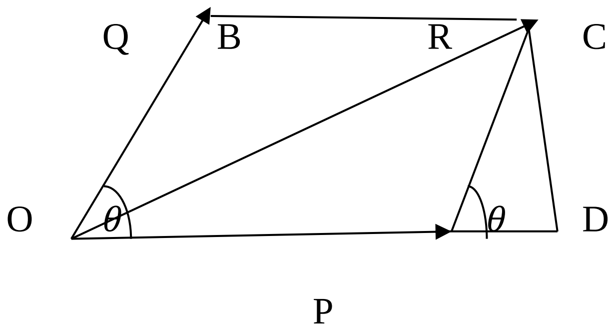
$$\frac{P}{sin\alpha} = \frac{Q}{sin\beta} = \frac{R}{sin\gamma}$$

Parallelogram Law of forces:

It states that "if the two forces acting simultaneously at a point represented in magnitude and direction by the two adjacent sides of the parallelogram, then the resultant of these two forces is represented in magnitude and direction by the diagonal of the parallelogram originating from that point.







Let pand Q are two concurrent force acting on a point O at an angle of θ.

The forces P and Q are graphically represented by the lines OA and OB respectively.

The parallelogram θACB is completed by drawing the lines BC and AC parallel to OA and OB respectively.

In parallelogram OACB, the diagonal OC represents the resultant force of P and q. by II Law of forces.

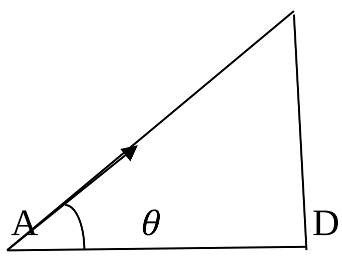
In order to prove the, parallelogram law of forces, extend the lines of action of force P, till its meet the perpendicular drawn from point C.

Let the point of intersection of these two lines be D. from the geometry of the parallelogram.

OB=AC

OA=BC

In triangle ACD



$\cos\theta = \frac{AD}{Q}$ $\sin\theta = \frac{CD}{Q}$

$AD = Q\cos\theta$ --------- (1) $CD = Q\sin\theta$ ------------- (2)

Also = $AD^2 + CD^2 = AC^2$

$= AD^2 + CD^2 = Q^2$--------------- (3)

In triangle OCD

$OC^2 = OD^2 + CD^2$

$= (OA + AD)^2 + CD^2$

$= OA^2 + AD^2 + 2 \times OA \times AD + CD^2$

$= OA^2 + (AD^2 + CD^2) + 2OA\ AD$

$= OA^2 + AC^2 + 2\ OA\ AD$

$R^2 = P^2 + Q^2 + 2 \times PQ\cos\theta$

$R = \sqrt{P^2 + Q^2 + 2 \times PQ\cos\theta}$

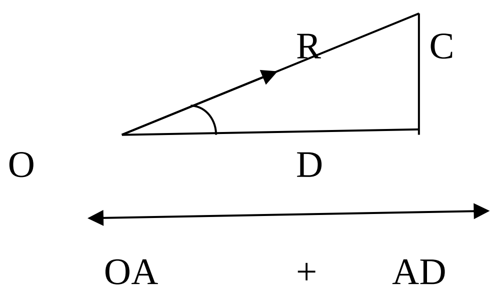
Inclination of the resultant force with the force P

Let the angle of inclinator of R with the line of action of the force P be $\propto$

In triangle OCD

$\tan\propto = \frac{CD}{OD} = \frac{CD}{OA+AD} = \frac{Q\sin\theta}{P+Q\cos\theta}$

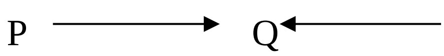
$\therefore \tan\propto = \frac{Q\sin\theta}{P+Q\cos\theta}$



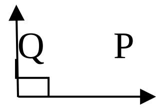
Important results:

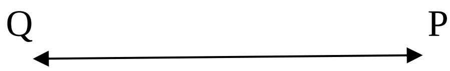
1. If $\theta = 0°$ then the resultant forces pand Q will be like collinear, then,

R=P +Q



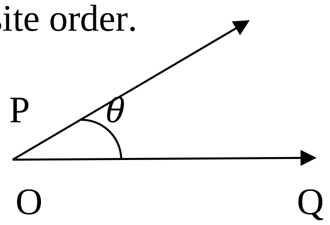
2. If $\theta = 90°$, the forces P and Q are at right angles then R= $\sqrt{P^2 + Q^2}$
3. If $\theta = 180°$, then the forces P and Q will be unlike collinear forces, then R=P-Q.

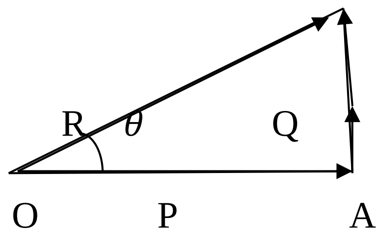




Triangle law of Forces:

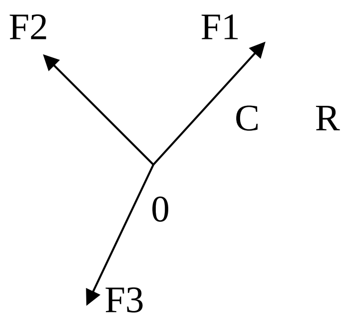
If two forces acting at a point are represented by two sides of a triangle taken in order, then their resultant force is represented by the third side taken in opposite order.

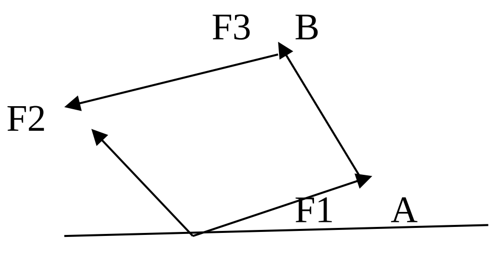




Polygon Law of forces:

Polygon Law of forces states that, 'if a number of coplanar concurrent forces are represented in magnitude and direction by the sides of a polygon taken in an order then their resultant force is represented by the closing side of the polygon taken in the opposite order.

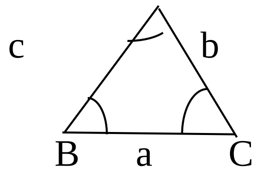




Sine Law:

The law of sines can be used when two angles and a side are known a technique known as triangulation.

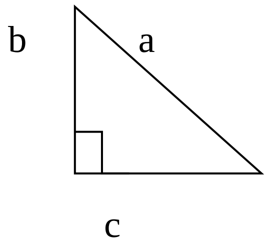
$$= \frac{a}{sin\alpha} = \frac{b}{sin\beta} = \frac{c}{sin\gamma}$$



Cosine Law:

It two side and the angle between the sides are known,

Then the third is given by



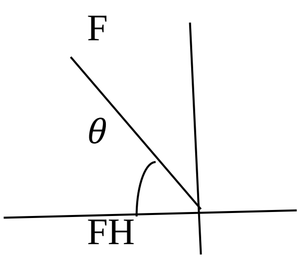
$$a^2 = b^2 + c^2 - 2bccos\alpha$$

$$b^2 = c^2 + a^2 - 2cacos\beta$$

$$c^2 = a^2 + b^2 - 2abcos\gamma$$

Resolution of a force in to its horizontal and vertical part.

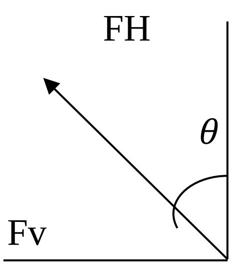
Method I



$$sin\theta = \frac{Fv}{F} => FV = Fsin\theta$$

$$cos\theta = \frac{FH}{F} => FH = Fcos\theta$$

Method II

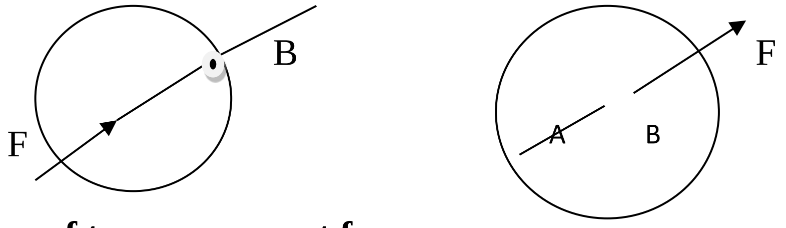


$$sin = \frac{FH}{F} => FH = Fsin\theta$$

$$cos\theta = \frac{Fv}{F} => Fv = Fcos\theta$$

Principle of transmissibility of Forces:

If a force act at any point of on a rigid body it may also be considered to act at any other point on its line of action.

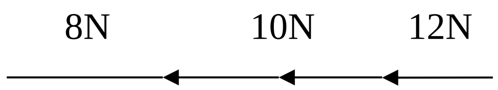


Resultant force of two concurrent forces:

1. Resultant force of two concurrent force
2. Resultant force of more than two concurrent force

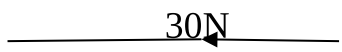
Problem based on parallelogram & Resultant forces:

1. Find the resultant force of the collinear forces shown in fig.

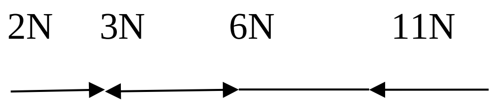


Soln:

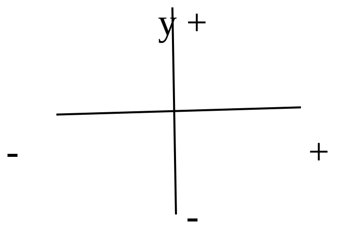
Resultant force R= 8+10+12=30N



2. Find the resultant force of the collinear forces, shown in fig



Soln:



Magnitude of resultant force

= 2-3+6-11

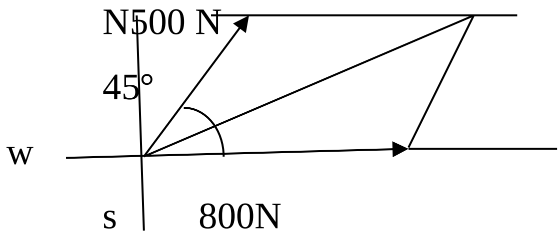
R = -6N

3. Find the resultant force an 800 N force acting towards eastern direction and a 500 n force acting towards north eastern direction.

by 1. Parallelogram Law

2. Triangle Law

Also find the direction



Given

P=800 N Q=500N θ=45°

To find

Resultant force & direction

Soln

1. Parallelogram Law

Resultant Force R= $\sqrt{P^2} + Q^2 + 2PQcos\theta$

R = $\sqrt{800^2} + 500^2 + 2 \times 800 \times 500 \times cos45$

R= 1206.52 N

Direction of magnitude

Q = $tan^{-1}\left[\frac{Qsin\theta}{P+Q\ cos\theta}\right]$

Q = $tan^{-1}\left[\frac{500sin45}{800+500\ cos45}\right]$

Q = 17°04'

Summing of components:

R = $\sqrt{FH^2 + \sum FV^2}$

∑FH = 800+500sin45=1153.55N

∑FH = 500 sin 45 = 353.55 N

R = $\sqrt{1153.55^2 + 353.55^2}$

R = 1206.52 N

$\alpha = tan^{-1}\frac{[\sum FV]}{\sum FH}$

$= tan^{-1}\frac{[1153.55]}{353.55}$

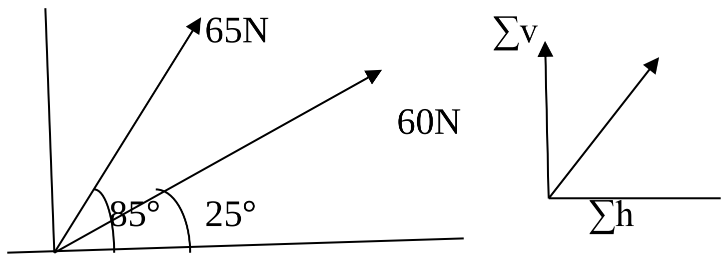
$\alpha = 17°04'$

4. Two forces 60 N and 65 N act on a screw at an angle of 25° and 85° from the base. Determine the magnitude and direction of their resultant.

Given:

P_1=60 N1,θ_1=25°

P_2=65 N, θ_2=85°



To find:

Magnitude & direction of their resultant

Soln:

1. Magnitude of resultant force

R $= \sqrt{\sum FH^2 + \sum FV^2}$

$\sum FH$ = 60 cosθ_1 +65 cosθ_2

= 60 cos25 +65 cos85

$\sum FH$ =60 N

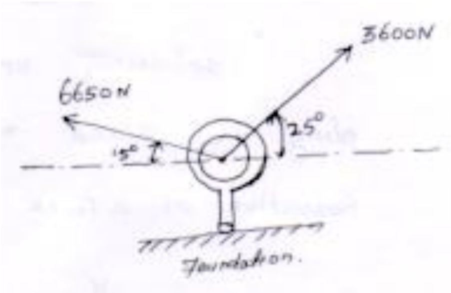
$\sum FV$ = 60 sinθ_1 +65 sinθ_2 = 60 sin25 +65 sin85

$\sum FV$ = 90 N

R = $\sqrt{60^2 + 90^2}$

R= 108.17 N

5. Two wires are attached to a bolt in a foundation as shown in fig. below. Determine the pull exerted by the bolt on the foundation.



<u>Soln:</u>

Resultant force R = $\sqrt{\sum FH^2 + \sum FV^2}$

$\sum FH$ = 3600 cos 25-6650 cos 15

$\sum FH$ = -3160 N

$\sum FV$ = 3600 sin25+ 6650 sin 15

$\sum FV$ = 3242N

R = $\sqrt{-3160^2 + 3242^2}$

R = 4527 N

$\alpha = tan^{-1}\left[\frac{\sum FV}{\sum FH}\right]$

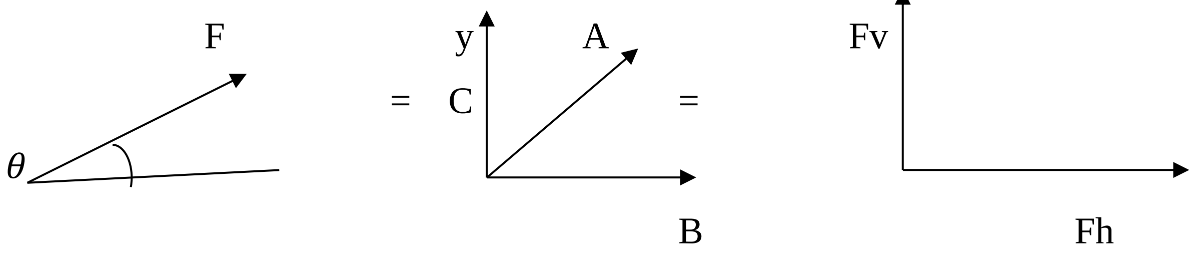
$\alpha = tan^{-1}\left[\frac{\sum 3242}{\sum 3160}\right]$

$\alpha = 45°73'$

⇒ **Resultant force of more than Two concurrent Forces:**

Resolution of Forces:

Splitting up a force into components along the fixed reference axis is called Resolution of a force.



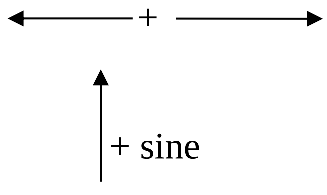
F_h = horizontal component = +F cosθ

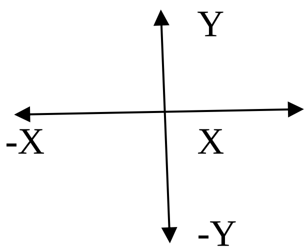
F_v = vertical component = +F sinθ

Sign conversion:

Horizontalcomponent -

Vertical component

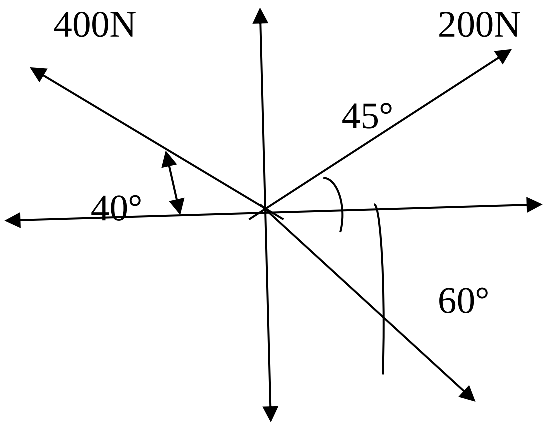




$$R = \sqrt{\sum FH^2 + \sum FV^2}$$

$$\alpha = tan^{-1}\left[\frac{\sum FV}{\sum FH}\right]$$

1. Three coplanar concurrent forces are acting at a point as shown in fig. Determine the resultant in magnitude of direction.



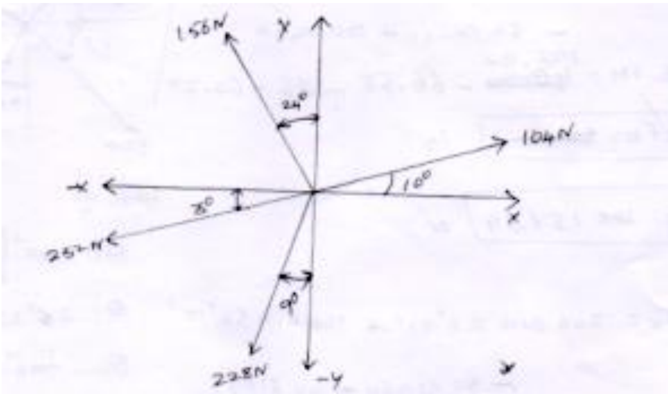
Soln:

Force & magnitude	θ	F$\cos\theta$	$Fsin\theta$
F_1=200	45°	200 cos45°=141.42	200 sin45°=141.42N
F_2=400	150°	400 cos150°=-326.41	400 sin150°=200
F_3=600	300°	600 cos300°=300 N	600 cos300°=-519.61
		$\sum FH$ = 95.01 N	$\sum FV$ = -178.19N

R = $\sqrt{\sum FH^2 + \sum FV^2}$=$\sqrt{95.01^2 + -178.19^2}$

R=201.95 N

2. The four coplanar forces acting at a point a as shown in fig. Determine the resultant in magnitude and direction.



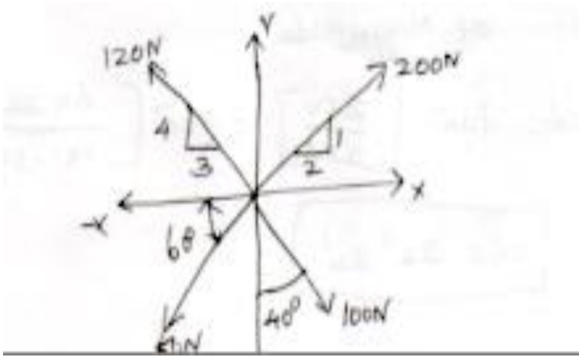
<u>Soln:2 nd method</u>

Note:θ_1=10° θ_2=90-24=66° θ_3=3° θ_4=90-9=81°

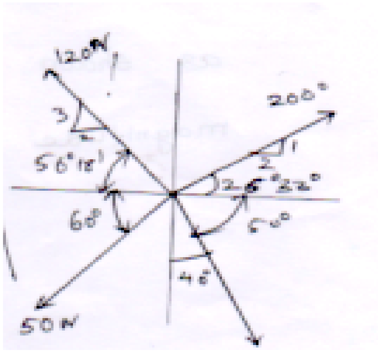
$\sum FH$ =-248.36 $\sum FV$ = -77.82

R=260.26N θ= 17.39°

3. A system of four forces acting on a body is shown in fig below. Determine the resultant force and direction.



Soln:



Resultant force R= $\sqrt{\sum FH^2 + \sum FV^2}$

$\sum FH$ =200 cos26°33$' - 120 \cos 56°18' - 50 \cos 60 + 100 cos 50$

$\sum FH$ =178.90-66.58-25+64.27

$\sum FH$ =151.59

$\sum FV = 200 sin 26°33' + 120 sin 56°18' - 50 sin 60 - 100 sin 50$

$\sum FV = 89.39 + 99.83 - 43.30 - 76.60$

$\sum FV$ =69.32N

R =$\sqrt{(151.59)^2 + (69.32)^2}$

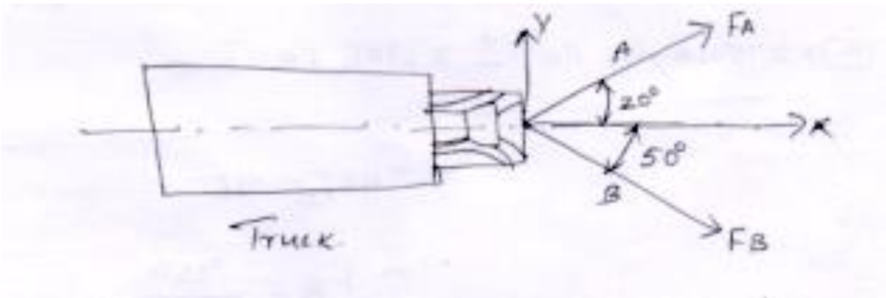
R = 166.68N

Direction of magnitude

$\alpha = tan^{-1}\left[\frac{\sum FV}{\sum FH}\right] = tan^{-1}\left[\frac{69.32}{151.59}\right]$

$\alpha = 24°34'$

4. The truck shown is to be toward using two ropes. Determine the magnitude of forces F_A& F_B acting on each rope in order to develop a resultant force of 950N directed along the positive X axis.



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Resultant force R= 950 N in positive × $direction$

∴Hence $\sum FH$ = 950

$\sum FH$ = 0

Resolving forces horizontally

$\sum FH$=FAcos20°+FB cos 50°

$FAcos20 + FBcos50 = 950$--------- (1)

Resolving forces vertically

$\sum FV = FAsin20 - FBsin50 = 0$

$FAsin20 - FBsin50 = 0$------------------- (2)

Solving eq(1) & (2)

$FAcos20 + FBcos50 = 950$

$$FAsin20 - FBsin50 = 0$$

$0.939FA + 0.642FB = 950$--------------- (1)

$0.342FA + 0.766FB = 0$--------------- (2)

$0.939FA + 0.642FB = 950$--------------- (1)

(2)$\times$ 2.75 $\quad 0.342FA \pm 0.766FB = \quad 0$

2.748FB = 950

$$FB = \frac{950}{2.748}$$

FB = 345.64 N

FB value sub in eqn(1)

$0.94\times$ FA+0.642$\times$345.64=950

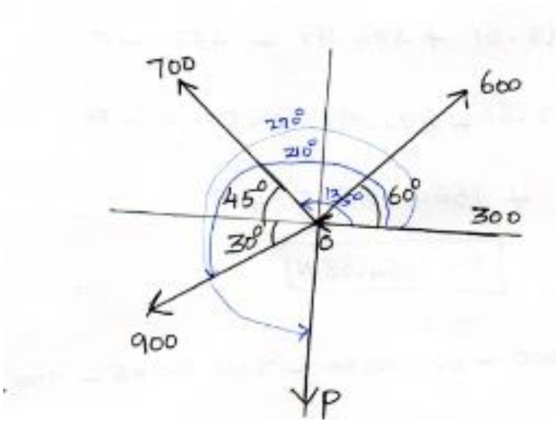
0.94 FA =950-221.9=728

$$FA = \frac{728.09}{0.94}$$

FA = 774.57 N

5. Five forces are acting on a particle. The magnitude of the forces are 300 N,600N,700N,900n and P and their respective angles with the horizontal are 0°,60°,135°,210°,270°. If the vertical component of all the force is -1000N, Find the value of P. Also calculate the magnitude and the direction of the resultant, assuming that the first force acts towards the point, while all the remaining forces act away from the point.

Given:



θ_1=0° θ_2=60° θ_3=180-135=45°

θ_4=180+[90-60]=210°=30°

θ_5=270=90°

F_1=300, F_2=600 F_3=700 F_4=900 F_5=P

$\sum FV$ = -1000N

Soln

Resultant force R=$\sqrt{(\sum FH)^2 + (\sum FV)^2}$

$\sum$FV= -1000N

To find the value of 'P'

Algebraic sum of vertical components

$\sum$Fv = 600 sin60 +700 sin45-900 sin30-P

-1000=519.61+494.97-450-P

-1000=-519.61-494.97+450=-P

- 1564.58 = - P

P=1564.58N

$\sum$FH=-300+600 cos60-700 cos45-900 cos30

$\sum$FH=-300 +300-494.97-779.42

$\sum$FH=-1274.39 N

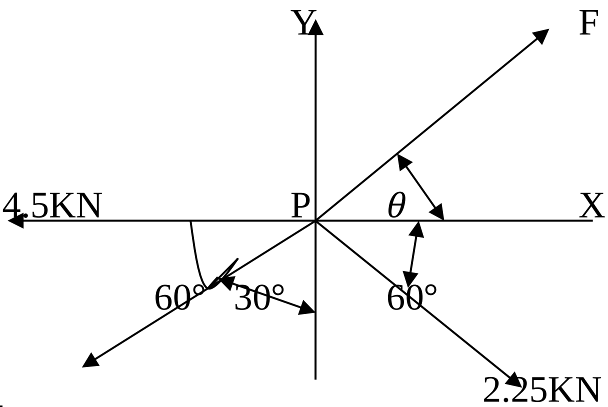
Resultant force R=$\sqrt{(-1274.39)^2 + (-1000)^2}$

R=1619.89 N

Direction $\alpha = tan^{-1}\left[\frac{\sum FV}{\sum FH}\right] = tan^{-1}\left[\frac{1000}{1274}\right]$

$\alpha = 38°7'$

6. Determine the magnitude and angle of f so that particle P shown in Fig



Soln:

$\sum$FH=F cosθ-4.5-7.5 cos60+2.25 cos60=0

Fcosθ-4.5-3.75+1.125=0---------------- (1)

$\sum$FV= F sinθ-7.5 sin60-2.25 sin 60=0

F sinθ-6.49-1.94=0-------------- (2)

Eqn (1) Rearrange

F cosθ-7.125=0

$F\ cos\theta=7.125$-----------------(3)

Eqn(2) Rearrange

$F\ sin\theta-8.43=0$

$F\ sin\theta=8.43$------------------ (4)

$\frac{Eqn(4)}{Eqn(3)}=\frac{Fsin\theta}{Fcos\theta}=\frac{8.43}{7.125}$

$\text{Tan}\theta = 1.183$

$\theta = tan^{-1}[1.183]$

$\theta = 49°47'$

Substitute $\theta = 49°47'$ in eqn(3)

$Fcos\theta=7.125 \Rightarrow F\ cos\ 49°47'=7.125$

$F=\frac{7.125}{cos49°47}$

F= 11.03 N

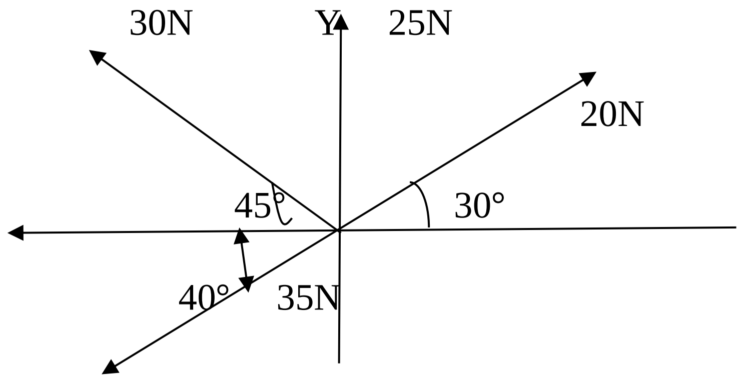
7. Particle 'O' is acted on by the following forces (HW)

   (i) 20 N inclined 30° to north of east

   (ii) 25 N towards the North

   (iii) 30 N towards the north west

   (iv) 35 N inclined 40° to south of west



Chapter -3

Equilibrium of Particles in Two Dimensions

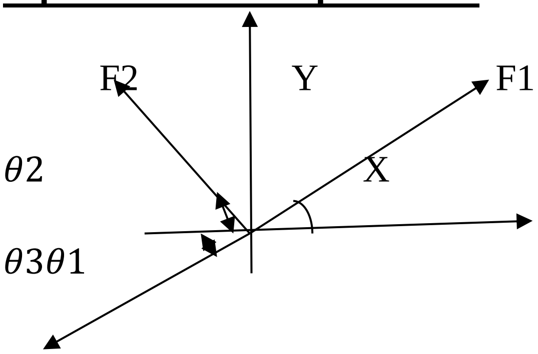
Equilibrium:

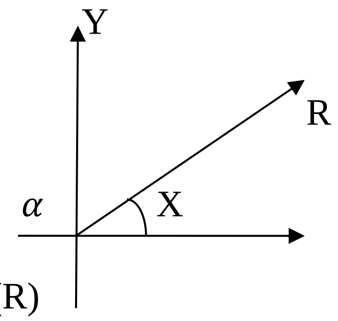
A body is said to be in a state of equilibrium, if the body is either at rest or moving at a constant velocity.

Equilibrium Force:

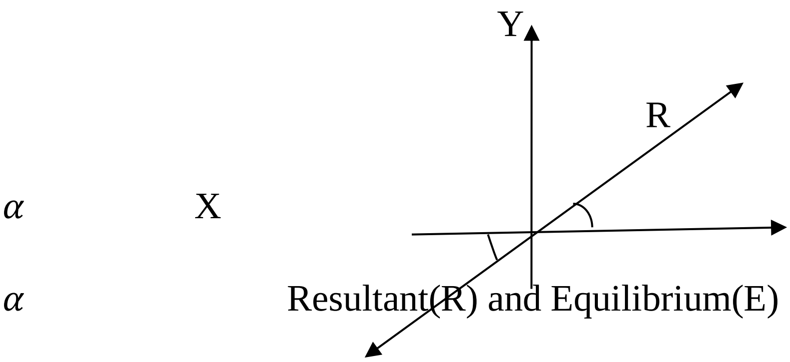
The set of forces where resultant is zero is called “Equilibrium Force”.

Equilibrium and Equilibrant:





Resultant(R)



Resultant(R) and Equilibrium(E)

Consider a particle subjected to three coplanar concurrent forces as shown in fig(1)

Let the resultant force of the force system R as shown in fig(2) with direction of α with horizontal. Due to this resultant force, the particle may starts moving in the direction of resultant force.

But if we apply an additional force of same magnitude and direction as that of resultant force, on the same line of action, but in opposite direction, then the movement of the particle will be arrested or the particle to said to be in Equilibrium.

The force E, which brings the particle (or set of force) to equilibrium, is called equilibrant.

Hence, Equilibrant (E) is Equal to the resultant force(R) in magnitude and direction, collinear but opposite in nature.

<u>Conditions of Equilibrium:</u>

For equilibrium condition of force system, the resultant is Zero.

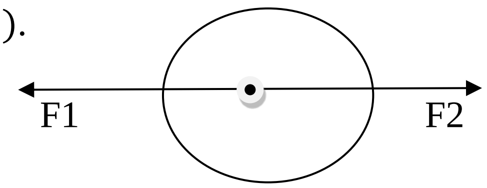
R=0

But R $= \sqrt{(\sum FH)^2 + (\sum FV)^2}$

$\sum FH=0$ $\sum FV=0$

<u>Principle of Equilibrium:</u>

Equilibrium principles are developed from the force Law of equilibrium ($\sum F = 0$).



1. Two force Equilibrium principle:

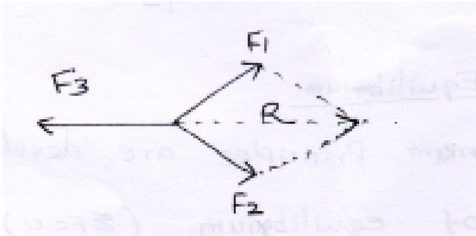
   If a body is subjected to two forces, then the body will be in equilibrium if the two forces are collinear, equal and opposite.

2. Three force equilibrium principle:

   If a body is subjected to three forces, then the body will be in equilibrium, if the resultant of any two forces is equal, opposite and collinear with the third force.

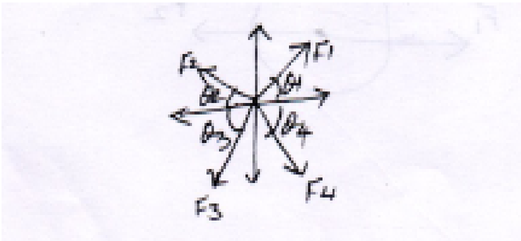
R is the resultant

F_1 and F_2 also $R=F_3$



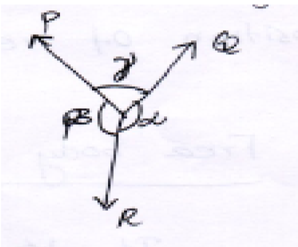
3. Four Force Equilibrium Principle:

   If a body is in equilibrium , acted upon by four forces, then the resultant of any two equal must be equal, opposite and collinear with the resultant of the other two.



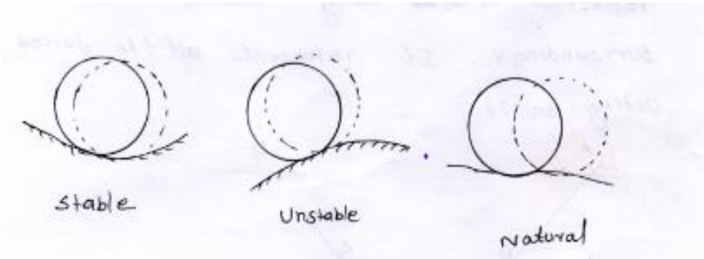
⇒Lami’s Theorem:

If three coplanar forces acting at a point be in equilibrium, than each force is proportional to the sine of the angle b/w the other two.



$$\frac{P}{sin\alpha} = \frac{Q}{sin\beta} = \frac{R}{sin\gamma}$$

Types of Equilibrium:



Stable Equilibrium:

A body is said to be in stable equilibrium, if it returns back to its original position atter it is slightly displaced from its position.

Unstable Equilibrium:

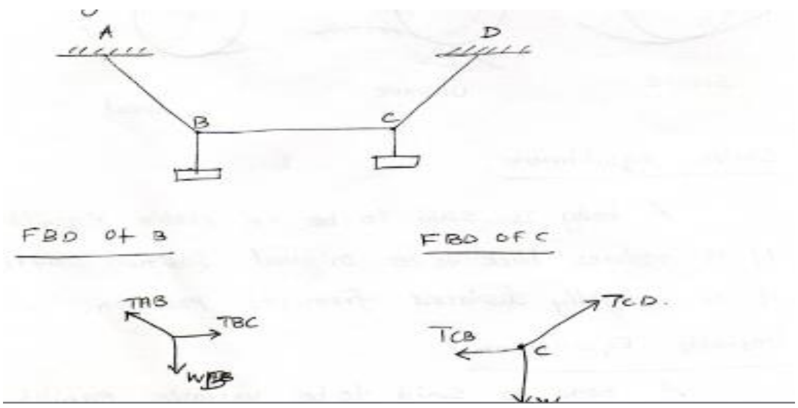
A body is said to be unstable equilibrium it does not return back to its original position and heals farther away after slightly displaced from its position of rest.

Natural Equilibrium:

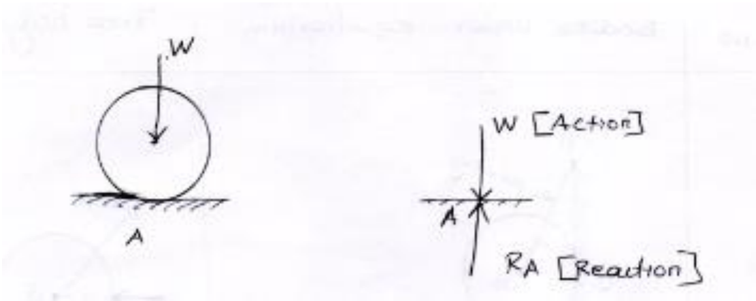
A body is said to be in natural equilibrium it in occupies a new position (also remain at rest) atter slightly displaced from its position of rest.

⇒Free body Diagram:

It is a sketch of the particle which represents it as being isolated from its surroundings. It represents all the forces acting on it.



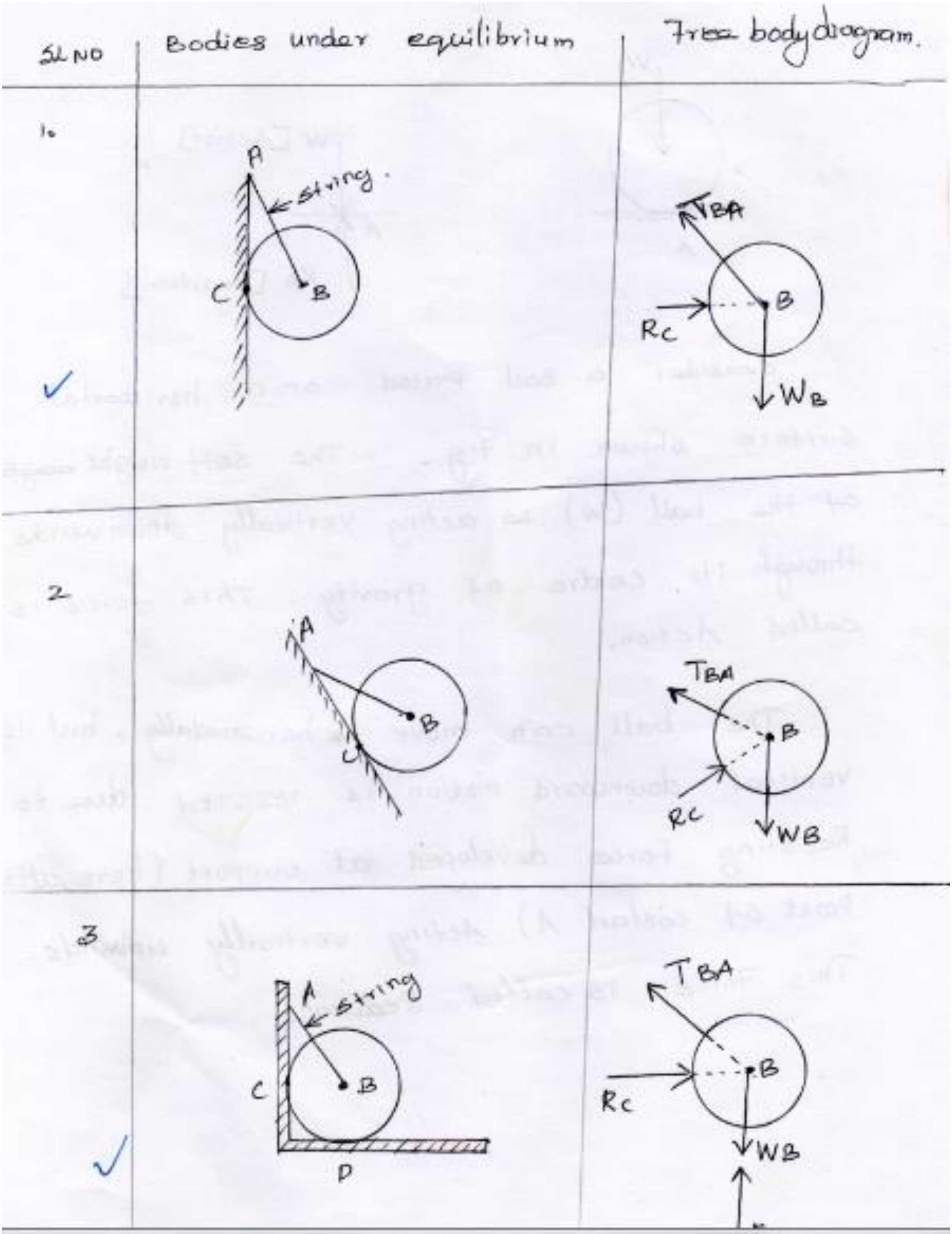
⇒Action and Reaction:



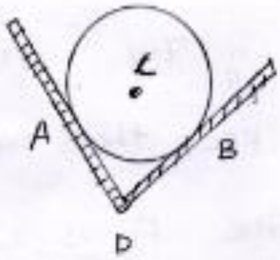
Consider a Ball placed on a horizontal surface shown in fig. The self weight of the ball (w) is acting vertically downwards through its centre of gravity. This force is called Action.

The ball can move horizontally, but its vertical downward motion is resisted due to resisting force developed at support (here, at the point of contact A) Acting vertically upwards. This force is called reaction.

⇒Free body diagram:



4.



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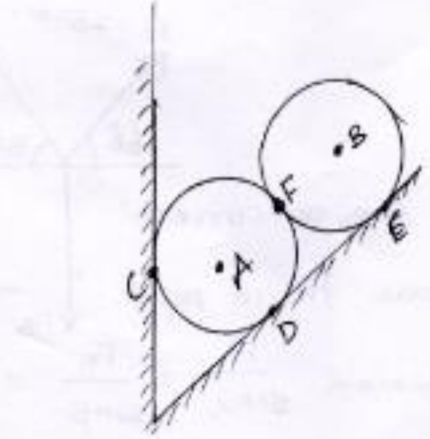
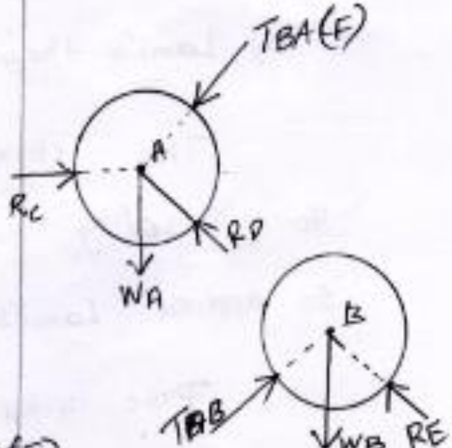
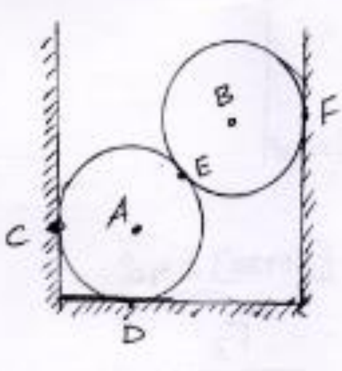
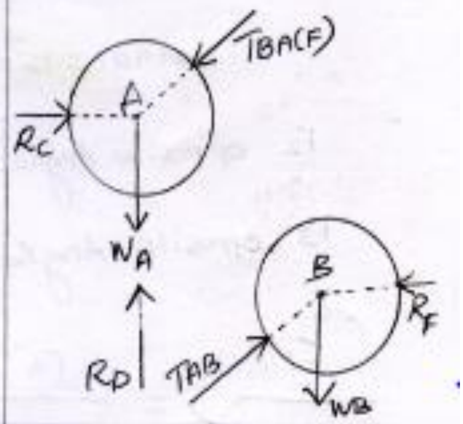




TBA(F)=TAB(F)

6





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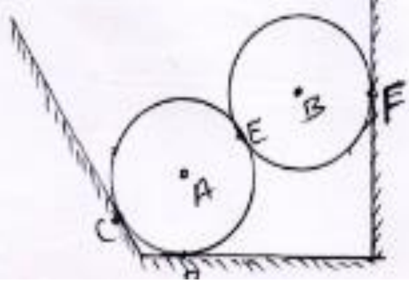
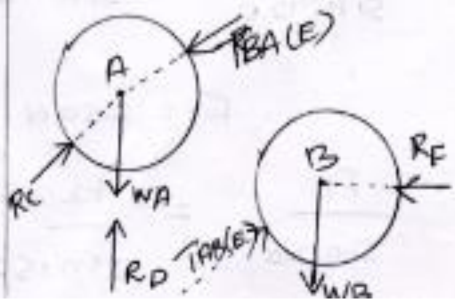
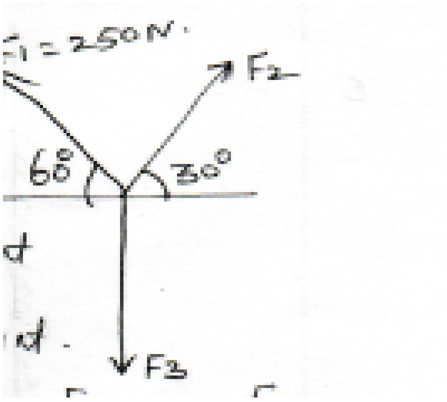




Problems:

1. The force shown in fig. is acting on a particle and keeps the particle in equilibrium. Then magnitude of force F1 is 250 N. Find the magnitude of forces F2 and F3.



Soln:

1. By Lami's theorem

The three concurrent force acting outwards from a point. So applied Lami's theorem. $\frac{F1}{sin\alpha} = \frac{F2}{sin\beta} = \frac{F3}{sin\gamma}$

First find the opposite angle of F1& F2&f3

F1 opposite angle

30+90=120°

F2 opposite angle (90+60)=150°

F3 opposite angle (180-[60+30]=90°

$$\frac{F1}{sin120} = \frac{F2}{sin150} = \frac{F3}{sin90}$$

F1=250 N

$$\frac{F1}{sin120} = \frac{F2}{sin150}$$

$\frac{250}{sin120} = \frac{F2}{sin150}$

F2=144.33 N

$\frac{F1}{sin120} = \frac{F3}{sin90}$

F3=288.67 N

2) By Equations of Equilibrium

∑FH=0 ∑FV=0 F3 is No horizontal force

1) ∑FH= F2 cos30-F1 cos60=0

F2 cos30-250 cos60=0

F2=$\frac{250cos60}{cos30}$

F2=144.33N

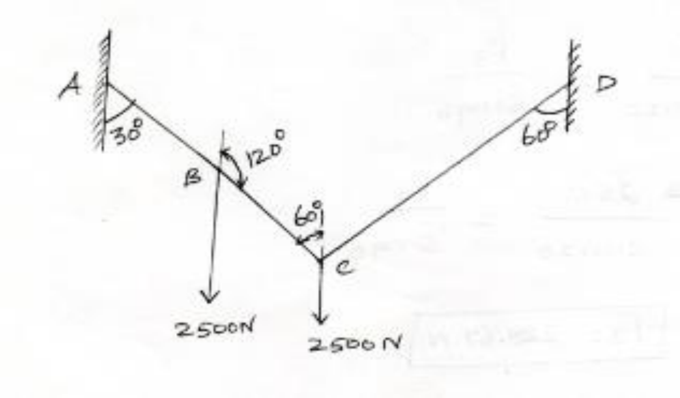
∑FH= F1 sin 30+ F2 sin 60-F3=0

250sin30+144.33 sin 60-F3=0

250 sin30+144.33 sin 60=F3

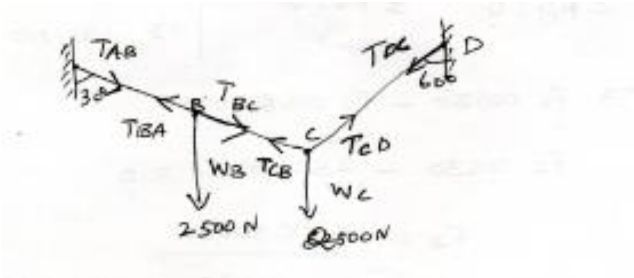
F3=228.67N

2. Two equal loads of 2500N are supported by a flexible string ABCD at points A &D. Find the tension in the portions AB,BC & CD of string.



Soln:

Free body diagram



$T_{AB} = T_{BA}$

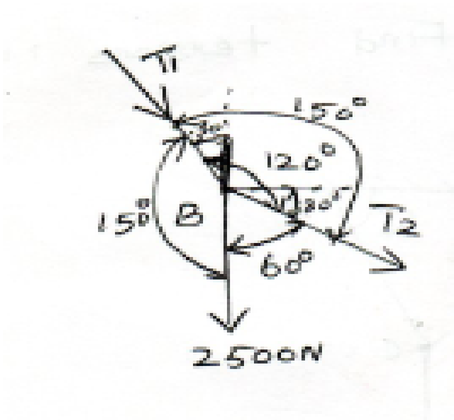
III ly$T_{BC} = T_{CB}$&$T_{CD} = T_{DC}$

⇒Let the tension in AB/BC &CD be T_1, T_2 & T_3 respectively

⇒ Let us split up the string

ABCD into two parts.

Consider A and B



By Lami's Theorem

$$\frac{TBA}{sin60} = \frac{TBC}{sin150} = \frac{Z500}{sin150}$$

$$\frac{TBA}{sin60} = \frac{2500}{sin150}$$

$$T_{BA} = \frac{2500}{sin150} \times sin60$$

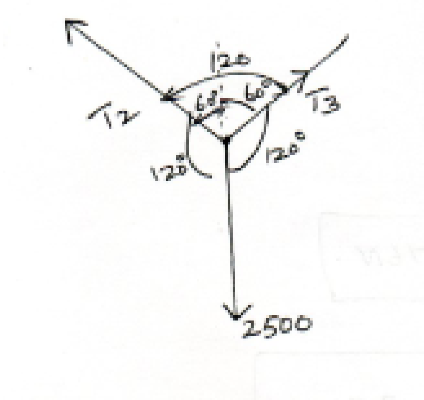
$$T1 = 4330.13\ N$$

$$\frac{T_{CB}}{sin150} = \frac{2500}{sin150}$$

$$T2 = \frac{2500}{sin150} \times sin150$$

$$T2 = 2500\ N$$

Consider a point c

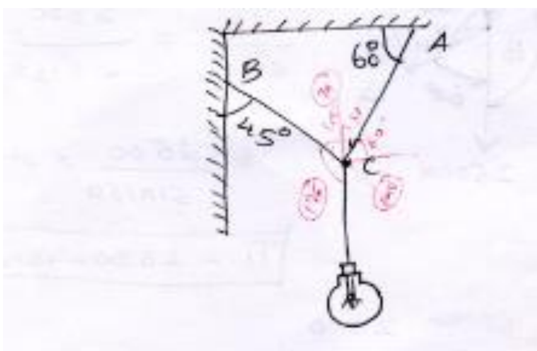


$$\frac{T2}{sin120} = \frac{T3}{sin120} = \frac{Z500}{sin120}$$

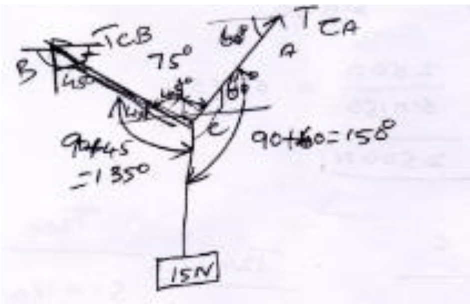
$$\frac{T3}{sin120} = \frac{2500}{sin120}$$

$$T3 = 2500\ N$$

3. An electric lamp weighting 15 N hangs from a point c, by $ two strings AC and Bc as shown in fig. find tensions in string Ac & BC



Soln:



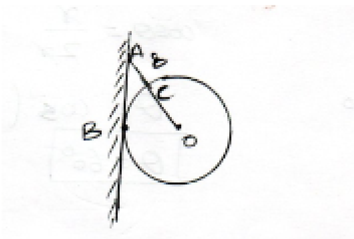
By Lami's theorem

$$\frac{TCB}{sin150} = \frac{TCA}{sin135} = \frac{15}{sin75}$$

$$\frac{TCB}{sin150} = \frac{15}{sin75} = TCB = 7.76\ N$$

$$\frac{TCA}{sin135} = \frac{15}{sin75} = TCA = 7.76N$$

4. A smooth sphere w is supported by a string fastened to a point A on the smooth vertical wall, the other end is in contact with point b, on the wall as shown in fig. If the length of the string AC is equal to the radius of the sphere, find the tension in the string & reaction of the wall.



Given:

Radius of sphere OB=OC=radius length of string, AC= radius of sphere=r , weight of sphere =w

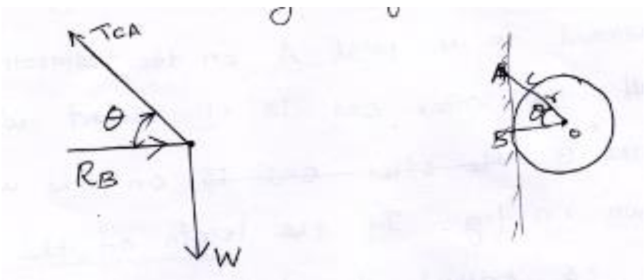
To find :

1. Tension in string

2. Reaction of the wall

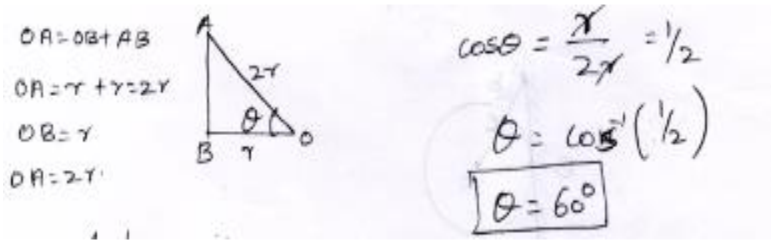
Soln:

Free body diagram



Find the angle b/w T_{CA}& R_B

From right angle triangle AOB



Apply ∑FH=0

R_B-T_{CA} cos 60 = 0-------------- (1)

∑FV=0

T_{CA} sin 60- w=0-------------- (2)

T_{CA} sin60=w

$T_{CA}=\frac{w}{sin60}$

T_{CA}= 1.155w

T_{CA} value sub in Eqn (1)

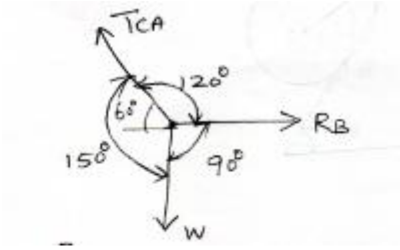
R_B-T_{CA} cos 60 = 0-------------- (1)

R_B-1.155w cos60=0

R_B=0.577w

Other method

By Lami's theorem

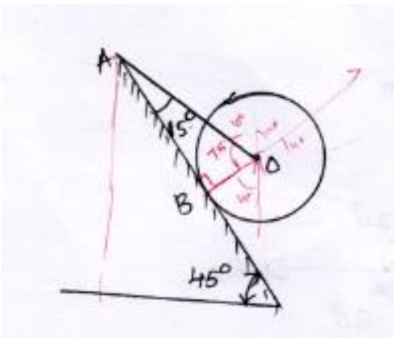


$$\frac{TCA}{sin90} = \frac{RB}{sin150} = \frac{w}{sin120}$$

$\frac{TCA}{sin90}$=$\frac{w}{sin120}$⇒R_B=$\frac{w}{sin120} \times sin150$

R_B=0.577 W

5. String AO holds a smooth sphere on an inclined plane ABC as shown in fig. the weight of the sphere is 1000 N and the plane is smooth. Calculate the tension in the string and the reaction at the point of contact B.



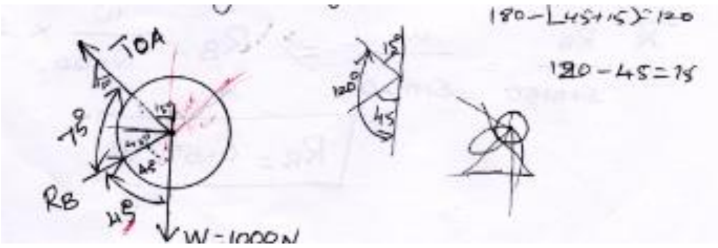
Given:

Weight of sphere W=1000N

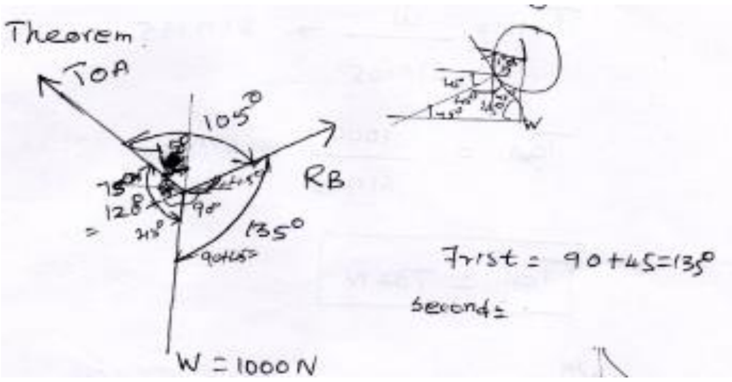
To find: Tension in string 7 Reaction

Soln:

Free body diagram



No of force is 3, so by using Lami's theorem



In right angled triangle OAB

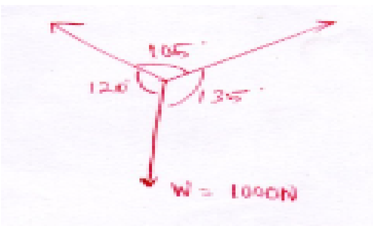
$\measuredangle OAB + \measuredangle ABO + \measuredangle BOA = 180°$ $\qquad \measuredangle OAB = 15°$

$15 + 90 + \measuredangle BOA = 180°$ $\qquad \measuredangle ABO = 90°$

BOA=180-(15+90)

$\measuredangle BOA = 75°$

Apply Lami's Eqn



$$\frac{TOA}{sin135} = \frac{RB}{sin120} = \frac{w}{sin105}$$

$$\therefore \frac{TOA}{sin135} = \frac{w}{sin105}$$

$T_{OA} = \frac{w}{sin105} \times sin135$

$T_{OA} = \frac{1000}{sin105} \times sin135$

T_{OA}= 732N

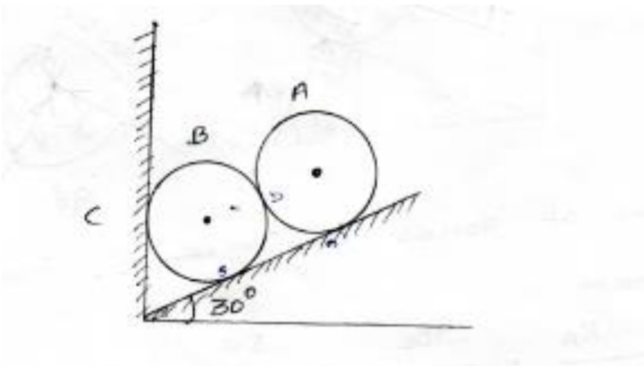
III $^{ly}\frac{R_B}{sin120} = \frac{w}{sin105}$

$\frac{R_B}{sin120} = \frac{1000}{sin105}$

$R_B = \frac{1000}{sin105} \times sin120$

R_B = 896.57N

6. Two identical rollers, each of weight 50 N, are supported by an inclined plane and a vertical wall as shown in fig. Find the reactions at the points of supports A,B and C. assume all the surface to be smooth.



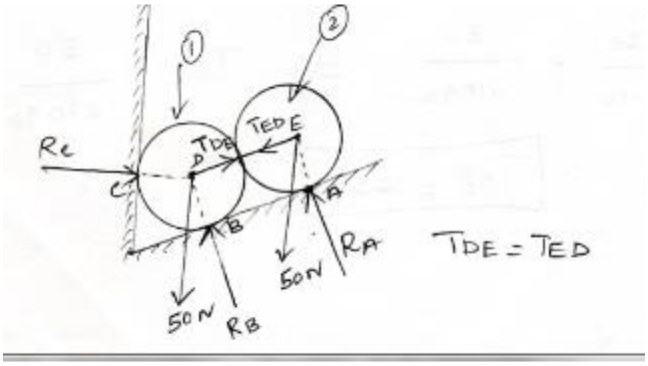
Given:

Weight of roller A & B = 50 N

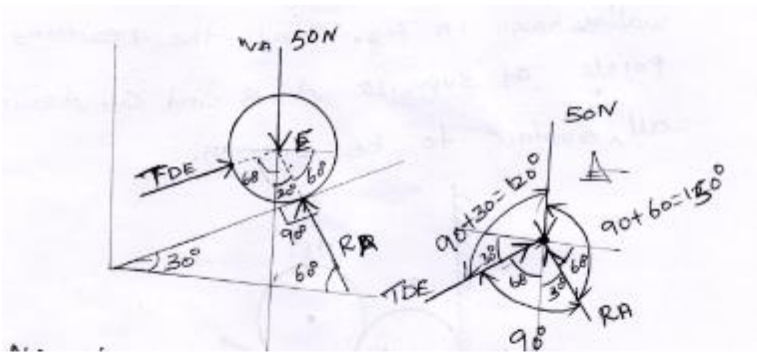
To find:

Reaction at the support A,B and C

Soln:



Free body diagram of roller 2



No of forces is three, apply Lami's Theorem

$$\frac{R_A}{sin120} = \frac{T_{DE}}{sin150} = \frac{50}{sin90}$$

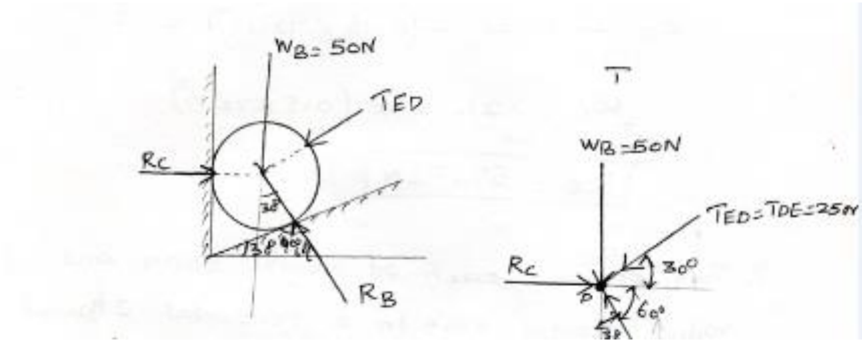
$$\frac{R_A}{sin120} = \frac{50}{sin90} \Rightarrow R_A = \frac{50}{sin90} \times sin120$$

$R_A = 43.3\ N$

$\frac{T_{DE}}{sin150} = \frac{50}{sin90} \Rightarrow T_{DE} = \frac{50}{sin90} \times sin150$

$T_{DE} = 25N$

Free body diagram of roller 1



All forces acting at point D.

In equilibrium condition

∑FH=0 &∑FH=0

∑FH=0 →+ - ←

R_C-TED cos 30-R_B cos 60 =0

R_C-25 cos 30-R_B cos 60 =0

R_C-21.65-0.5R_B =0-------- (1)

∑FH=0 ↓- ↑+

$R_B$sin60- TED sin 30-50 =0

R_B-sin60-25 sin30-50=0

R_B-sin 60 =25 sin 30+ 50=62.5

$R_B = \frac{62.5}{sin60}$

$R_B = 72.17$ N

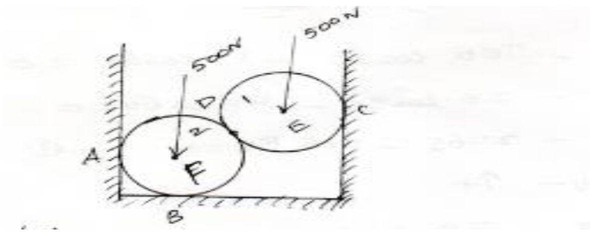
R_B value substitute Eqn(1)

$R_C - 21.65 - (0.5 \times 72.17) = 0$

$R_c = 21.65 + (0.5 \times 72.17)$

$R_C = 57.73\ N$

7. Two spheres each of weight 500 N and of radius 100mm rest in a horizontal channel of width of 360mm as shown in fig. find the reactions on the points of contact A ,B and C. Assume all the surface of contact are smooth.



Given data:

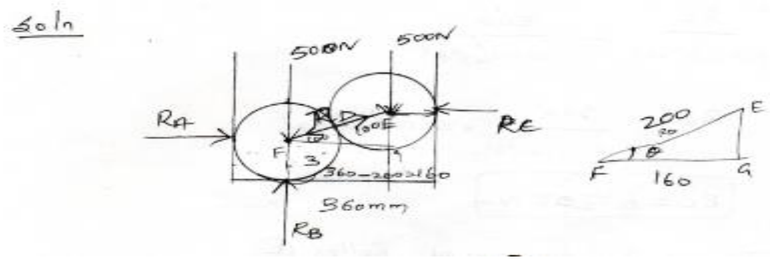
Weight of each Roller w = 500 N

Width of channel = 360 mm

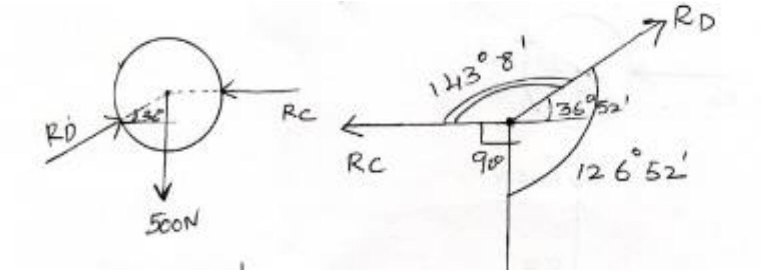
Radius of rollers r = 100mm

To find:

Reaction on the points of A,B & C.



Free body diagram of roller (1)



$\cos\theta = \frac{FG}{EF}$

$$\theta = cos^{-1}\left(\frac{FG}{EF}\right)$$

$$\theta = cos^{-1}\left(\frac{160}{200}\right)$$

$$\theta = 36°52'$$

By Lami's Theorem

$$\frac{R_D}{sin90} = \frac{R_C}{sin126°52} = \frac{500}{sin143°8}$$

$$\frac{R_D}{sin90} = \frac{500}{sin143°8}$$

$$R_D = \frac{500}{sin143°8} \times sin90$$

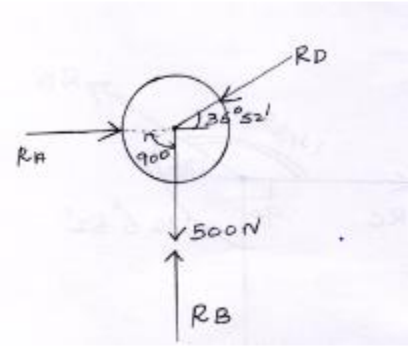
$$R_D = 833.39\ N$$

$$\frac{R_C}{sin126°52} = \frac{500}{sin143°8}$$

$$R_C = \frac{500}{sin143°8} \times sin126°52'$$

$$R_C = 673.08N$$

Free body diagram of roller(2)



∑FH = $R_A - R_D \cos 36°52'$=0

→+ - ← $R_A = R_D \cos 36°52'$

$R_A - 833.39 \times \cos 36°52'$

$R_A = 666.74\ N$

∑FV =0 ↓- ↑+

$R_B - R_D \sin 36°52' - 500$=0

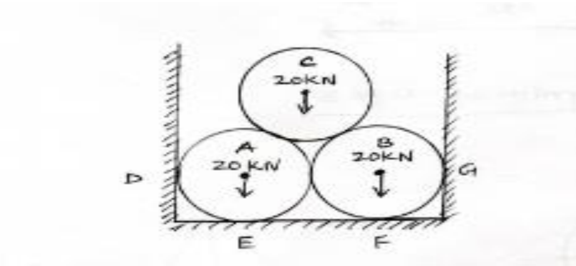
$R_B - 833.39 \sin 36°52'$-500=0

$R_B - 499.99 - 500$=0

$R_B = 999.99N$

8. Three smooth pipes each weighting 20 KN and of diameter 60 cm are to be placed in a rectangular channel with horizontal base as shown in fig.

   Calculate the reactions at the points of contact b/w the pipes and b/w the channel and the pipes. Take width of the channel as 160 cm.



Given:

Weight of each pipe = $w_A = w_B = w_C = 20\ KN$

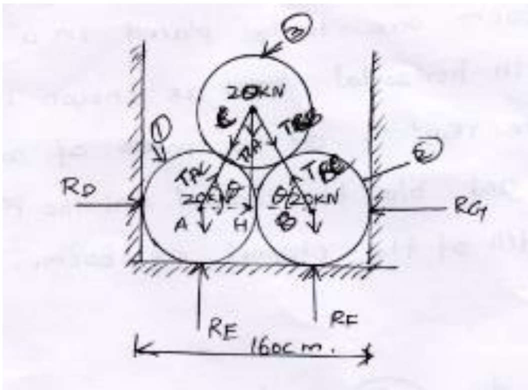
Diameter of each pipe = $D_A = D_B = D_C = 60cm$

Width of channel = 160 cm

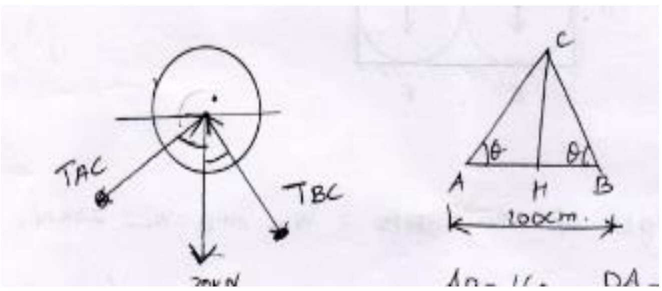
To find:

Reaction at the points : D,E,f,G

Soln:



Free body diagram of pipe 3



AB=160-DA-BG diameter

From triangle HAC AB=160-30-30 DA=Db=60cm

$\cos\theta = \frac{AH}{AC}$ AB=100cm

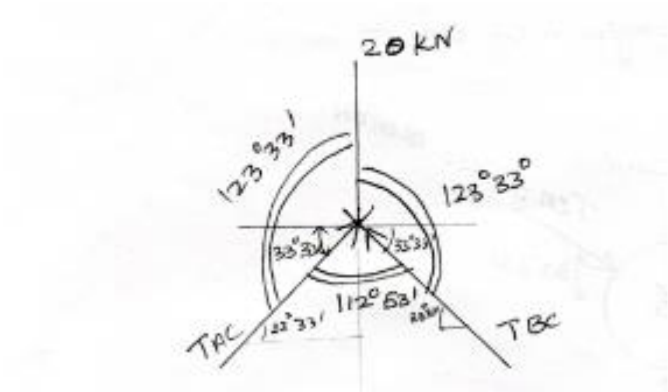
$\theta = cos^{-1}\left[\frac{AH}{AC}\right]$ AC=BC BG=DB/2

AC=BC=2×radius DA=BG=radius

$= cos^{-1}\left[\frac{50}{60}\right]$ AC=BC=2× 30

$\theta = 33°33'$ AC=BC=60 cm

AH=$\frac{AB}{2} = \frac{100}{2} = 50cm$



By Lami's theorem

$$\frac{T_{AC}}{sin123°33\prime} = \frac{T_{BC}}{sin123°33\prime} = \frac{20}{sin112°53\prime}$$

$$\frac{T_{AC}}{sin123°33\prime} = \frac{20}{sin112°53\prime}$$

$$T_{AC} = \frac{20}{sin112°53\prime} \times sin123°33'$$

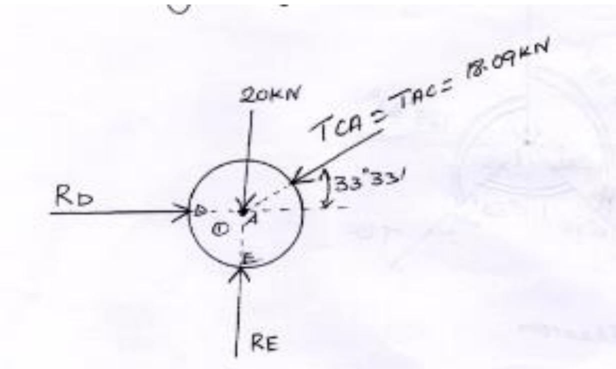
T_{AC} =18.09 KN

$$\frac{T_{BC}}{sin123°33\prime} = \frac{20}{sin112°53\prime}$$

$$T_{BC} = \frac{20}{sin112°53\prime} \times sin123°33'$$

$T_{BC} = 18.09KN$

Free body diagram of pipe(1)



∑FH = 0 →+ - ←

$R_D - T_{CA}\cos 33°33'$=0

$R_D = 18.09\cos 33°33' = 0$

$R_D - 15.07 = 0$ $T_{CA} = T_{Ac}$

$R_D = 15.07\ KN$

∑FV =0 ↓- ↑+

$R_E - 20 - T_{CA}\sin 33°33'$=0

$R_E - 20 - 18.09\sin 33°33'$=0

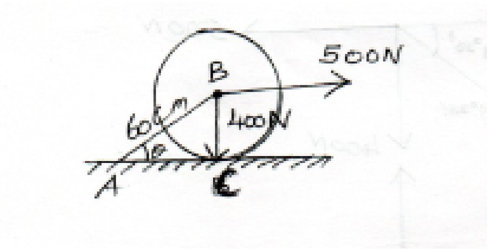
$R_E - 29.99$ =0

$R_E = 29.99KN$

Similarly the free body diagram of pipe (2) is analyzed for pipe (1)

$\therefore R_E = 29.99KN$ &$R_D = 15.07\ KN$

9. A circular roller of radius 20 cm and of weight 400 N resets on a smooth horizontal surface and is held in position by an inclined bar AB of length 60 cm as shown in fig. a horizontal force of 500 n is acting at b. Find the Tension in bar AB and the reaction at C.



Given:

Radius of circular roller r=20 cm

Weight of roller w = 400 N

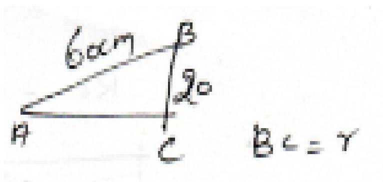
Horizontal force =500 N

To find:

1) Tension in AB
2) Reaction at C.

Soln:

From triangle ABC

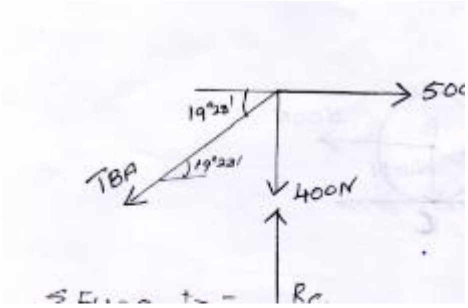


$\sin\theta = \frac{BC}{AB} \Rightarrow \theta = sin^{-1}\left(\frac{BC}{AB}\right)$

$\theta = sin^{-1}\left(\frac{20}{60}\right)$

$\theta = 19°28'$

Free body diagram



$\sum FH = 0 \quad \rightarrow + \quad - \leftarrow$

$\sum FH = -T_{BA} cos 19°28' + 500 = 0$

$-T_{BA} = cos 19°28' = -500$

$-T_{BA} = \frac{-500}{cos 19°28}$

$-T_{BA} = 530.31N$

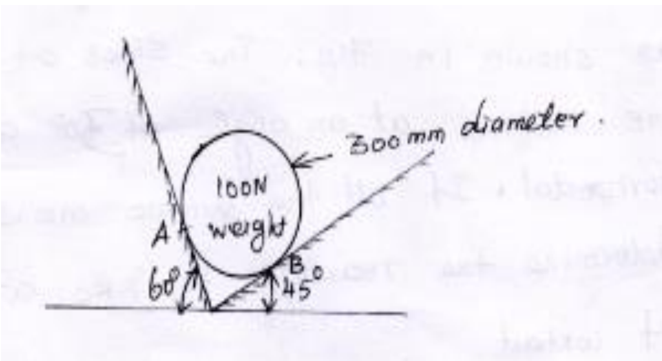
$\sum FV = 0 \ \downarrow - \ \uparrow +$

$-400 + R_c - T_{BA} \sin 19°28' = 0$

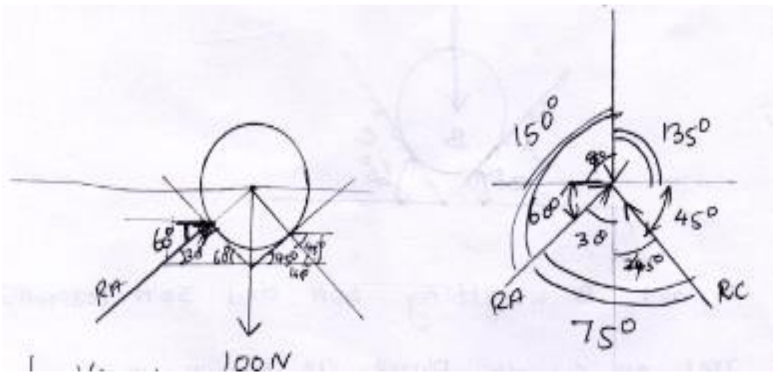
$R_c = T_{BA} sin 19°28' + 400 = 530.31 \times sin 19°28' + 400$

$R_C - 576.62N$

10.Determine the reaction at a and B



Soln:



By Lami’s theorem

$$\frac{R_A}{sin135} = \frac{R_C}{sin150} = \frac{100}{sin75}$$

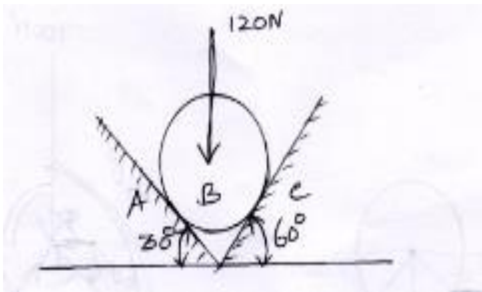
$$\frac{R_A}{sin135} = \frac{100}{sin75} \Rightarrow R_A = \frac{100}{sin75} \times sin135$$

$$R_A = 73.2N$$

$$\frac{R_C}{sin135} = \frac{100}{sin75} \Rightarrow R_C \frac{100}{sin75} \times sin150$$

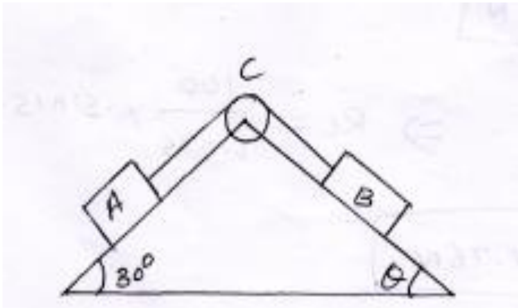
$$R_C = 51.76N$$

11. A Ball weighht120N in a right angle groove as shown in fig. The sides of the groove are inclined at an angle of $30° and\ 60°$ to the horizontal. If all the surface are smooth,then determine the reaction RA&RC at the point of contact



12. A and B weighting 40 N and 30 N respectively rest on smooth planes as shown in fig. They are connected by a weight less chord passing over

a friction less pulley. Determine the angleθ &the tension in the chord for equilibrium. Also find the reaction of Block A&B



Given:

WA=40N

WB=30N

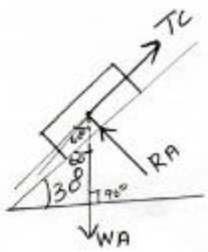
To find

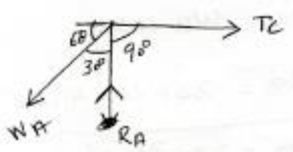
1.θ

2. Reaction of Block A&B

Soln

F B D of Block A





$\sum FH = 0$

TC-WA cos 60=0

TC = WA cos 60

TC = 40 cos 60

TC = 20N

$$\sum FV = 0$$

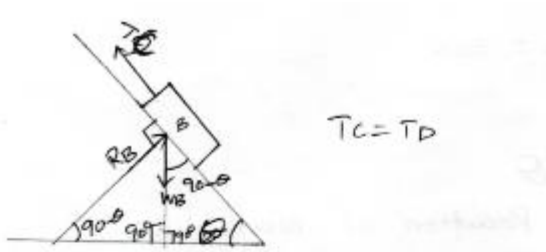
+RA - WA sin 60=0

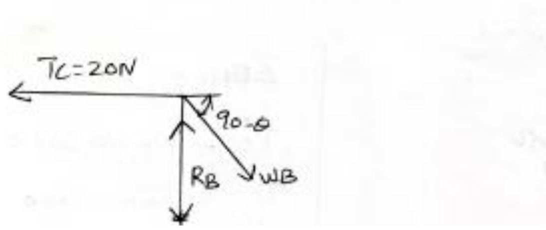
+ RA = WA sin 60

RA = 40 sin 60

RA = 34.64N

FBD of Block B





$\sum FH = 0$

$-T_C + W_B \times \cos(90-\theta)=0$

$-T_C = -W_B \cos(90-\theta)$

$\sum FV = 0$

$R_B - W_B \sin(90-\theta)=0$

$R_B = W_B \sin(90-\theta)$

$T_C = W_B \sin\theta$

$20 = 30\sin\theta$

$\sin\theta = \frac{20}{30}$

$\theta = sin^{-1}\left(\frac{20}{30}\right)$

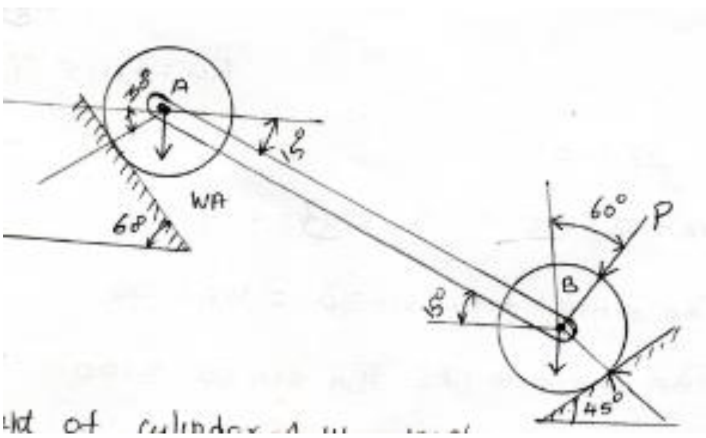
$\theta = 41°48'$

$R_B = W_B \cos\theta$

$R_B = 30\times cos41°48'$

$R_B = 22.36N$

13. The following fig shows cylinders, A of weight 100 N and B weight 50 N resting on smooth inclined planes. They are connected by a bar of negligible weight hinged to each cylinder at their geometric centers by smooth pins. Find the force P, that can hold the system in the given position.



Given:

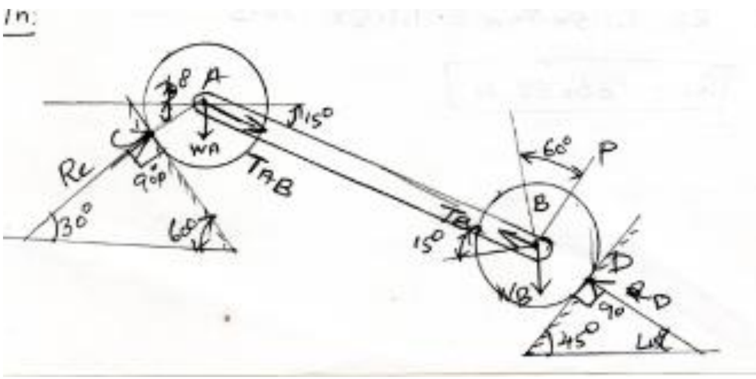
Weight of cylinder A $W_A = 100\ N$

Weight of cylinder B $W_B = 50\ N$

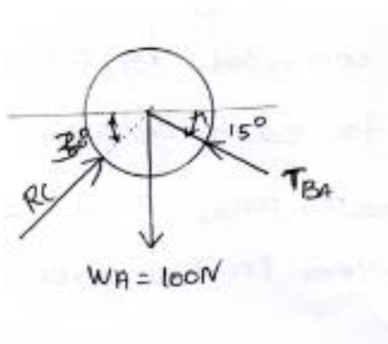
To find:

Force ‘P’

Soln:



Free body diagram for cylinder A



$\sum$FH = 0

$\sum$FH $= R_C cos30 - -T_{BA} cos15$=0

$R_C cos3 = T_{BA} cos15$

$R_C = \frac{T_{BA} cos15}{cos30}$

$R_C = 1.115 T_{BA}$---------------- (1)

$\sum$FV = 0

$\sum$FV - $W_A + T_{BA} sin15 + R_C sin30 = 0$

$T_{BA} sin 15 + R_C sin 30 = W_A = 0$

$T_{BA} sin 15 + 1.115\, T_{BA} sin 30 = 0$

$T_{BA}[sin 15 + 1.115 sin 30] = 100$

$T_{BA} = \frac{100}{sin 15 + 1.115 sin 30}$

$T_{BA} = 122.5\ N$

$R_C = 1.115 \times T_{BA} = 1.115 \times 122.5 R_C = 136.58 N$

<u>Free body diagram of cylinder B</u>

$\sum$FH = 0 →+ - ←

$T_{AB} cos 15 - P cos 30 - R_D 45 = 0$

$122.5\ cos 15° - P cos 30 - R_D cos 45 = 0$

$118.32 - 0.866P - 0.707 R_D$=0

$-0.866P - 0.707 R_D = -118.32$-------- (1)

$\sum$FV =0 ↓- ↑+

-$W_{B\_} P sin 30 - T_{AB} sin 15 + R_D sin 45 = 0$

$-50 - P \times sin 30 - 122.5 sin 15 + R_D sin 45 = 0$

-$50 - 0.5P - 31.7 + 0.707\ R_D = 0$

$-81.7 - 0.5P + 0.707\ R_D = 0$

$0.707\ R_D - 0.5P = 81.7$

$-0.5P + 0.707 R_D = 81.7$------------------ (2)

(1)⇒ $-0.866P - 0.707R_D = -118.32$

(2)⇒ $-0.5P + 0.707R_D = 81.7$

-0.366P = -36.62

P=$\frac{36.62}{0.366}$

P=100 N

Substitute in (1)

$-0.866P - 0.707R_D = -118.32$

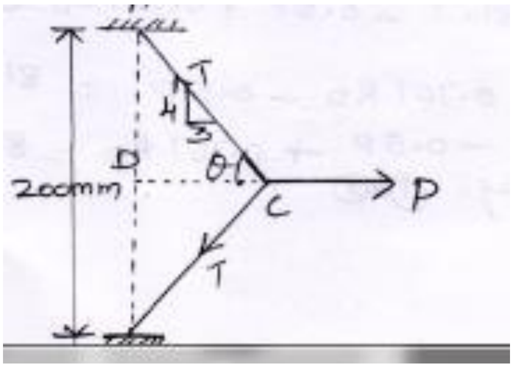
$-86.6 - 0.707 \times R_D = -118.32$

$-0.707R_D = -118.32 + 86.6 = -32$

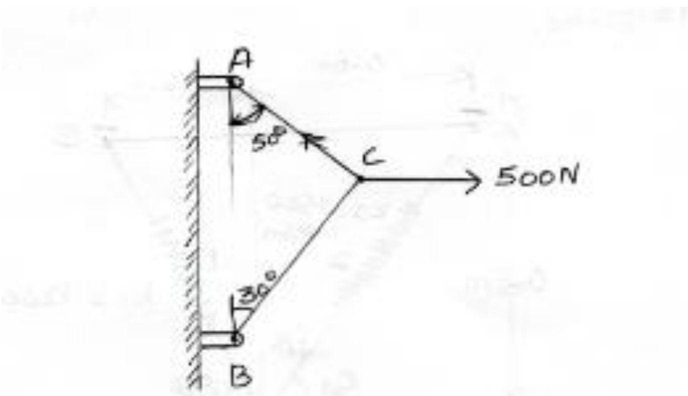
$R_D = \frac{-32}{-0.707}$

$R_D = 45.26N$

14. A rubber band has an unstretuched length of 200mm.It is pulled until its length is 250mm. as shown in fig. the horizontal force P is 1.75 n. what is the tension in the band (HW)



15. Two cables are tied together at c and are loaded as shown in fig below. Determine the tension in the cable AC and BC



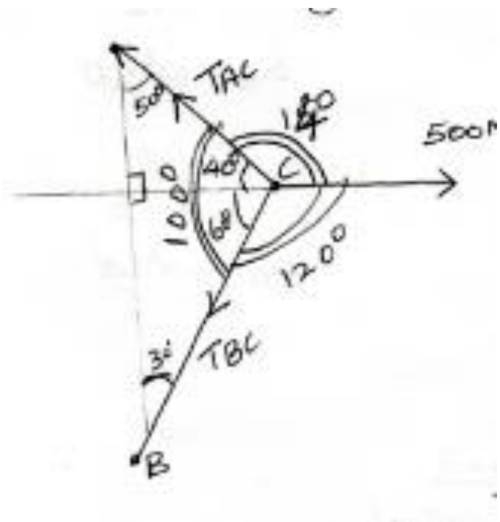
Given:

Force on C=500 N

To find: Tension of cable AC & Bc

Soln:

Free body Diagram



By using Lami's Theorem

$$\frac{T_{AC}}{sin120} = \frac{T_{BC}}{sin140} = \frac{500}{sin100}$$

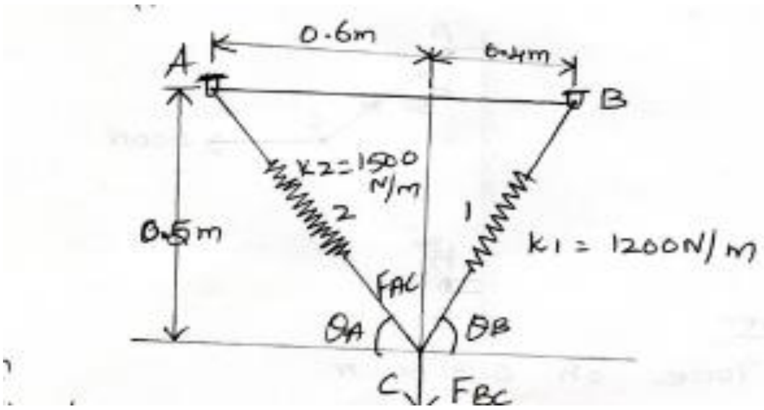
$$\frac{T_{AC}}{sin120} = \frac{500}{sin100} \Rightarrow T_{AC} = \frac{500}{sin100} \times sin120$$

$$T_{AC} = 439.69N$$

$$\frac{T_{BC}}{sin140} = \frac{500}{sin100} \Rightarrow T_{BC} \frac{500}{sin100} \times sin140$$

$$T_{BC} = 326.35N$$

16. A 30kg block is suspended by two spring having stiffness as shown. Determine the instructed length of each spring after the block is removed.

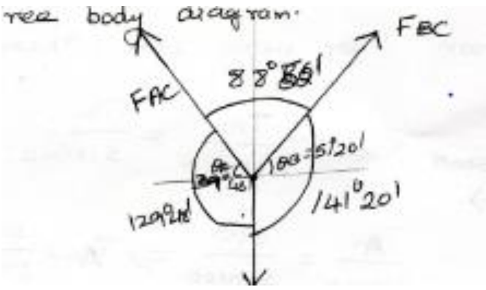


Unknown

Length of each spring L1&L2

Soln:

Free body diagram



$$= tan\theta_B = \frac{0.5}{0.4}$$

$$\theta_B = 51°20'$$

$$= tan\theta_A = \frac{0.5}{0.6}$$

$$\theta_A = 39°48'$$

F=30 kg=30× 9.81

=294.3N

By Lami's theorem $\frac{F_{AC}}{sin141°20\prime} = \frac{F_{BC}}{sin129°48\prime} = \frac{294.3}{sin88°52\prime}$

$$\frac{F_{AC}}{sin141°20'} = \frac{294.3}{sin88°52'} \Rightarrow F_{AC} = \frac{294.3}{sin88°521} \times sin141°20'$$

$$F_{AC} = 183.91N$$

$$\frac{F_{BC}}{sin129°48\prime} = \frac{294.3}{sin88°52\prime}$$

$$F_{BC} = \frac{294.3}{sin88°52\prime} \times sin129°48'$$

$$F_{BC} = 226.15N$$

Stiffness =$\frac{Force}{Deflection}$

$$k_2 = \frac{F_{AC}}{\delta_2}$$

1500=$\frac{183.91}{\delta_2}$

$$\delta_2 = 0.122m$$

$$k_1 = \frac{F_{BC}}{\delta_1}$$

1200=$\frac{226.15}{1}$

$$\delta_2 = 0.188m$$

L_1=$\sqrt{(0.4)^2 + (0.5)^2} = 0.6403m$

L_2 =$\sqrt{(0.6)^2 + (0.5)^2} = 0.7810m$

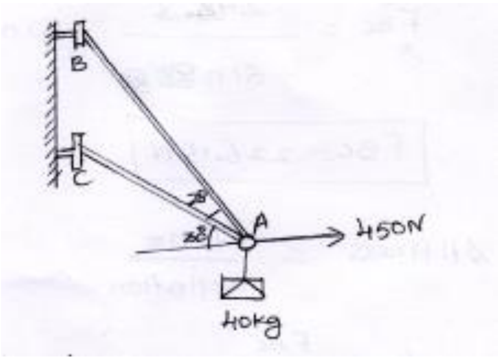
$$l_1 = L_1 - \delta_1 = 0.6403 - 0.188$$

$$l_1 = 0.452m$$

$$l_2 = L_2 - \delta_2 = 0.7810 - 0.122$$

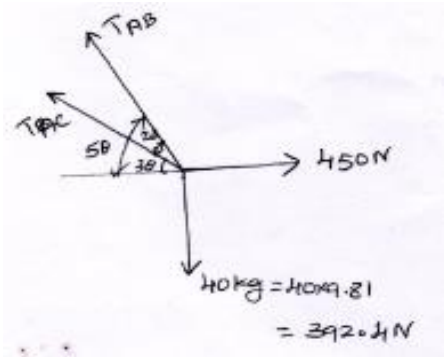
$l_2 = 0.728m$

17. Determine th3e tension in cables AB and AC required to hold the 40 kg crate shown in fig. below.



To find: T_{AB} & T_{AC}

Soln:



$\sum FH = 0 \quad \rightarrow + \quad - \leftarrow$

$450 - T_{AB}cos50 - T_{AC}cos30 = 0$

$T_{AB}cos50 + T_{AC}cos30 = 450$

$0.64T_{AB} + 0.86T_{AC} = 450$----------(1)

$\sum FV = 0 \;\downarrow - \;\uparrow +$

$-392.4 + T_{AB} sin50 + T_{AC} sin30 = 0$

$T_{AB} sin50 + T_{AC} sin30 = 392.4$

$0.76T_{AB} + 0.5T_{AC} = 392.4$---------- (2)

Solving Eq(1)&(2)

(1) $\Rightarrow 0.64T_{AB} + 0.86T_{AC} = 450$

(2)$\times$ 1.72 $\Rightarrow 0.76T_{AB} + 0.85T_{AC} = 392.4$

-0.46T_{AB} = -224.92

$$T_{AB} = \frac{224.92}{0.46}$$

$$T_{AB} = 488.95N$$

T_{AB} *sub in eqn*(1)

$0.64 \times 488.95 + 0.86T_{AC} = 450$

$$T_{AC} = 159.38N$$

Chapter-4

Forces in space –Resultant and Equilibrium of particles in Three Dimensions [Vector approach]

Quantities:

Physical Quantities are

i) Scalar quantity
ii) Vector quantity

Scalar Quantity:

Scalar quantity are those which are completely defined by their magnitude only.

Ex. 2kg of mass

25℃ of temperature

10 m/s acceleration

Vector Quantity

The Quantity which are defined by their magnitude and direction is known as vector quantity.

Ex. 10 n force acting vertically downward direction

9.81 m/s^{2}acceleration directed towards is centre of the earth.

Types of Vectors

1. Free Vector
2. Fixed Vector
3. Sliding Vector

4. Unit Vector
5. Zero(or) Null Vector
6. Equal Vector
7. Like Vector

1. **Free Vector:**

If the vector may act at any point in space maintaining some magnitude and direction with no specific point of action is called Free vector.

2. **Fixed Vector:**

The vector whose point of action is same is called Fixed vector.

3. **Sliding vector:**

The vector may be applied at any point along its Line of action is called sliding vector.

4. **Unit vector:**

A vector whose magnitude is unity is called unit vector.

AB, n= $\frac{\overrightarrow{AB}}{|\vec{A}||\vec{B}|}$

5. **Zero (or) Null vector**

It is defind as the vector whose magnitude is zero.

6. **Equal vector**

Those vector which are similar to each other but have same magnitude and direction in same and equal is called equal vector.

7. **<u>Like vector:</u>**

These vector each are slimier to each other and have same direction and unequal magnitude is called like vector.

8. **<u>Vector Addition:</u>**

We Law of vector addition are

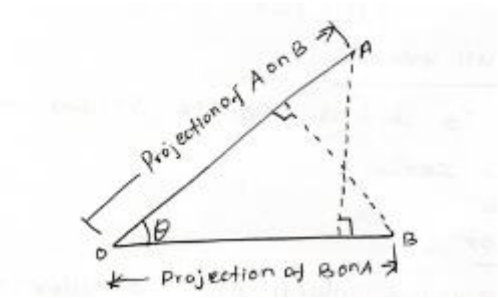
A+B=B+A [commutative Law]

A+ [B+C] =A+B+C [associative Law]

<u>Vector Product</u>

1. Scaler product(or) dot product
2. Vector product(or) cross product

1. **<u>Scaler product (or) dot product:</u>**



$$\vec{A}.\vec{B} = |\vec{A}||\vec{B}|\, cos\theta$$

When the angle b/w two vector $\vec{A}$&$\vec{B}$

$$cos\theta = \frac{\vec{A}.\vec{B}}{|\vec{A}|\,.|\vec{B}|}$$

(i) When $\theta = 0°$

Then $\vec{A}.\vec{B} = |\vec{A}||\vec{B}|$

That is the two vector are in same direction.

(ii) when $\theta = 90° \vec{A}.\vec{B} = 0$

so the vectors are perpendicular

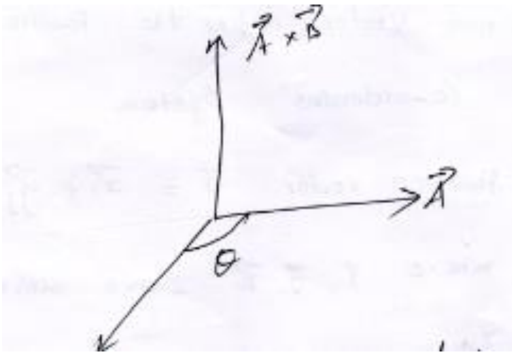
(iii) $\vec{A}.\vec{B} = |A|$

When the projection B or A

If the angle b/w the A & B given

$cos\theta = \frac{\vec{A}.\vec{B}}{AB}$

2. Vector Product (or) cross product



In forms of Rectangular component A=$A_{xi} + A_{yi} + A_{zk}$

B= $B_{xi} + B_{yi} + B_{zk}$

$$A\times B=\begin{vmatrix} i & j & k \\ Ax & Aj & Az \\ Bx & By & Bz \end{vmatrix}$$

The Angle b/w the vector is given by

$$sin\theta = \frac{|\vec{A} \times \vec{B}|}{|\vec{A}||\vec{B}|}$$

Dot product of force and displacement given workdone.

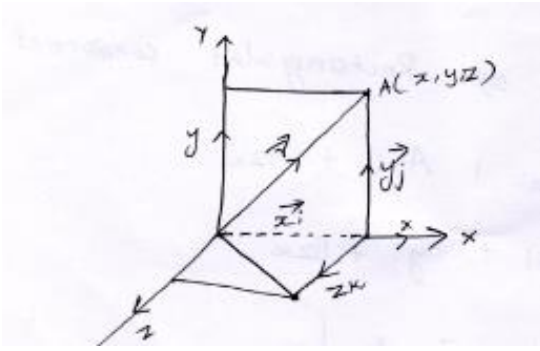
∴ Workdone=F_d

Position vector:

Position vector defines the position of points in any co-ordinate system.

Position vector $\vec{r}$= $\overrightarrow{x\imath}$+$\overrightarrow{y\jmath}$+$\overrightarrow{zk}$

Where $\vec{\imath}$, $\vec{\jmath}$, $\vec{k}$ – are unit vector



Magnitude $r = \vec{r} = \sqrt{x^2 + y^2 + z^2}$

Formula

Resultant vector $r = \vec{A} + \vec{B} + \vec{C}$

Unit vector to resultant vector $n = \frac{\vec{R}}{|\vec{R}|}$

$|R| = \sqrt{x^2 + y^2 + z^2}$

Magnitude $=\sqrt{x^2 + y^2 + z^2}$

Unit vector $n = \frac{\overrightarrow{AB}}{|\overrightarrow{AB}|}$

$\overrightarrow{AB} = (x_2 - x_1)\vec{\imath} + (y_2 - y_1)\vec{\jmath} + (z_2 - z_1)\vec{k}$

$|AB| = \sqrt{x^2 + y^2 + z^2}$

Dot product vector $\vec{A}.\vec{B} = [\vec{\imath} + \vec{\jmath} + \vec{k}].[\vec{\imath} + \vec{\jmath} + \vec{k}]$

Angle b/w the vector $cos\theta = \frac{\vec{A}.\vec{B}}{|\vec{A}||\vec{B}|}|$

Cross Product vector $A \times B = \begin{vmatrix} i & j & k \\ x1 & y1 & z1 \\ x2 & y2 & z2 \end{vmatrix}$

Angle b/w the vector $sin\theta = \frac{|\vec{A}\times\vec{B}|}{|\vec{A}||\vec{B}|}$

Problems:

1. Three vectors A, B, C are given as $A = 3\vec{\imath} + 2\vec{\jmath} + 4\vec{k}, B = 4\vec{\imath} - 2\vec{\jmath} + 6\vec{k}$

   $C = 2\vec{\imath} - 3\vec{\jmath} - \vec{k}$, find

   1. The resultant vector
   2. A unit vector || er top resultant vector

Given:

$A = 3\vec{\imath} + 2\vec{\jmath} + 4\vec{k}$

$B = 4\vec{\imath} - 2\vec{\jmath} + 6\vec{k}$

$C = 2\vec{\imath} - 3\vec{\jmath} - \vec{k}$

To find:

1. The resultant vector
2. A unit vector || ^{er} top resultant vector

Solution:

1. The resultant vector

$R = \vec{A} + \vec{B} + \vec{C}$

$R = 3\vec{i} + 2\vec{j} + 4\vec{k} + 4\vec{i} - 2\vec{J} + 6\vec{k} + 2\vec{i} - 3\vec{j} - \vec{k}$

$R = 9\vec{i} - 3\vec{j} + 9\vec{k}$

2. A unit vector || er top resultant vector

Unit Vector $n = \frac{\vec{R}}{|\vec{R}|}$

$\vec{R} = 9\vec{i} - 3\vec{j} + 9\vec{k}$

$|R| = \sqrt{9^2 + (-3)^2 + 9^2} = \sqrt{81 + 9 + 81}$

$|R| = \sqrt{171}$

$R = 13.08$

$$n = \frac{9\vec{i} - 3\vec{j} + 9\vec{k}}{13.08}$$

$$n = \frac{9}{13.08}\vec{i} - \frac{3}{13.08}\vec{j} + \frac{9}{13.08}\vec{k}$$

Unit vector$n = 0.68i - 0.22j + 0.68k$

2.If $A = \vec{i} - \vec{j} - 2\vec{k}, B = 3\vec{i} + 2\vec{J} - 2\vec{k}C = 2\vec{i} + 3\vec{j} - 4\vec{k}$, find

$2A - 2B + 3C$ n terms of i, j, k and its magnitude.

Given :

$A = \vec{i} - \vec{j} - 2\vec{k}$

$B = 3\vec{i} + 2\vec{J} - 2\vec{k}$

$C = 2\vec{i} + 3\vec{j} - 4\vec{k}$

To find:

$2A - 2B + 3C =$? magnitude

Solution:

$$2A - 2B + 3C = ?$$

$$2A = 2[\vec{i} - \vec{j} - 2\vec{k}]$$

$$2A = 2\vec{i} - 2\vec{j} - 4\vec{k}$$

$$2B = 2[3\vec{i} + 2\vec{j} - 2\vec{k}]$$

$$2B = 6\vec{i} + 4\vec{j} - 4\vec{k}$$

$$3C = 3[2\vec{i} + 3\vec{j} - 4\vec{k}]$$

$$3C = 6\vec{i} + 9\vec{j} - 12\vec{k}$$

$$2A - 2B + 3C = \left[2\vec{i} - 2\vec{j} - 4\vec{k}\right] - \left[6\vec{i} + 4\vec{j} - 4\vec{k}\right] + 6\vec{i} + 9\vec{j} - 12\vec{k}$$

$$= 2\vec{i} - 2\vec{j} - 4\vec{k} - 4\vec{j} + 4\vec{k} + 9\vec{j} - 12\vec{k}$$

$$2A - 2B + 3C = 2\vec{i} + 3\vec{j} - 12\vec{k}$$

$$2A - 2B + 3C = \sqrt{2^2 + 3^2 + (-12)^2} = \sqrt{4 + 9 + 144}$$

$$|2A - 2B + 3C| = \sqrt{157} = 12.53$$

$$|2A - 2B + 3C| = 12.53$$

3. Find the unit vector along the line which ordinates at point (2,3,-2) and passes through the point(1,0,5)

Given:

At point (2,3,-2)=(x_1,y_1,z_1)(1,0,5) = (x_2,y_2,z_2)

To find:Unit vector 'n'=?

Soln:

Unit vector $n = \frac{\overrightarrow{AB}}{|AB|}$

$AB = -1$

$A = x_1\vec{\imath} + y_1\vec{\jmath} + z_1\vec{k}$

$B = x_2\vec{\imath} + y_2\vec{\jmath} + z_2\vec{k}$

$$AB = (x_2 - x_1)\vec{\imath} + (y_2\text{-}y_1)\vec{\jmath} + (z_2 - z_1)\vec{k}$$

$$AB = (1 - 2)\vec{\imath} + (0\text{-}3)\vec{\jmath} + (5 - (-2))\vec{k}$$

$$AB = -1\vec{\imath} - 3\vec{\jmath} + 7\vec{k}$$

$$|AB| = \sqrt{(-1)^2 + (-3)^2 + (7)^2}$$

$$|AB| = \sqrt{1 + 9 + 49} = \sqrt{59}$$

$$|AB| = 7.68$$

$$n = \frac{\overrightarrow{AB}}{|\overrightarrow{AB}|} \frac{= -1\vec{\imath} - 3\vec{\jmath} + 7\vec{k}}{7.68}$$

$$n = \frac{-1}{7.68}\vec{\imath} - \frac{3}{7.68}\vec{\jmath} - \frac{7}{7.68}\vec{k}$$

$$\boldsymbol{ans}\; n = -0.13\vec{\imath} - 0.39\vec{\jmath} + 0.91\vec{k}$$

4. Find the dot product of two vector $A = 2\vec{\imath} - 6\vec{\jmath} - 3\vec{k}$, $B = 4\vec{\imath} + 3\vec{\jmath} - \vec{k}$ also find the angle b/w the angle b/w them.

Given Data:

$A = 2\vec{\imath} - 6\vec{\jmath} - 3\vec{k}$

$b = 4\vec{\imath} + 3\vec{\jmath} - \vec{k}$

To find:

1. Dot product of Two vector
2. Angle b/w the vector

Soln:

1. Dot product of two vector

$A.B = [2\vec{\imath} - 6\vec{\jmath} - 3\vec{k}].[4\vec{\imath} + 3\vec{\jmath} - \vec{k}]$

$= 2 \times 4 + [-6] \times 3 + [-3] \times [-1]$

$= 8 - 18 + 3$

$A.B = -7$

2. Angle b/w two vector

$Cos\theta = \frac{A.B}{|A||B|}$

$|\vec{A}| = \sqrt{(2)^2 + (-6)^2 + (-3)^2}$

$|\vec{A}| = \sqrt{4 + 36 + 9}$

$|\vec{A}| = 7$

$|\vec{B}| = \sqrt{(4)^2 + (3)^2 + (-1)^2} = 16 + 9 + 1 = \sqrt{16 + 9 + 1}$

$$\left|\vec{B}\right| = \sqrt{26}$$

$$cos\theta = \frac{-7}{7\sqrt{26}} = \frac{-1}{\sqrt{26}}$$

$$\theta = cos^{-1}\left[\frac{-1}{\sqrt{26}}\right]$$

5. Find the cross product of vector $A = 2\vec{\imath} - 6\vec{\jmath} - 3\vec{k}, B = 4\vec{\imath} + 3\vec{\jmath} - \vec{k}$ and the angle b/w them.

Given:

$A = 2\vec{\imath} - 6\vec{\jmath} - 3\vec{k}$

$B = 4\vec{\imath} + 3\vec{\jmath} - \vec{k}$

To find:

1. Cross product of vector
2. Angle b/w hem two vector

Soln:

Cross product:$A \times B$

$$\vec{A} \times \vec{B} = \begin{vmatrix} \vec{\imath} & \vec{\jmath} & \vec{k} \\ 2 & -6 & -3 \\ 4 & 3 & -1 \end{vmatrix}$$

$$= \vec{\imath}[(-6 \times -1) - (-3 \times 3)] - \vec{\jmath}[(2 \times -1) - (4 \times -3)]$$

$$+\vec{k}[(2 \times 3) - (4 \times -6)$$

$$\vec{\imath}[6 + 9] - \vec{\jmath}[-2 + 12] + \vec{k}[6 + 24]$$

$$\vec{A} \times \vec{B} = 15\vec{\imath} - 10\vec{\jmath} + 30\vec{k}$$

$$sin\theta = \frac{|\vec{A} \times \vec{B}|}{|\vec{A}||\vec{B}|}$$

$$\left|\vec{A} \times \vec{B}\right| = \sqrt{15^2 + (-10)^2 + (30)^2}$$

$$\left|\vec{A} \times \vec{B}\right| = \sqrt{1225}$$

$$\left|\vec{A} \times \vec{B}\right| = 35$$

$$\left|\vec{A}\right| = \sqrt{2^2 + (-6)^2 + (-3)^2} = \sqrt{4 + 36 + 9}$$

$$\left|\vec{A}\right| = \sqrt{49}$$

$$\left|\vec{A}\right| = 7$$

$$\left|\vec{B}\right| = 4^2 + 3^2 + (-1)^2 = \sqrt{16 + 9 + 1} = \sqrt{26}$$

$$\left|\vec{B}\right| = \sqrt{26}$$

$$sin\theta = \frac{|\vec{A}\times\vec{B}|}{|\vec{A}||\vec{B}|} = \frac{35}{7\times\sqrt{26}}$$

$$sin\theta = \frac{5}{\sqrt{26}}$$

$$\theta = sin^{-1}\frac{5}{\sqrt{26}}$$

$$\theta = 78.69'$$

Formula used for three dimension force analysis

Force vector $\vec{F} = \lambda \times F$

$$\lambda = \frac{\overrightarrow{OA}}{|\overrightarrow{OA}|}$$

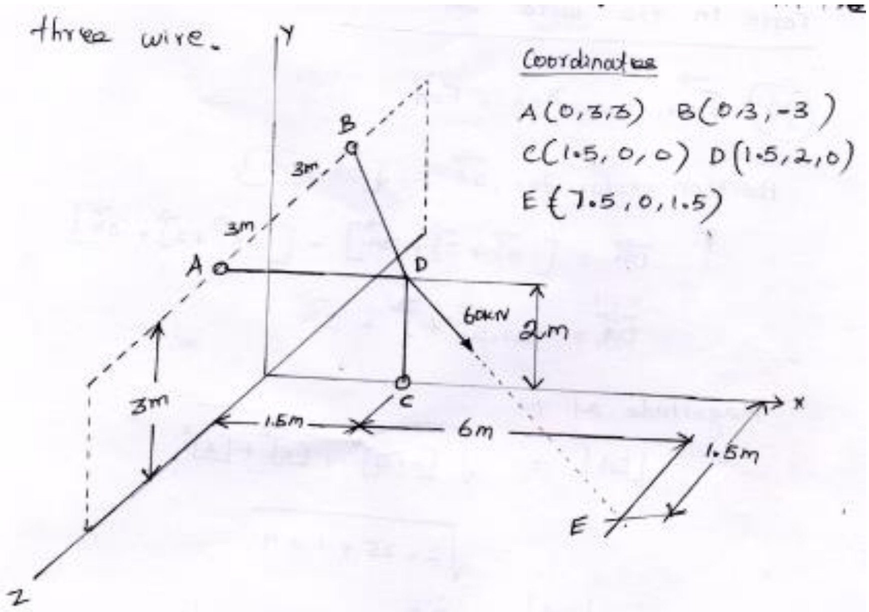
Magnitude $|OA| = \sqrt{(x)^2 + (y)^2 + (z)^2}$

$$\vec{R} = \overrightarrow{F_A} + \overrightarrow{F_B} + \overrightarrow{F_C}$$

$$R = \sqrt{{F_x}^2 + {F_y}^2 + {F_z}^2}$$

$$\theta = cos^{-1}\left(\frac{Rx}{R}\right)$$

1. In the figures shown, three wire jointed at D. The Two ends A and B are on the wall and the other end C is on the ground. The wire CD is vertical. A force of 60 KN is applied at 'D' and it passes through a point E on the ground as shown in fig. Find the forces in all the three wire.



Given:

$$F_{DE} = 60KN$$

To find:

$$F_{DA} = ?\ F_{DB} = ?\ F_{DC} = ?$$

Soln:

$$OA = 0\vec{\imath} + 3\vec{\jmath} + 3\vec{K}$$

$$OB = 0\vec{\imath} + 3\vec{\jmath} - 3\vec{k}$$

$$OC = 1.5\vec{\imath} + 0\vec{\jmath} + 0\vec{k}$$

$$OD = 1.5\vec{\imath} + 2\vec{\jmath} + 0\vec{k}$$

$$OE = 7.5\vec{\imath} + 0\vec{\jmath} + 1.5\vec{k}$$

Force in the wire DA 'F_{DE}'

$\overrightarrow{F_{DA}} = \lambda_{DA} \times F_{DA}$

Position vector for $\overrightarrow{DA} = [\overrightarrow{OA} - \overrightarrow{OD}]$

$\overrightarrow{DA} = [0\vec{i} + 3\vec{j} + 3\vec{k}] - [1.5\vec{i} + 2\vec{j} + 0\vec{k}]$

$\overrightarrow{DA} = -1.5\vec{i} + \vec{j} + 3\vec{k}$

Magnitude of DA

$|DA| = \sqrt{(-1.5)^2 + (1)^2 + (3)^2}$

$= \sqrt{2.25 + 1 + 9}$

$|DA| = 3.5$

$\lambda_{DA} = \frac{\overrightarrow{DA}}{|DA|} = \frac{-1.5\vec{i}+\vec{j}+3\vec{k}}{3.5}$

$\lambda_{DA} = -0.428\vec{i} + 0.285\vec{j} + 0.857\vec{k}$

$\overrightarrow{F_{DA}} = \lambda_{DA} \times F_{DA}$

$$\overrightarrow{F_{DA}} = -0.428\vec{i}F_{DA} + 0.285\vec{j}F_{DA} + 0.857\vec{k}F_{DA}$$

$\overrightarrow{F_{DA}} = -0.428\vec{i}F_{DA} + 0.285\vec{j}F_{DA} + 0.857\vec{k}F_{DA}$------- (1)

Force in the wire DB

Force from D to B coordinates

$\overrightarrow{F_{DB}} = \lambda_{DB} \times F_{DB}$

$$\lambda_{DB} = \frac{\overrightarrow{DB}}{|DB|}$$

$$\overrightarrow{DB} = [\overrightarrow{OB} - \overrightarrow{OD}]$$

$= [0\vec{i} + 3\vec{j} - 3\vec{k}] - [1.5\vec{i} + 2\vec{j} + 0\vec{k}]$

$\overrightarrow{DB} = -1.5\vec{i} + \vec{j} - 3\vec{k}$

$$|\overrightarrow{DB}| = \sqrt{(1.5)^2 + (1)^2 + (-3)^2}$$

$$\left|\overrightarrow{DB}\right| = 3.5$$

$$\lambda_{DB} = \frac{DB}{|DB|} = \frac{-1.5\vec{\imath} + \vec{\jmath} - 3\vec{k}}{3.5}$$

$$\lambda_{DB} = 0.428\vec{\imath} + 0.285\vec{\jmath} - 0.857\vec{k}$$

$$\overrightarrow{F_{DB}} = \lambda_{DB} \times F_{DB}$$

$F_{DB} = -0.428F_{DB}\vec{\imath} + 0.285F_{DB}\vec{\jmath} - 0.857F_{DB}\vec{k}$-------------- (2)

Force in the wire DC

Force from D to C coordinate

$$\overrightarrow{F_{DC}} = \lambda_{DC}.F_{DC}$$

$$\lambda_{DB} = \frac{\overrightarrow{DC}}{|\overrightarrow{DC}|}$$

$$\overrightarrow{DC} = [\overrightarrow{OC} - \overrightarrow{OD}]$$

$$= \left[1.5\vec{\imath} + 0\vec{\jmath} - 0\vec{k}\right] - [1.5\vec{\imath} + 2\vec{\jmath} + 0\vec{k}]$$

$$\overrightarrow{DC} = 0\vec{\imath} - 2\vec{\jmath} + 0\vec{k}$$

$$\left|\overrightarrow{DC}\right| = \sqrt{(0)^2 + (-2)^2 + (0)^2} = \sqrt{4}$$

$$\left|\overrightarrow{DC}\right| = 2$$

$$\lambda_{DC} = \frac{\overrightarrow{DB}}{|\overrightarrow{DB}|} = \frac{o\vec{\imath} - 2\vec{\jmath} + 0\vec{k}}{2}$$

$$\lambda_{DC} = \overrightarrow{-\jmath}$$

$$\overrightarrow{F_{DC}} = \lambda_{DC} \times F_{DC}$$

$\overrightarrow{F_{DC}} = -F_{DC}\vec{\jmath}$------------------ (3)

Force in the wire DE

Force from D to E

$\overrightarrow{F_{DE}} = \lambda_{DE}.F_{DE}$

$$\lambda_{DE} = \frac{\overrightarrow{DE}}{|\overrightarrow{DE}|}$$

$$\overrightarrow{DE} = [\overrightarrow{OE} - \overrightarrow{OD}]$$

$= [7.5\vec{\imath} + 0\vec{\jmath} - 1.5\vec{k}] - [1.5\vec{\imath} + 2\vec{\jmath} + 0\vec{k}]$

$\overrightarrow{DE} = 6\vec{\imath} - 2\vec{\jmath} + 1.5\vec{k}$

$$|\overrightarrow{DE}| = \sqrt{(6)^2 + (-2)^2 + (1.5)^2} = \sqrt{36 + 4 + 2.25}$$

$|\overrightarrow{DE}| = 6.5$

$$\lambda_{DE} = 0.923\vec{\imath} + 0.307\vec{\jmath} + 0.23\vec{k}$$

$\overrightarrow{F_{DE}} = \lambda_{DE} \times F_{DE}$

$F_{DE} = -0.923\vec{\imath} \times F_{DE} - 0.307\vec{\jmath} \times F_{DE} + 0.23\vec{k} \times F_{DE}$

$F_{DE} = -0.923F_{DE}\vec{\imath} - 0.307F_{DE}\vec{\jmath} + 0.23F_{DE}\vec{k}$

$F_{DE} = 60KN$

$$F_{DE} = -0.923 \times 60\vec{\imath} - 0.307 \times 60\vec{\jmath} + 0.23 \times 60\vec{k}$$

$\overrightarrow{F_{DE}} = 55.38\vec{\imath} - 18.42\vec{\jmath} + 13.84\vec{k}$------------ (4)

$F_{DA} = -0.428\vec{\imath}F_{DA} + 0.285\vec{\jmath}F_{DA} + 0.857\vec{k}F_{DA}$

$F_{DB} = -0.428F_{DB}\vec{\imath} + 0.285F_{DB}\vec{\jmath} - 0.857F_{DB}\vec{k}$

$F_{DC} = -F_{DC}\vec{\jmath}$

$F_{DE} = 55.38\vec{\imath} - 18.42\vec{\jmath} + 13.84\vec{k}$

Applying equilibrium condition

$\sum F_x = 0, F_{DA}.F_{DB}.F_{DC}.F_{DE} = 0$

$-0.428F_{DA} - 0.428F_{DB} + 55.38 = 0$

$$-0.428[F_{DA} + F_{DB}] + 55.38 = 0$$

$F_{DA} + F_{DB} = -55.38$

$F_{DA} + F_{DB} = \frac{-55.38}{-0.428}$

$F_{DA} + F_{DB} = 129.39$-------------- (5)

$\sum F_y = 0$

$-0.285F_{DA} + 0.285F_{DB} - F_{DC} - 18.42 = 0$

$-0.285F_{DA} + 0.285F_{DB} - F_{DC} = 18.42$

$\sum F_Z = 0$

$-0.857F_{DA} - 0.857F_{DB} + 18.42 = 0$

$0.857[F_{DA} - F_{DB}] + 13.84 = 0$

$F_{DA} - F_{DB} = \frac{-13.84}{0.857}$

$F_{DA} - F_{DB} = -16.15$------------- (7)

(5)⇒ $F_{DA} + F_{DB} = 129.39$

(7)⇒ $F_{DA} - F_{DB} = -16.15$

$2F_{DA} = 113.24$

$F_{DA} = \frac{113.24}{2}$

$F_{DA} = 56.62\ KN$

(5) ⇒$F_{DA} + F_{DB} = 129.39$

$56.62 + F_{DB} = 129.39$⇒$F_{DB} = 129.39 - 56.62$

$F_{DB} = 72.76\ KN$

(6) ⇒$-0.285 \times F_{DA} + 0.285F_{DB} - F_{DC} = 18.42$

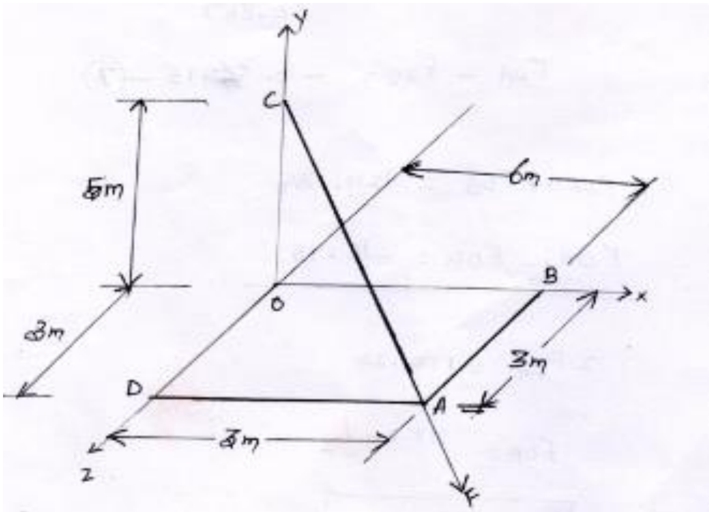
$$-0.285 \times 56.62 - 0.285 \times 72.76 - F_{DC} = 18.42$$

$$16.138 + 20.71 - F_{DA} = 18.42$$

$$F_{DA} = 16.138 + 20.71 - 18.42$$

$$F_{DA} = 18.428\ N$$

2. Fig shows three cables AB, AC,& AD that are used to support the end of a sign which exerts a force of $F = (250\vec{\imath} + 450\vec{\jmath} - 150\vec{k})N$ at A. Determine the force develop in each cable.



Given:

$F = (250\vec{\imath} + 450\vec{\jmath} - 150\vec{k})N$ at A

To find:

Force in AB, AC &AD

Soln:

$A = (3,0,3)$

$B = (6,0,0)$
$C = (0,5,0)$

$D = (0,0,3)$

$OA = 3\vec{\imath} + 0\vec{\jmath} + 3\vec{K}$

$OB = 6\vec{\imath} + 0\vec{\jmath} + 3\vec{k}$

$OC = 0\vec{\imath} + 5\vec{\jmath} + 0\vec{k}$

$OD = 0\vec{\imath} + 0\vec{\jmath} + 3\vec{k}$

Force of AB

$F_{AB} = \lambda_{AB} \times F_{AB}$

$\lambda_{AB} = \frac{AB}{|AB|}$

Position vector for AB

$$AB = OB - OA = [6\vec{\imath} + 0\vec{\jmath} + 0\vec{k}] - [3\vec{\imath} + 0\vec{\jmath} + 3\vec{k}]$$

$$|\overrightarrow{AB}| = 3\vec{\imath} + 0\vec{\jmath} - 3\vec{k}$$

$|\overrightarrow{AB}| = \sqrt{3^2 + 0^2 + [-3]^2} = \sqrt{9 + 0 + 9}$

$|\overrightarrow{AB}| = \sqrt{18}$

$|\overrightarrow{AB}| = 4.2$

$\lambda_{AB} = \frac{3\vec{\imath} + 0\vec{\jmath} - 3\vec{k}}{4.2}$

$\lambda_{AB} = 0.714\vec{\imath} + 0\vec{\jmath} - 0.714\vec{k}$

$\overrightarrow{F_{AB}} = \lambda_{AB} \times F_{AB}$

$\overrightarrow{F_{AB}} = 0.714F_{AB}\vec{\imath} + 0\vec{\jmath} - 0.714F_{AB}\vec{k}$-------------------- (1)

Force on AC

$$\overrightarrow{F_{AC}} = \lambda_{AC} \times F_{AC}$$

$$\lambda_{AC} = \frac{\overrightarrow{AC}}{\left|\overrightarrow{AC}\right|}$$

Position vector of $AC = OC - OA$

$$\left[0\vec{\imath} + 5\vec{\jmath} + 0\vec{k}\right] - [3\vec{\imath} + 0\vec{\jmath} + 3\vec{k}]$$

$$AC = -3\vec{\imath} + 5\vec{\jmath} - 3\vec{k}$$

$$\left|\overrightarrow{AC}\right| = \sqrt{(-3)^2 + (5)^2 + (-3)^2} = \sqrt{9 + 25 + 9} = \sqrt{43}$$

$$\left|\overrightarrow{AC}\right| = 6.5$$

$$\lambda_{AC} = \frac{\overrightarrow{AC}}{\left|\overrightarrow{AC}\right|} = \frac{-3\vec{\imath} + 5\vec{\jmath} - 3\vec{k}}{6.5}$$

$$\lambda_{AC} = -0.461\vec{\imath} + 0.769\vec{\jmath} - 0.461\vec{k}$$

$$\overrightarrow{F_{AC}} = \lambda_{AC} \times F_{AC}$$

$\overrightarrow{F_{AC}} = -0.461F_{AC}\vec{\imath} + 0.769F_{AC}\vec{\jmath} - 0.461F_{AC}\vec{k}$ ------------ (2)

Force on AD

$$\overrightarrow{F_{AD}} = \lambda_{AD} \times F_{AD}$$

$$\overrightarrow{AD} = \overrightarrow{OA} - \overrightarrow{OA} = \left[0\vec{\imath} + 0\vec{\jmath} + 3\vec{k}\right] - [3\vec{\imath} + 0\vec{\jmath} + 3\vec{k}]$$

$$\overrightarrow{AD} = -3\vec{\imath} + 0\vec{\jmath} + 0\vec{k}$$

Magnitude of AD

$$\left|\overrightarrow{AD}\right| = \sqrt{(-3)^2} = \sqrt{9}$$

$$\left|\overrightarrow{AD}\right| = 3$$

$$\lambda_{AD} = \frac{\overrightarrow{AD}}{|\overrightarrow{AD}|} = \frac{-3\vec{i} + 0\vec{j} + 0\vec{k}}{3}$$

$\lambda_{AD} = \overrightarrow{-i}$

$\overrightarrow{F_{AD}} = -F_{AD}\vec{i}$--------------- (3)

$F = 250\vec{i} + 450\vec{j} - 150\vec{k}$ [$is\ given$]

Applying the equilibrium Eqn

$\sum F_x = 0$

$0.714F_{AB} - 0.461F_{AC} - F_{AD} + 250 = 0$

$0.714F_{AB} - 0.461F_{AC} - F_{AD} = -250$-------------- (4)

$\sum F_y = 0$

$0F_{AB} + 0.769F_{AC} + 450 = 0$---------------- (5)

$0.769F_{AC} = -450$

$F_{AC} = \frac{-450}{0.769}$

$F_{AC} = -585.17\ N$

$\sum F_z = 0$

$-0.714F_{AB} - 0.461\ F_{AC} - 150 = 0$---------------- (6)

$-0.714F_{AB} - 0.461\ \times (-585.17) - 150 = 0$

$-0.714F_{AB} + 269.76 - 150 = 0$

$-0.714F_{AB} + 119.76 = 0$

$F_{AB} = \frac{-119.76}{-0.714}$

$F_{AB} = 167.73N$

(4)$\Rightarrow$ $0.714F_{AB} - 0.461F_{AC} - F_{AD} = -250$

$$0.714 \times 167.73 - 0.461 \times -585.17 - F_{AD} = -250$$

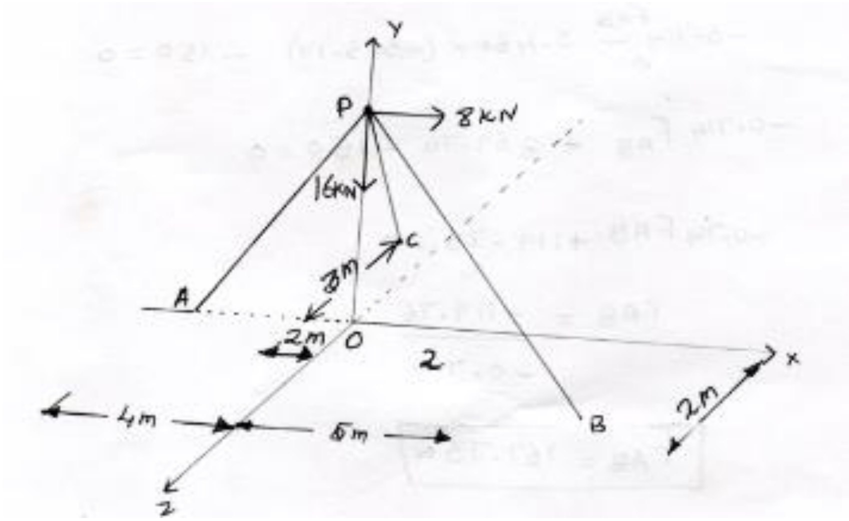
$$119.76 + 269.76 - F_{AD} = -250$$

$$389.52 - F_{AD} = -250$$

$$-F_{AD} = -250 - 389.52$$

$$-F_{AD} = -639.52 F_{AD} = 639.52\ N$$

3. Two force act upon the tripod at point P as shown in fig. The force 8 KN is parallel to X axis & the force 16 KN is parallel to Y axis. Determine the force acting at the legs of tripod if the rest on legs on ground at A, B, &C whose coordinates with respect to O are given the height of the P above the origin is 10m.



Given:

8 KN at point 'P' in horizontal

16 KN at point 'P' in vertical

Height of point P=10m from 0

To Find:

F_{PA}, F_{PB}, F_{PC}

Soln:

Coordinates

$A = (-4,0,0), B = (5,0,2), C = (-2,0,-3), P(0,10,0)$

$OA = -4\vec{i} + 0\vec{j} + 0\vec{k}, OB = 5\vec{i} + 0\vec{j} + 2\vec{k}, OC = -2\vec{i} + 0\vec{j} - 3\vec{k}$

$OP = 0\vec{\imath} + 10\vec{\jmath} + 0\vec{k}$

Force on F_{PA}

$$\overrightarrow{F_{PA}} = \lambda_{PA} \times F_{PA} \lambda_{PA} = \frac{\vec{A}}{|\overrightarrow{PA}|}$$

$$\left|\overrightarrow{PA}\right| = \overrightarrow{OA} - \overrightarrow{OP}$$

$$= -4\vec{\imath} - [10\vec{\jmath}]$$

$$\left|\overrightarrow{PA}\right| = -4\vec{\imath} - 10\vec{\jmath}$$

$$\left|\overrightarrow{PA}\right| = \sqrt{(-4)^2 + (-10)^2} = \sqrt{16 + 100} = \sqrt{116}$$

$$\left|\overrightarrow{PA}\right| = 10.77$$

$$\lambda_{PA} = \frac{\overrightarrow{PA}}{|\overrightarrow{PA}|} = \frac{-4\vec{\imath} - 10\vec{\jmath}}{10.77}$$

$$\lambda_{PA} = -0.371\vec{\imath} - 0.928\vec{\jmath}$$

$$\overrightarrow{F_{PA}} = \lambda_{PA} \times F_{PA} = -0.371\vec{\imath} \times F_{PA} - 0.928\vec{\jmath} \times F_{PA}$$

$$\overrightarrow{F_{PA}} = -0.371 F_{PA}\vec{\imath} - 0.928 F_{PA}\vec{\jmath}$$ ------------ (1)

Force of PB

$$\overrightarrow{F_{PB}} = \lambda_{PB} \times F_{PB} \lambda_{PB} = \frac{\overrightarrow{PB}}{|\overrightarrow{PB}|}$$

$$\overrightarrow{PB} = \overrightarrow{OB} - \overrightarrow{OP}$$

$$= \left[5\vec{\imath} + 2\vec{k}\right] - [10\vec{\jmath}]$$

$$\overrightarrow{PB} = 5\vec{\imath} - 10\vec{\jmath} + 2\vec{k}$$

$$\left|\overrightarrow{PB}\right| = 11.35$$

$$\lambda_{PB} = \frac{\overrightarrow{PB}}{|\overrightarrow{PB}|} = \frac{5\vec{\imath} - 10\vec{\jmath} + 2\vec{k}}{11.35}$$

$$\lambda_{PB} = 0.44\vec{\imath} - 0.88\vec{\jmath} + 0.176\vec{k}$$

$$\overrightarrow{F_{PB}} = \lambda_{PB} \times F_{PB}$$

$$\overrightarrow{F_{PB}} = 0.44F_{PB}\vec{\imath} - 0.88F_{PB}\vec{\jmath} + 0.176F_{PB}\vec{k}$$-------- (2)

Force on PC

$$\overrightarrow{F_{PC}} = \lambda_{PC} \times F_{PC}\lambda_{PC} = \frac{\overrightarrow{PC}}{|\overrightarrow{PC}|}$$

$$\overrightarrow{PC} = \overrightarrow{OC} - \overrightarrow{OP}$$

$$= [2\vec{\imath} - 3\vec{k}] - 10\vec{\jmath}$$

$$\overrightarrow{PC} = -2\vec{\imath} - 10\vec{\jmath} - 3\vec{k}$$

$$|\overrightarrow{PC}| = \sqrt{(-2)^2 + (-10)^2 + (-3)^2} = \sqrt{4 + 100 + 9} = \sqrt{113}$$

$$|\overrightarrow{PC}| = 10.63$$

$$\lambda_{PC} = \frac{\overrightarrow{PC}}{|\overrightarrow{PC}|} = \frac{-2\vec{\imath} - 10\vec{\jmath} - 3\vec{k}}{10.63}$$

$$\lambda_{PC} = -0.188\vec{\imath} - 0.94\vec{\jmath} - 0.282\vec{k}$$

$$\overrightarrow{F_{PC}} = \lambda_{PC} \times F_{PC}$$

$$\overrightarrow{F_{PC}} = -0.188F_{PC}\vec{\imath} - 0.94F_{PC}\vec{\jmath} - 0.282\vec{k}$$

$$P = 0\vec{\imath} + 10\vec{\jmath} + 0\vec{k}$$------------ (4)

Apply Equilibrium condition

$$\sum F_x = 0$$

$$-0.371F_{PA} + 0.44F_{PB} - 0.188F_{PC} = 0$$------------- (5)

$$\sum F_y = 0$$

$$-0.928F_{PA} - 0.88F_{PB} - 0.94F_{PC} + 10 = 0$$

$$-0.928F_{PA} - 0.88F_{PB} - 0.94F_{PC} = -10$$

$$0.928F_{PA} - 0.88F_{PB} - 0.94F_{PC} = 10$$ -------- (6)

$$\sum F_Z = 0$$

$$-0.282F_{PC} + 0.178F_{PB} = 0$$

$0.178F_{PB} - 0.282F_{PC} = 0$--------- (7)

Solve Eqn(5)&(6)

(5)× 0.928 $\quad - 0.344\, F_{PA} + 0.4F_{PB} - 0.174F_{PC} = 0$

(6) × 0.371 $\quad 0.344F_{PA} + 0.326F_{PB} + 0.348F_{PC} = 3.71$

$0.726F_{PB} - 0.174F_{PC} = 3.71$--------- (8)

Solve Eqn (7) &(8)

(7)⇒0.726⇒ $\quad 0.127F_{PB} - 0.2F_{PC} = 0$

(8)⇒0.176⇒ $\quad 0.127F_{PB} + 0.03F_{PC} = 0.652$

$-0.23F_{PC} = -0.652$

$F_{PC} = \frac{-0.652}{-0.23}$

Eqn(7) becomes $0.176F_{PB}$-$0.282F_{PC} = 0$

$0.176 \times F_{PB} - 0.282 \times 2.834 = 0$

$F_{PB} = \frac{0.282 \times 2.834}{0.176}$

$F_{PB} = 4.539N$

Eqn (5) becomes

$-0.371F_{PA} + 0.44F_{PB} - 0.188F_{PC} = 0$

$-0.371F_{PA} + 0.44 \times 4.539 - 0.188 \times 2.834 = 0$

$-0.371 \times F_{PA} + 1.997 - 0.532 = 0$

$-0.371 \times F_{PA} + 1.465 = 0$

$F_{PA} = \frac{-1.465}{-0.371}$

$F_{PA} = 3.94N$